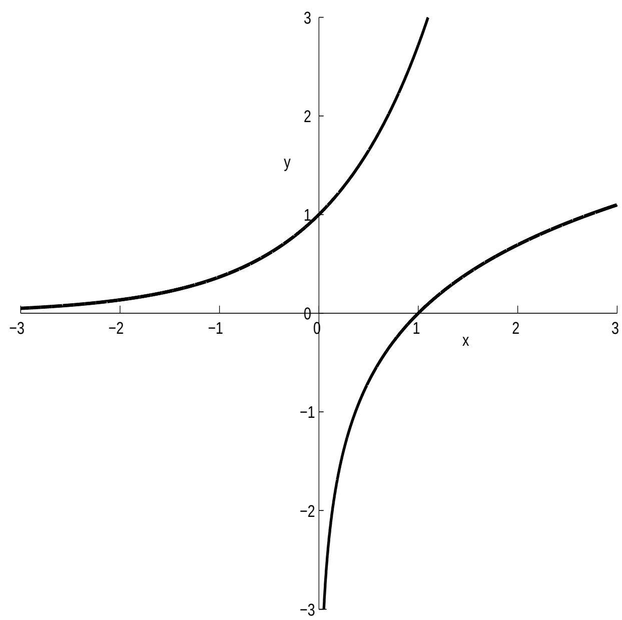
Department of Mathematical Sciences
Rensselaer Polytechnic Institute

Ready... Set...

CALCULUS



May 8, 2013

Ready... Set... Calculus

This document was prepared by the faculty of the Department of Mathematical Sciences at Rensselaer Polytechnic Institute in the interest of enriching the Freshman Calculus experience.

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Preface

Experience shows that calculus students who begin their study with substantial “initial” mathematical skills are better able to gain a mastery of the subject. Because Rensselaer attracts students with a spectrum of mathematical experiences, the Department of Mathematical Sciences has compiled a list of questions dealing with “initial” skills. These questions can be used by entering students to self-assess individual strengths.

Rensselaer does not set aside substantial (calculus) course time for “initial” skill development. A calculus course may begin with a brief review of a few of the topics covered in the problems that follow, such as natural logarithms, exponential functions and inverse trigonometric functions. Due to time constraints, the syllabus does not allocate time for starting-from-scratch skill development.

It is believed that almost all entering students will find at least one topic (perhaps more) in which skills can be enhanced. While some skills will be sharpened while the calculus course is in progress, students may benefit from improving their “initial” skills in advance. It is believed that there is not one strategy suitable for all entering students.

The problem list that follows consists entirely of mathematical constructs that are believed to have been addressed in a good high school mathematics program. However, Rensselaer Mathematics faculty are aware that high school mathematics programs are not uniformly focused. Some entering students will be more thoroughly grounded in “initial” skills than others.

Students who anticipate a rich calculus experience are encouraged to have a look at the problem list and make their own self-assessments.

Independent of calculus preparation, this list can provide a challenge for the human intellect.

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Contents

1 RULES OF THE GAME **1**
- 1.1 Welcome . . . 1
- 1.2 Pencil and Paper . . . 1
- 1.3 Self-Assessment . . . 1
- 1.4 A Pep Talk . . . 2
- 1.5 Notation/Vocabulary . . . 2

2 ARITHMETIC/ALGEBRAIC OPERATIONS **5**
- 2.1 Hand Calculations without Calculators . . . 5
- 2.2 True/False . . . 8

3 INEQUALITIES **11**
- 3.1 Hand Calculations without Calculators . . . 11
- 3.2 True/False . . . 15

4 TRIGONOMETRIC FUNCTIONS **19**
- 4.1 Hand Calculations without Calculators . . . 19
- 4.2 True/False . . . 22

5 LOGARITHMS/EXPONENTIALS **27**
- 5.1 Hand Calculations without Calculators . . . 27
- 5.2 True/False . . . 30

6 GRAPH RECOGNITION **33**

7 Examples and Help **39**
- 7.1 Introduction . . . 39
- 7.2 Arithmetic/Algebraic Operations . . . 40
- 7.3 Inequalities . . . 42
- 7.4 Trigonometric Functions . . . 44

7.5 Logarithms/Exponentials 46
7.6 Graph Recognition 47

8 ANSWERS **53**

Chapter 1

RULES OF THE GAME

1.1 Welcome

The faculty of the Department of Mathematical Sciences welcomes the freshman class to Rensselaer Polytechnic Institute. We designed this book to help guide Science and Engineering majors in assessing and practicing their mathematical skills.

1.2 Pencil and Paper

The problems that follow can be solved without the use of hand-held-calculators.[1] They offer introductory calculus students a chance to self-assess their "pencil and paper" skills.

1.3 Self-Assessment

Students seeking to calibrate their "initial" mathematical skills, are invited to try their hands at the problem list without using a calculator, and then check their answers with those provided in Chapter 8. This is not meant as a formal test, but rather as a way for individual students to "self-assess" their own mathematical preparation for college-level Calculus. In making this self-assessment, students taking Calculus in the Fall should be aware that at the beginning of the course there is only a brief review of just a few of the topics covered in this problem set.

[1] At many universities, including Rensselaer, students are asked to write exams without the use of calculators or computers.

