Name: _____ Name-code: _____

University of Michigan Physics Department
Graduate Qualifying Examination

Part I - Classical Physics
Saturday, May 6, 2006 9:00am-1:00pm

Please write your name-code on each page of the exam.

This is a closed book exam – but you may use the "Constants, Conversions, and Formulas" sheet we provide for this exam.

Show your work in an organized manner to receive partial for it. If you need to make an assumption or an estimate, indicate it clearly.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions.

Please do not write on this page. The hatched column to the right and the total score will be filled in by the graders.

Good Luck!

Total Score:

1	
2	
3	
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12	

"Constants, Conversions, and Formulas"

University of Michigan Physics Department Graduate Qualifying Examination

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$
\begin{aligned}
\text{speed of light} \quad c &= 2.998 \times 10^{8}\ \mathrm{m/s} \\
\text{electron charge} \quad e &= 1.602 \times 10^{-19}\ \mathrm{C} \\
\text{Planck's constant} \quad h &= 6.626 \times 10^{-34}\ \mathrm{J \cdot s} = 4.136 \times 10^{-15}\ \mathrm{eV \cdot c} \\
\hbar = h/2\pi &= 1.055 \times 10^{-34}\ \ \mathrm{J \cdot s} = 0.658 \times 10^{-15}\ \mathrm{eV \cdot s} \\
\text{Rydberg constant} \quad R_\infty &= 1.097 \times 10^{6}\ \mathrm{m}^{-1} \\
\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} &= 8.988 \times 10^{9}\ \mathrm{N \cdot m^2/C^2} \\
\text{Universal gas constant} \quad R &= 8.3\mathrm{J/K \cdot mol} \\
\text{Avogadro's number} \quad N_A &= 6 \times 10^{23}\ \mathrm{mol}^{-1} \\
\text{Boltzmann's constant} \quad k_B = R/N_A &= 1.38 \times 10^{-23} J/K = 8.617 \times 10^{-5}\mathrm{eV/K} \\
\text{Stefan} - \text{Boltzmann constant} \quad \sigma &= 5.6703 \times 10^{-8}\ \mathrm{W/m^2K^4} \\
\text{radius of the sun} \quad R_{sun} &= 6.96 \times 10^{8}\ \mathrm{m} \\
\text{radius of the moon} \quad R_{moon} &= 1.74 \times 10^{6}\ \mathrm{m} \\
\text{radius of the earth} \quad R_{earth} &= 6.37 \times 10^{6}\ \mathrm{m} \\
G_N &= 6.67 \times 10^{-11} m^3/kg/s^2 = 6.71 \times 10^{-39} GeV^{-2}
\end{aligned}
$$

SOME USEFUL CONVERSIONS AND COMBINATIONS

$$
\begin{aligned}
\text{fine structure constant} \quad \alpha &= ke^2/\hbar c = 1/137 \\
\text{Bohr magneton} \quad e\hbar/2m_e &= 9.27 \times 10^{-24}\mathrm{J/T} = 5.79 \times 10^{-5}\mathrm{eV/T} \\
hc &= 19.865 \times 10^{-26}\ \mathrm{J \cdot m} = 12.41 \times 10^{3}\ \mathrm{eV \cdot \AA} = 1241\ \mathrm{MeV \cdot fm} \\
\hbar c &= 3.165 \times 10^{-26}\ \mathrm{J \cdot m} = 1973\ \mathrm{eV \cdot \AA} = 197.3\ \mathrm{MeV \cdot fm} \\
ke^2 &= 1.44\ \mathrm{MeV \cdot fm} \\
& 1\mathrm{\AA} = 10^{-10}\ \mathrm{m} = 10^{5}\ \mathrm{fm} \qquad 1\mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}
\end{aligned}
$$

SOME USEFUL RELATIONS

$$\frac{C_V}{Nk} = 9\,(T/\theta_D)^3 \int_0^{\theta_D/T} \frac{x^4 e^4 dx}{(e^x - 1)^2} \quad \text{(Debye formula)}$$

$$= 3\left[1 - \frac{1}{20}\left(\frac{\theta_D}{T}\right)^2 + \ldots\right] = \frac{12\pi^4}{5}\left(\frac{T}{\theta_D}\right)^3 (1 + \ldots)$$

$$\frac{U}{N} = \frac{3}{5}E_F\left[1 + \frac{5\pi^2}{12}\left(\frac{kT}{E_F}\right)^2 - \frac{\pi^4}{16}\left(\frac{kT}{E_F}\right)^4 + \ldots\right] \quad \text{(degenerate electron gas)}$$