If you get stuck, Chapter 7 provides a few examples with solutions, references, and links to web sites where more detailed help can be found.

1.4 A Pep Talk

For most of us, pencil and paper computational skills tend to evaporate over time (even a few weeks). So, it may be that a mid-summer, first-time view of the problem list will be a "scary" event. But experience has shown that skills lost over time will resurface during a concentrated review process. A few hours of review, off and on, during the weeks before the first Calculus class will provide large dividends. Go for it!

1.5 Notation/Vocabulary

We realize that entering students may not be familiar with all the notation and phrasing of problems herein. We believe it is important to learn different ways of expressing mathematical ideas and will endeavor to address this during first semester Calculus.

- In what follows we use the symbol $\mathbb{R}$ to denote the real numbers. These are the numbers with which we are all familiar; e.g., the numbers $-5, \frac{6}{7}, 123.4, \pi, \sqrt{2}$ are all real numbers. The number $\sqrt{-1}$ is not a real number; it can be thought of as a complex number. We frequently use letters to denote real numbers and may indicate this with symbols such as "$x \in \mathbb{R}$" which is usually read "x is an element of $\mathbb{R}$" and is sometimes written out in words as "x is a real number" or "x denotes a real number."

- In some of the problems, a solution *as an irreducible quotient of two integers* is sought. This means a fraction whose numerator and denominator are both integers, and have no common factors. This translates to a representation of the form, e.g., $-\frac{17}{20}$, or $\frac{1}{2}$, but not of the form $\frac{2}{6}$ (which is reducible to $\frac{1}{3}$), or of the form $\frac{\pi}{2}$ (because π is not an integer).

- For two real numbers a and b, with $a < b$, we may use the notation (a, b) to denote the set of all real numbers x such that $a < x < b$. Similarly, we may use the notation $[a, b]$ to denote the set of all real numbers x such that $a \leq x \leq b$. We may write

(a, ∞) to denote the set of all real numbers x such that $a < x$, etc.

Chapter 2

ARITHMETIC/ALGEBRAIC OPERATIONS

2.1 Hand Calculations without Calculators

Sample Problem. Express the number $\frac{5}{6} - \frac{1}{3}$ as an irreducible quotient of two integers.

> The sum can be computed by noticing that $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6}$. However, in order to represent the sum in the requested form, one writes $\frac{3}{6} = \frac{1}{2}$, so that $\frac{1}{2}$ is the solution to the problem.

1. Express the value of $\frac{3}{\frac{9}{3}}$ as an integer.

2. Express the number $\frac{3}{4} + \left(-\frac{1}{2}\right)$ as an irreducible quotient of two integers.

3. Express the number
$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{1}{5}}$$
in simplest form (as an irreducible quotient of two integers).

4. Express the number $\frac{1}{2}+\frac{1}{3+\frac{3}{4}}$ in simplest form (as an irreducible quotient of two integers).

5. Express as an integer the value of
$$\frac{1}{\left(\frac{1}{2}-\frac{3}{5}\right)^2}.$$

6. Express as an integer the value of 2^3.

7. Express as an integer the value of 2^0.

$a =$
$b =$

8. By factoring, the function $f(x) = x^2 - 2x - 35$ can be written in the form $f(x) = (x-a)(x-b)$. Express a and b as integers.

9. Multiply the expressions $(z^2 + 2z + 1)$ and $(z - 3)$ and express the answer in the form $az^3 + bz^2 + cz + d$. Express $a,\ b,\ c,\ d$ as integers.

$a =$
$b =$
$c =$
$d =$

10. If $(x - a)^2 = x^2 - 34x + 289$, express as an integer the value of a.

$a =$

11. When $(z^5 - 4z^4 + 5z^3 - 2z^2 - 6z + 6)$ is divided by $(z - 1)$ the answer can be expressed in the form $az^4 + bz^3 + cz^2 + dz + e$. Find integer values for a, b, c, d, e. Hint: Recall long division of polynomials.

$a =$
$b =$
$c =$
$d =$
$e =$

12. By completing the square, the polynomial $x^2 - 6x + 4$ can be written in the form $(x - a)^2 + b$. Express a and b as integers.

$a =$
$b =$

13. Solve the equation $x^2 + \frac{5}{2}x = \frac{3}{2},\ x \in \mathbb{R}$, and write each of the solutions in the form of an integer or as an irreducible quotient of two integers.

14. The expression

$$\frac{x+1-\frac{1}{x+1}}{\frac{1}{x+1}},$$

such that $x \neq -1$, can be written in the form $ax^2 + bx$. Express a and b as integers.

$a =$
$b =$

2.2 True/False

Sample Problem. If x and y denote non-zero real numbers, then

$\frac{\frac{y}{x}}{\frac{x}{y}} = \left(\frac{y}{x}\right)^2$. Is this statement True or False?

This statement can be shown to be true by noticing that

$$\frac{\frac{y}{x}}{\frac{x}{y}} = \frac{y}{x}\frac{y}{x} = \frac{y^2}{x^2} = \left(\frac{y}{x}\right)^2.$$

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

______(1) If x and y are positive real numbers, then $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$.

______(2) If x is a positive real number, then $\frac{1}{\frac{1}{x}} = x$.

______(3) If x and y are positive real numbers, then $\frac{y}{\frac{y}{x}} = x$.

______(4) If x is a real number, then $\sqrt{x^2} = x$.

______(5) If a and b are positive real numbers, then $\sqrt{a^2 + b^2} = a + b$.

______(6) If a and b denote positive real numbers, then $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$.

______(7) If x and y denote non-zero real numbers, then $\frac{x^2y^3}{x^3y^2} = \left(\frac{x}{y}\right)^{-1}$.

______(8) If x and y denote positive real numbers, then $\left(x^3y^{-4}\right)^2 = x^5y^{-2}$.

______(9) Let a, b, and c denote real numbers such that $a \neq 0$. If $ax^2 + bx + c = 0$, then $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$.

______(10) If a, b, and c denote real numbers and $p(x) = ax^2 + bx + c,\ x \in \mathbb{R}$, then there exists a real number x_0 such that $p(x_0) = 0$.

Chapter 3

INEQUALITIES

3.1 Hand Calculations without Calculators

Sample Problem. The set of all real numbers x such that $|x-1| \leq 2$ is an interval of the form $a \leq x \leq b$. Express a and b as integers. Hint: Check the graphs of $f(x) = |x-1|,\ x \in \mathbb{R}$, and $g(x) = 2,\ x \in \mathbb{R}$, plotted on the same coordinate axes.

> The graphs of f and g are sketched in the accompanying figure. It is clear that $f(x) \leq g(x)$ for $-1 \leq x \leq 3$. Thus $a = -1$ and $b = 3$. Note: The inequality $-1 \leq x \leq 3$ represents the closed interval [-1,3]. See section 7.1 for another way to solve this problem.

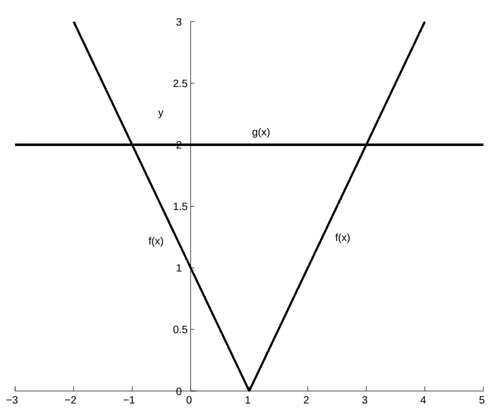

Figure 3.1: Graphs of f and g.

$a =$
$b =$

1. The set of real numbers x such that $x^2 < 1$ is an interval of the form $a < x < b$. Express a and b as integers.

$a =$

2. The set of real numbers x such that $x + 4 > 0$ is an interval of the form $x > a$. Express a as an integer.

$a =$
$b =$

3. The set of real numbers x such that $|x| < 2$ is an open interval of the form (a, b). Express a and b as integers.

$a =$

4. The inequality $-1 < x < 5$ is satisfied by all real numbers x such that $|x - a| < 3$. Express a as an integer.

5. The set of all non-zero real numbers x such that $\frac{6}{x} > 2$ can be expressed as an open interval of the form (a, b). Express a and b as integers.

$a =$
$b =$

6. The set of real numbers x such that $|x - 3| \le 4$ is an interval of the form $a \le x \le b$. Express a and b as integers. Hint: See the sample problem.

$a =$
$b =$

7. The set of real numbers y such that $|y + 2| > 2$ consists of two open intervals, one of the form $(-\infty, a)$ and one of the form (b, ∞) where $a < b$. Express a and b as integers. Hint: Check the graphs of $f(y) = |y + 2|,\ y \in \mathbb{R}$ and $g(y) = 2,\ y \in \mathbb{R}$, plotted on the same coordinate axes.

$a =$
$b =$

8. The set of real numbers z such that $(z - 1)^2 < 1$ is an open interval of the form (a, b). Express a and b as integers.

$a =$
$b =$

$a =$
$b =$

9. The set of real numbers z such that $(z-1)^2 \not> 1$ is an interval of the form $a \le z \le b$. Express a and b as integers.

$a =$
$b =$

10. The set of real numbers z such that $z^2 - z < 6$ is an open interval of the form (a, b). Express a and b as integers.

$a =$
$b =$

11. The set of real numbers z such that $z < 0$ and $\frac{1}{z} + \frac{z}{2} > \frac{-3}{2}$ is an open interval of the form (a, b). Express a and b as integers. Hint: Check the graph of an appropriate quadratic function of z, remaining aware of the sign of z.

$a =$
$b =$

12. The subset of real numbers x for which $f(x) = x^2 - 3x + 2$ assumes negative values is an open interval of the form (a, b). Express a and b as integers. Hint: Sketch a graph of f.

13. The subset of real numbers such that $x < 3$ and $f(x) = x^3 - 2x^2 - 5x + 6 > 0$ is an open interval of the form (a, b). Express a and b as integers. Hint: $f(1) = 0$. Now use long division of polynomials and factor the cubic.

a =
b =

14. The subset of real numbers x for which $|x+1| - |x-1| > 0$ is an open interval of the form (a, ∞). Express a as integer. Hint: Sketch the graphs of $f(x) = |x+1|,\ x \in \mathbb{R}$ and $g(x) = |x-1|,\ x \in \mathbb{R}$, plotted on the same coordinate axes.

a =

3.2 True/False

Sample Problem. If a and b are real numbers such that $a < b$ and if $c \leq 0$, then $ac \leq bc$. Is this statement True or False?