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \quad \text{(nonrelativistic Fermi energy)}$$

$$r = r_\circ A^{1/3}, \qquad r_\circ = 1.2 \times 10^{-15} m \quad \text{(approximate average nuclear radius)}$$

$$E_b = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A} \quad \text{(semiempirical binding energy of a nucleus)}$$

$$\frac{n}{s} + \frac{n\prime}{s\prime} = \frac{n\prime - n}{R} \text{(refraction of paraxial rays)}$$

MASSES OF SOME ELEMENTARY PARTICLES

	Rest Mass, m_0 (kg)	m_0c^2 (MeV)
Electron	9.109×10^{-31}	0.511
Proton	1.673×10^{-27}	938.3
Neutron	1.675×10^{-27}	939.6
Atomic mass unit (1 amu)	1.661×10^{-27}	931.5

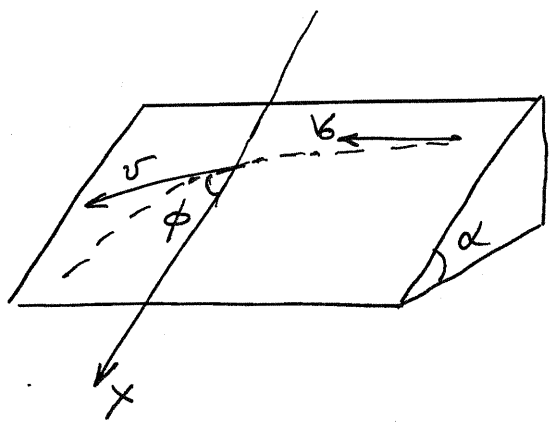
VISIBLE LIGHT SPECTRUM

300	400	500		600		700(Nanometer)	
← Ultraviolet	Violet	Blue	Green	Yellow	Orange	Red	Infrared →

Name-code: _____

Part A: Obligatory Problems

1. On an inclined plane with angle α with the horizontal a small body was given an initial velocity v_0. Find the relation between the speed of the body and the angle ϕ (at the initial moment, $\phi_0 = \pi/2$) if the friction coefficient is $\mu = \tan\alpha$.



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2. To the ends of a massless thread that goes around a pulley one fixes two masses m_1 and m_2. There is friction between the thread and the pulley such that the thread starts sliding over the pulley when the ratio between the masses is $m_2/m_1 = \eta_0$.
(i) Find the friction coefficient μ.
(ii) Find the acceleration of the masses if $m_2/m_1 = \eta > \eta_0$.

Name-code: _____

3. Consider a spherical density distribution of the form

$$\rho(r) = \rho_0 \frac{1}{(r/r_0)(1 + r/r_0)^3},$$

where ρ_0 is a (constant) density scale and r_0 is a (constant) length scale. Find the mass profile $M(r)$ and the gravitational potential $\Psi(r)$ for this extended mass distribution. In particular find the total mass M_∞ and total depth Ψ_0 of the gravitational potential well:

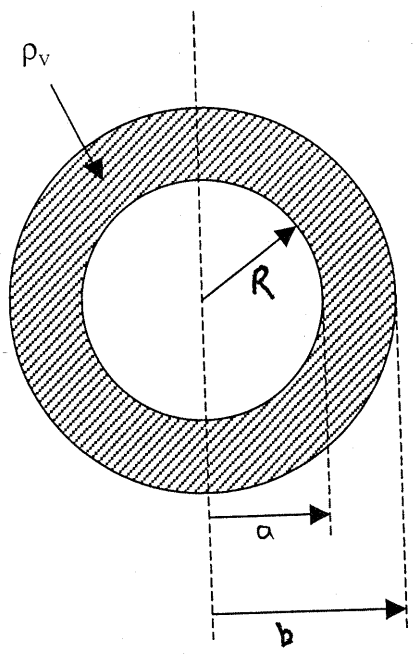
$$M_\infty = \lim_{r\to\infty} M(r) \quad \text{and} \quad \Psi(0) = \lim_{r\to 0} \Psi(r).$$