In order for the statement to be true, it must be true that $ac \leq bc$ for all real numbers a, b, c that satisfy $a < b$ and $c \leq 0$. So, while we can find numbers like $a = 3,\ b = 5$ and $c = 0$ so that $ac \leq bc$ (because $0 \leq 0$), this does not mean that the statement is true in general. The statement can be shown to be false by choosing appropriate values for a, b and c. Substituting $a = 1$, $b = 2$ and $c = -1$ into the inequality $ac < bc$, it now reads $(1)(-1) < (2)(-1)$, or $-1 < -2$, which is false.

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

_____(1) If the real number x is such that $x \neq 0$, then $x^2 > 0$.

_____(2) If the real number x is such that $|x+1| > 0$, then $x < 0$.

_____(3) For every two real numbers a and b it follows that $|a|\,|b| = |ab|$.

_____(4) If for two real numbers a and b it is known that $a < 0$ and $b > 0$, then $ab < 0$.

_____(5) If for two real numbers a and b it is known that $ab > 0$, then either $a > 0$ or $b > 0$.

_____(6) If the real number x satisfies $x < -1$, then $x^2 > 1$.

_____(7) If the two real numbers a and b are both less than the real number c, then $ab < c^2$.

_____(8) If the two real numbers a and b are both less than the real number c, then $a + b < 2c$.

_____(9) If the real number x satisfies $x > -1$, then $x^2 > 1$.

_____(10) If the two real numbers a and b are such that $a < b$, then $a^2 < b^2$.

_____(11) If the two positive real numbers a and b are such that $a < b$, then $\frac{1}{b} < \frac{1}{a}$.

_____(12) If the two non-zero real numbers a and b are ordered by $a < b$, then $\frac{1}{b} < \frac{1}{a}$.

_____(13) If the two real numbers a and b are ordered by $a < b$, and if $c > 0$, then $ac < bc$.

_____(14) The set of real numbers x that satisfy (both) $|x + 2| < 4$ and $|x - 5| < 4$ is the empty set.

______(15) If x, y and z are real numbers such that $xz < yz$ and $z < 0$, then $x > y$.

______(16) Every two real numbers a and b satisfy $|a + b| \leq |a| + |b|$.

______(17) Every two real numbers a and b satisfy $|a - b| \geq ||a| - |b||$.

Chapter 4

TRIGONOMETRIC FUNCTIONS

4.1 Hand Calculations without Calculators

Sample Problem. For some value of θ in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the value of $\sin(\theta)$ is $-\frac{\sqrt{3}}{2}$. The value of θ can be written in the form $\alpha\pi$. Express the value of α as an irreducible quotient of two integers.

> One first notes that the value of $\sin(\theta)$ is less than zero. This occurs when θ lies in the third and fourth quadrants. The third quadrant is not included in the domain of θ, so θ must lie in the fourth quadrant. One recognizes that the angle θ is $-\frac{\pi}{3}$. Therefore, α is $-\frac{1}{3}$.

1. The value of $\sin(\theta)$ is $\frac{1}{5}$. Express the value of $\csc(\theta)$ as an integer.

2. The value of $\sec(\theta)$ is $\frac{5}{4}$ and the value of $\tan(\theta)$ is positive. Express the value of $\sin(\theta)$ in simplest form (as an irreducible quotient of two integers).

3. The value of $\cot(\theta)$ is $\frac{3}{4}$ and the value of $\cos(\theta)$ is negative. Express the value of $\sin(\theta)$ in simplest form (as an irreducible quotient of two integers).

4. The value of $\sin(\theta)$ is $\frac{1}{5}$. The value of $\cos^2(\theta)$ can be written in the form $6(\alpha^2)$. Express the value of α^2 in simplest form (as an irreducible quotient of two integers).

5. For some value of θ in the interval $[0, \pi]$, the value of $\cos(\theta)$ is $-\frac{1}{2}$. The value of θ can be written in the form $\alpha\pi$. Express the value of α in simplest from (as an irreducible quotient of two integers).

6. The value of $\tan(\theta)$ is $\sqrt{3}$ and the value of $\sin(\theta)$ is positive. Express the value of $\sec(\theta)$ as an integer.

7. Express as an integer the value of $\tan\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$. Hint: Read this as "the tangent of the angle whose sine is $\frac{1}{\sqrt{2}}$." Recall that the range of the inverse sine function is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

8. Express as an integer the value of $\tan\left(\cot^{-1}\left(\frac{1}{3}\right)\right)$. Hint: Read this as "the tangent of the angle whose cotangent is $\frac{1}{3}$."

9. The value of $\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$ can be written in the form $\frac{\sqrt{2}(\sqrt{3}+1)}{\alpha}$. Express α as an integer.

10. The value of $\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$ can be written in the form $\frac{\sqrt{2}(\sqrt{3}-1)}{\alpha}$. Express α as an integer.

11. For some real number θ, $0 \leq \theta \leq 2\pi$, it is known that $\sin(\theta) = \frac{\sqrt{3}}{2}$ and $\tan(\theta) = -\sqrt{3}$. The value of θ can be written in the form $\alpha\pi$. Express α as an irreducible quotient of two integers.

12. For some real number θ, it is known that $\sin(\theta) = \frac{1}{4}$ and $\cos(\theta) > 0$. The value of $\cos(\theta)$ can be written in the form $\frac{\sqrt{\alpha}}{4}$. Express α as an integer.

13. For some real number θ, it is known that $\sin(\theta) = \frac{1}{4}$ and $\cos(\theta) > 0$. The value of $\sin(2\theta)$ can be written in the form $\frac{\sqrt{15}}{\alpha}$. Express α as an integer.

14. For some real number x, $\cos(x) = \frac{1}{3}$ and $\sin\left(\frac{x}{2}\right) > 0$. The value of $\sin\left(\frac{x}{2}\right)$ can be written as $\frac{1}{\sqrt{\alpha}}$. Express α as an integer.

15. For some real number x, $\sin(x) = \frac{4}{5}$. Express the value of $\sin(-x)$ as an irreducible quotient of two integers.

4.2 True/False

Sample Problem. For $0 < x < \pi$, it follows that $\cos(x)\csc(x) = \cot(x)$. Is this statement True or False?

This can be shown to be true by noticing that $\cos(x)\csc(x) = \cos(x)\frac{1}{\sin(x)} = \frac{\cos(x)}{\sin(x)} = \cot(x)$.

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

______(1) The function $\sin(x)$ is defined for all real values of x.

______(2) For all real values x, it follows that $\sin^2(x) = \cos^2(x) - 1$.

______(3) For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, it follows that $\tan(x)\cos(x) = \sin(x)$.

______(4) For $0 < x < \frac{\pi}{2}$, it follows that $\tan(x)\cot(x) = 1$.

______(5) For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, it follows that $\cos(x)\sec(x) = 1$.

______(6) For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, it follows that $\tan^2(x) - \sec^2(x) = 1$.

______(7) For $0 < x < \pi$, it follows that $\csc(x)\sin(x) = 1$.

______(8) For all $x \in \mathbb{R}$, it follows that $0 \leq \sin(x) \leq 1$.

______(9) The function $\sin^{-1}(x)$ is defined for all real values of x.

______(10) For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, it follows that $1 + \tan(x) > 1$.

______(11) For $-1 \leq x \leq 1$, it follows that $\sin(\sin^{-1}(x)) = x$.

______(12) The function $\tan^{-1}(x)$ is defined for all real values of x.

______(13) For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, it follows that $1 + \tan^2(x) = \sec^2(x)$.

______(14) For all real values of x, it follows that $\sin(x + \frac{\pi}{2}) = \cos(x)$.

______(15) For all real values of x, it follows that $\sin(x) + \cos(x) = \cos(2x)$.

______(16) For all real values of x, it follows that $2\sin(x)\cos(x) = \sin(2x)$.

______(17) For all real values of x, it follows that $1 - 2\sin^2(x) = \cos(2x)$.

______(18) For all real values of x, it follows that $2\cos^2(x) - 1 = \cos(2x)$.

______(19) For all real values of θ, it follows that $\cos\left(\frac{\theta}{2}\right) = \frac{\cos(\theta)+1}{2}$.

Chapter 5

LOGARITHMS/EXPONENTIALS

5.1 Hand Calculations without Calculators

Sample Problem. Express as an integer the value of $4^{\frac{3}{2}}$.

The solution can be computed by noticing that $4^{\frac{3}{2}} = (4^3)^{\frac{1}{2}} = 64^{\frac{1}{2}} =$ 8.

1. Express as an integer the value of $(64)^{\frac{2}{3}}$.

2. Express as an integer the value of $\log_2 8$.

3. Express as an integer the value of $\ln 1$.

4. Express as an integer the value of $(\frac{1}{16})^{\frac{-1}{4}}$.

5. Express as an integer the value of $\frac{1}{\log_8 2}$.

6. Express as an integer the value of $3^{\log_2 4}$.

7. Express as an integer the value of $3^{\log_3 6}$.

8. Express as an integer the value of $e^{\ln 1}$.

9. Express as an integer the value of $\sqrt[3]{(27)(125)}$.

10. Express as an integer the value of $\log_{10} 2 + \log_{10} 50$.

11. Express as an integer the value of $(\sqrt[3]{9})(3)^{-2/3}$.

12. Express as a positive integer a value of x that satisfies the equation $x^2 = 2^6$.

13. Express as an integer the value of $16^{\frac{1}{2}\log_4 2}$.

14. Express as an integer the value of $16^{\log_4 \sqrt{2}}$.

15. Express as an integer the value of $\frac{1}{2}\log_{\frac{1}{4}} 80 - \frac{1}{2}\log_{\frac{1}{4}} 5$.

16. Express as an integer the value of α that satisfies $(2^{\log_4 2})(4^{\log_2 4}) = 2^{\alpha/2}$.

17. Express as an integer the smallest real number x such that $2^x \leq 3^x$.

5.2 True/False

Sample Problem. For positive real numbers a and b

$$(\ln b)^a = a \ln b.$$

Is this statement True or False?