Name-code: _____

4. A long cylindrical conducting rod (non-magnetic material) of radius R is rotating with an angular velocity $\vec{\omega}$ along a uniform and constant magnetic field $\vec{B}_0$ (parallel to the cylindrical axis). Neglecting any transient effect and assuming $\omega R \ll c$,
(a) determine the electric field inside and outside the rod.
(b) describe the charge distribution on the rod and find relevant charge densities.
(c) find the induced emf.

Name-code: _____

5. A hollow, non-conducting sphere centered on the origin carries a uniform charge per unit volume of ρ_v in the region $a < R < b$ between its inner and outer wall radii a and b respectively. Determine the electric field E as a function of radial distance R at an arbitrary point
(a) inside the inner radius of the sphere ($R < a$),
(b) at positions inside the thick wall ($a < R < b$),
(c) and outside the sphere ($R > b$).



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6. An optical diffraction grating has many grooves on it separated by a constant distance h. The Frauenhofer diffraction pattern consists of bright fringes of different orders m, $m+1$, $m+2$, ... that emerge at different angles with respect to the grating normal. Let the angle of the mth-order bright fringe from the grating normal be θ.
(a) Derive the "grating" equation that relates the optical wavelength λ to m, h, and θ at intensity maxima.
(b) What is the angular separation $\Delta\theta$ of the intensity maxima for two different wavelengths λ_1 and λ_2 observed in the same order m?

Name-code: _____

7. The equation of state for a van der Waals gas can be written in the form

$$\left(p + \frac{A}{V^2}\right)(V - B) = NkT,$$

where p is the pressure, V is the volume, N is the number of particles, k is the Boltzmann constant, T is the temperature, and where A, B are constants that determine the departure of the equation of state from the ideal gas law. Assume that the atmosphere of Earth is replaced by a gas with this equation of state. In the limit of an isothermal atmosphere ($T \to$ constant) and no short range forces ($B \to 0$), find the differential equation that describes the variation of the density $\rho = \mu N/V$ with elevation z. First solve the equation to find an implicit solution for the density of the atmosphere and then evaluate the implicit solution in the limit of large heights ($z \to \infty$).

Name-code: _____

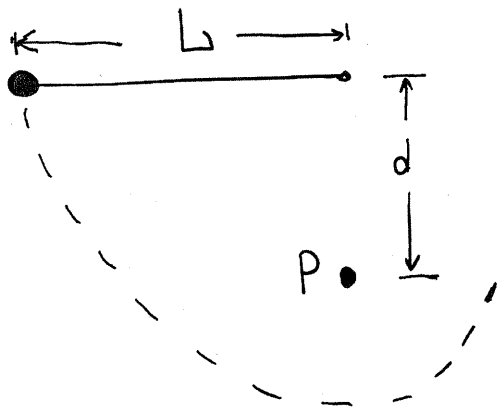
8. Consider a refrigerator operating in a cyclical manner. Each cycle takes heat Q_L from a reservoir kept at low temperature T_L and deposits heat Q_H in a reservoir kept at high temperature T_H. The amount of work done by an external engine to cause these heat transfers in one cycle is W. Let $\omega = \frac{Q_L}{W}$ so that ω is a measure of efficiency of the refrigerator.

   Express the maximum possible efficiency in terms of T_L and T_H.

Name-code: _____

Part B: Optional Problems

9. Consider a string with length L. There is a fixed peg P at a distance d from the origin of the string. When the initially stationary ball is released with the string horizontal it will swing along the dashed trajectory.
(i) What is the speed when it reaches its lowest point?
(ii) What is the speed when it reaches its highest point after the string catches on the peg?
(iii) What is the minimal value of d that allows the ball to swing completely around the fixed peg?



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10. A very long cylinder of radius R extends along the z-axis and carries a constant electric polarization $\vec{P}$ pointing in the $+\hat{\mathbf{z}}$-direction. There are no free charges. Calculate the magnitude of the electric field at locations along the axis of the cylinder at a distance d from the top surface ($d \ll$ length). Provide results for the cases of both inside and outside the cylinder. Sketch the direction of the field vector in a drawing.