> One can see this statement is false by referring to the log property $\ln b^a = a \ln b$. (Check $b = e$ and $a = 2$.) Notice the property states only the value b is raised to the power a, not the value $\ln b$.

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

_____(1) For positive real numbers a and b,

$$\log_{10} a + \log_{10} b = \log_{10}(ab).$$

_____(2) For positive real numbers a and b,

$$\log_3 a - \log_3 b = \log_3 \frac{a}{b}.$$

_____(3) For positive real numbers a and b,

$$(\ln a)(\ln b) = \ln(ab).$$

_____(4) If $\ln a < 0$ for some real number a, then $a < 0$.

_____(5) For positive real numbers a and b,

$$\frac{\log_{10} b}{a} = \log_{10}(b^{1/a}).$$

_____(6) $(3)^\pi = e^{\pi \ln 3}$.

_____(7) Even though the function e^x is defined for all real numbers x, its inverse function $\ln x$, is defined for only positive real numbers.

_____(8) The function $e^x, \;\; x \in \mathbb{R}$, never assumes a negative value.

_____(9) The function $\log_2 x, \;\; x \in \mathbb{R}$, never assumes the value 0.

_____(10) Even though the function $\ln e^x$ is defined for all real numbers x, the function $e^{\ln x}$ is not defined for all real numbers x.

_____(11) The graphs of the functions $e^x, \;\; -\infty < x < \infty$, and $\ln x, \; 0 < x < \infty$, never intersect each other.

_____(12) $(4)^\pi = 3^{\pi \log_3 4}$.

_____(13) Let x denote a positive real number. If a and b denote real numbers, then $(x^a)(x^b) = x^{ab}$.

_____(14) Let x denote a positive real number. If a and b denote real numbers, then $(x^a)^b = x^{ab}$.

_____(15) Let x denote a positive real number. If a denotes a real number, then $x^{-a} = (\frac{1}{x})^a$.

_____(16) Let x and y denote positive real numbers with $x < y$. If a denotes a real number, then $x^a < y^a$.

_____(17) Let x denote a positive real number. If a and b denote real numbers, then $x^a + x^b = x^{a+b}$.

Chapter 6

GRAPH RECOGNITION

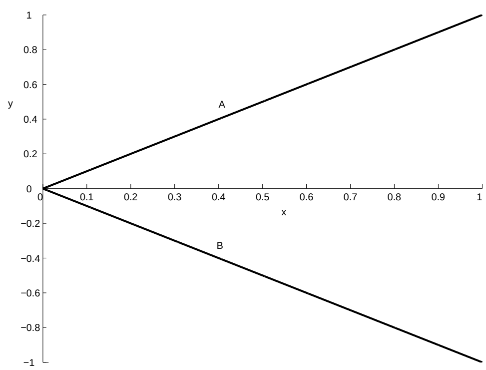

Figure 6.1: Two Curves

Sample Problem. In Figure 6.1, one of the curves is the graph of $f(x) = -x,\ 0 \leq x \leq 1$, and the other is the graph of $g(x) = x,\ 0 \leq x \leq 1$. Which of the two curves is the graph of f, A or B?

> The graph of $f(x) = -x$ has negative slope and the graph of $g(x) = x$ has positive slope. The graph B has negative slope, which can be seen by using two points on the line, $(0, 0)$ and $(1, -1)$, and computing the slope. The change in y-values is $-1 - 0 = -1$ and the change in x-values is $1 - 0 = 1$. Thus, one can see the slope of graph B is -1, and that the graph of f is B.

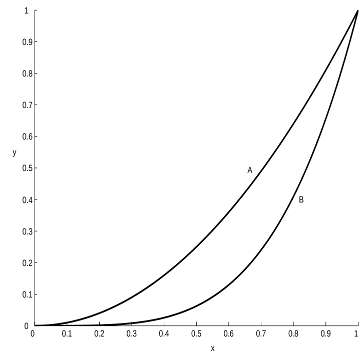

Figure 6.2: Two Curves

1. In Figure 6.2, one of the curves is the graph of $f(x) = x^2$, $0 \leq x \leq 1$, and the other is the graph of $g(x) = x^4$, $0 \leq x \leq 1$. Which of the two curves is the graph of f, A or B?

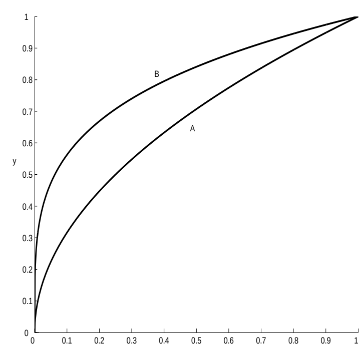

Figure 6.3: Two Curves

2. In Figure 6.3, one of the curves is the graph of $f(x) = x^{\frac{1}{2}}$, $0 \leq x \leq 1$, and the other is the graph of $g(x) = x^{\frac{1}{4}}$, $0 \leq x \leq 1$. Which of the two curves is the graph of f, A or B?

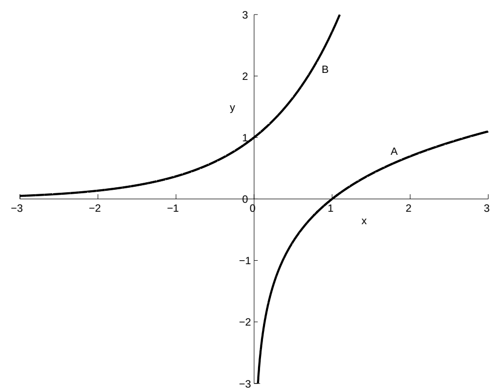

Figure 6.4: Two Curves

3. In Figure 6.4, one of the curves is the graph of $f(x) = e^x$, defined on a particular domain, and the other is the graph of $g(x) = \ln x$, defined on another domain. Which of the two curves is the graph of f, A or B?

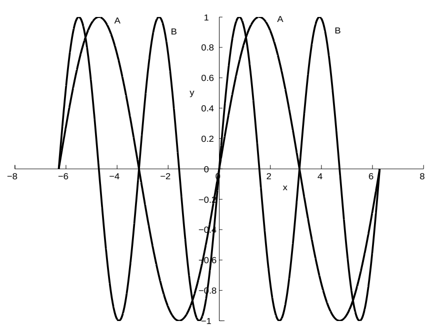

Figure 6.5: Two Curves

4. In Figure 6.5, one of the curves is the graph of $f(x) = \sin(x),\ -2\pi \leq x \leq 2\pi$ and the other is the graph of $g(x) = \sin(2x),\ -2\pi \leq x \leq 2\pi$. Which of the two curves is the graph of f, A or B?

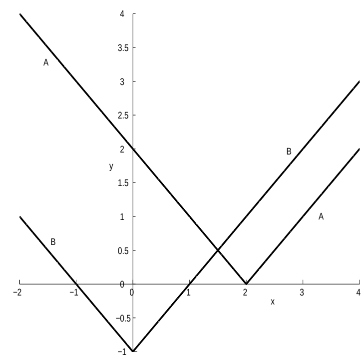

Figure 6.6: Two Curves

5. In Figure 6.6, one of the curves is the graph of $f(x) = |x| - 1,\ -2 \leq x \leq 4$, and the other is the graph of $g(x) = |x-2|,\ -2 \leq x \leq 4$. Which of the two curves is the graph of f, A or B?

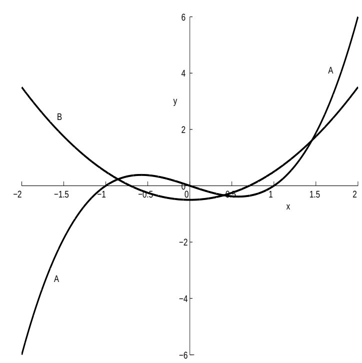

Figure 6.7: Two Curves

6. In Figure 6.7, one of the curves is the graph of a quadratic polynomial $f(x)$ and the other is the graph of a cubic polynomial $g(x)$. Which of the two curves is the graph of f, A or B?

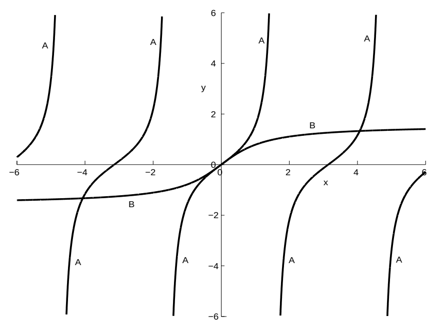

Figure 6.8: Two Curves

7. In Figure 6.8, one of the curves is the graph of $f(x) = \tan(x)$, defined on a suitable domain, and the other is the graph of $g(x) = \arctan(x)$, defined on a suitable domain. Which of the two curves is the graph of f, A or B?

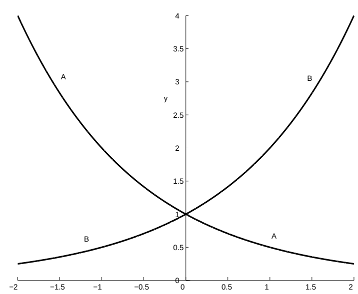

Figure 6.9: Two Curves

8. In Figure 6.9 one of the curves is the graph of $f(x) = 2^x,\ -2 \leq x \leq 2$, and the other is the graph of $g(x) = 2^{-x},\ -2 \leq x \leq 2$. Which of the two curves is the graph of f, A or B?