    You may test your result by considering the limits $d \to 0$ and $d \to \infty$.
    Note: $\int \frac{x}{(a^2+x^2)^{3/2}} dx = -\frac{1}{(a^2+x^2)^{1/2}}$.

Name-code: _____

11. An "axicon" lens is constructed by cutting the tip of a smooth glass cone off, perpindicular to the cone axis along z. An optical element with two flat, circular faces is thereby formed (Fig. 1 below). The diameter of the cone base is b and the diameter of the face near the former tip of the cone is a ($a < b$). Face a is opaque due to black paint applied as shown in Fig. 2. A parallel beam of light impinges on the b face from the left, travelling parallel to the z-axis, and rays farther off-axis than $a/2$ and less than $b/2$ refract at the second interface, where the index changes from $n_{glass} = 1.5$ to $n_{air} = 1.0$. If the cone angle is $\alpha = 120°$, and the dimensions are $a = 2\,\text{cm}$, $b = 6\,\text{cm}$ and $h = 2/\sqrt{3} = 1.555$,
(a) find the angle β, and
(b) find the length f of the region where light is concentrated in a "line focus" after passing through the axicon.

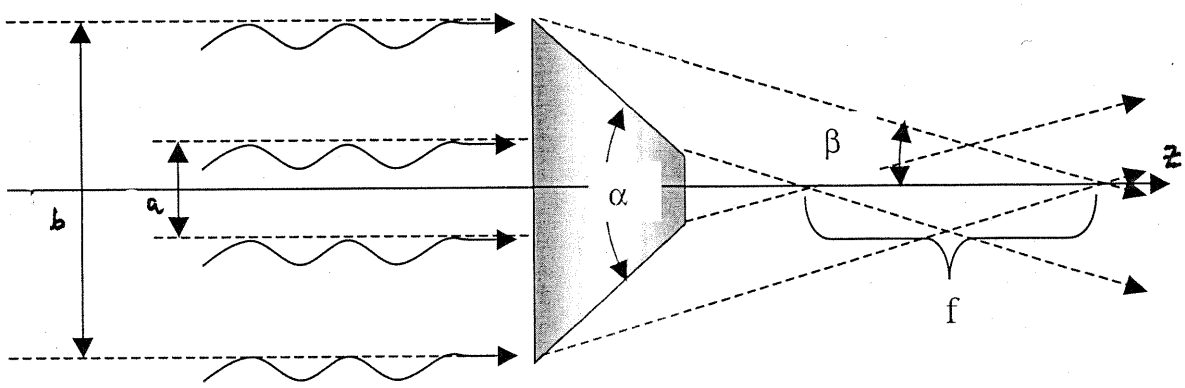

Fig. (a). Side view of the axicon.

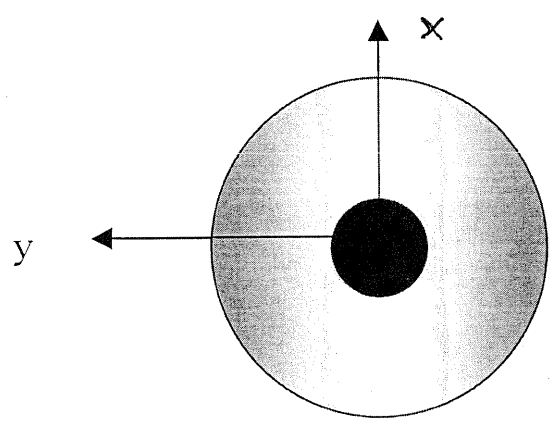

Fig. (b). View of the axicon down the z-axis.

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Space to work out problem 11:

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12. Consider a satellite orbiting a planet of mass M. Assume that the satellite has a bound orbit, with negative specific energy E. Define $\epsilon = |E| \geq 0$ and assume that the orbit has specific angular momentum j. (a) Find the turning points of the orbit in terms of M, ϵ, and j. (b) Find the maximum allowed value of specific angular momentum j for a given specific energy ϵ. (c) Find the value of the angular momentum for which the outer turning point is twice the inner turning point.