Chapter 7

Examples and Help

7.1 Introduction

The examples in the following sections are meant to help clarify some of the problems in the previous chapters and point out some common mistakes. The examples are not an extensive review of “initial” skills. Following each example is a list of online references where more help can be found. Each web page reference leads to a specific topic. Eight web sites are referenced which contain a review of “initial” skills, examples and problems to be worked. The URLs for the web sites are:

- http://mathmistakes.info/
- http://www.algebrahelp.com/
- http://www.purplemath.com/modules/index.htm
- http://www.khanacademy.org/math/trigonometry
- http://www.sosmath.com/
- http://www.themathpage.com/

In addition to the web sites, more help can be found in any pre-calculus textbook or in the following books which are available at most book stores:

- *Schaum’s Easy Outline of Precalculus* by F. Safier and K. Kirkpatrick
- *Schaum’s Outline of Precalculus* by Fred Safier

7.2 Arithmetic/Algebraic Operations

Facts:

(i) $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ (See example 7.2.4)

(ii) $\sqrt{x^2} = |x|$ (See example 7.2.4 and section 7.3)

Common Mistakes:

(i) $\frac{1}{x+y}$ is not equivalent to $\frac{1}{x} + \frac{1}{y}$ (See example 7.2.2)

(ii) $\frac{ax+y}{az}$ is not equivalent to $\frac{x+y}{z}$ (See example 7.2.3)

(iii) $\sqrt{a+b}$ is not equivalent to $\sqrt{a} + \sqrt{b}$ (See example 7.2.4)

(iv) $(ax+by)^2$ is not equivalent to $a^2x^2 + b^2y^2$ (See example 7.2.5)

Example 7.2.1. Express the value of $\frac{4}{(\frac{16}{4})}$ as an integer.

> The value of $\frac{4}{(\frac{16}{4})}$ can be read as $4 \div \frac{16}{4}$ or $4 \cdot \frac{4}{16} = \frac{16}{16}$. Expressed as an integer, the solution is 1.
>
> More help on fractions can be found at
> http://www.purplemath.com/modules/fraction.htm

Example 7.2.2. Suppose that $x > 0,\ y > 0$. The expression

$$\frac{x+y}{\frac{1}{x} + \frac{1}{y}}$$

can be written in the form axy. Express a as an integer.

> In order to simplify the fraction, the denominator needs to be expressed as one fraction. The fraction $\frac{x+y}{\frac{1}{x}+\frac{1}{y}}$ can be written as $\frac{x+y}{\frac{x+y}{xy}}$ or $\frac{xy(x+y)}{(x+y)}$. Canceling the common factor, the fraction is simplified to xy. Therefore, the solution to the problem is $a = 1$.
>
> More help on simplifying rational expressions can be found at
> http://www.purplemath.com/modules/rtnldefs.htm

Example 7.2.3. Suppose that $x \neq 0$, $y \neq 0$. The expression

$$\frac{6x^2y + 12xy^2}{3xy}$$

can be written in the form $ax + by$. Express a and b as integers.

In order to simplify the expression, one can completely factor the numerator. The fraction $\frac{6x^2y+12xy^2}{3xy}$ can be written as $\frac{6xy(x+2y)}{3xy}$. Canceling the common factor, the fraction is simplified to $2(x + 2y)$ or $2x + 4y$. Therefore, the solution to the problem is $a = 2$ and $b = 4$.

More help on simplifying rational expressions can be found at
http://www.purplemath.com/modules/rtnldefs.htm

Example 7.2.4. If a and b denote real numbers less than zero, then $\sqrt{a^2b^2} = ab$. True or False?

From Fact (ii) it follows that $\sqrt{a^2b^2} = \sqrt{(ab)^2} = |ab|$. Since $a < 0$ and $b < 0$ we have that $ab > 0$ and so by Fact (i), $ab = |ab|$. Therefore $\sqrt{a^2b^2} = ab$, and the statement is True.

More help on simplifying radical expressions can be found at
http://www.purplemath.com/modules/radicals.htm

Example 7.2.5. The quadratic polynomial $2x^2-12x+6$ can be written in the form $a(x-b)^2+c$. Express a, b and c as integers.

To rewrite the expression in the requested form, the method of completing the square can be used. To complete the square, the expression $2x^2-12x+6$ can first be written as $2(x^2-6x)+6$ or $2(x^2-6x+9)+6-18$. Note, the $+9$ creates a perfect square, but in order not to change the value of the expression -18 must also be added. Factoring, the expression becomes $2(x-3)^2-12$. Thus the solution is $a=2, b=3$ and $c=-12$.

More help on completing the square can be found at
http://www.purplemath.com/modules/sqrquad.htm,
http://www.themathpage.com/aPreCalc/complete-the-square.htm and
http://www.sosmath.com/algebra/factor/fac07/fac07.html

7.3 Inequalities

Facts:

(i) $|x|<a$ is equivalent to $-a<x<a$ (See example 7.3.1)

(ii) $|x|>a$ is equivalent to $x<-a$ or $x>a$ (See example 7.3.2)

Common Mistakes:

(i) $-x>a$ is not equivalent to $x>-a$, but $-x>a$ is equivalent to $x<-a$ (See example 7.3.2)

Example 7.3.1. The set of real numbers x such that $|\frac{1}{2}x-1|\leq 1$, is a closed interval of the form $[a,b]$. Express a and b as integers.

The inequality $|\frac{1}{2}x-1|\leq 1$ can be written as $-1\leq\frac{1}{2}x-1\leq 1$. By adding one to both sides and then multiplying both sides by two, the inequality becomes $0\leq x\leq 4$, which can be written as [0,4]. Therefore, the solution is $a=0$ and $b=4$.

More help on solving inequalities can be found at
http://www.sosmath.com/algebra/inequalities/ineq03/ineq03.html and
http://www.purplemath.com/modules/ineqsolv.htm

Example 7.3.2. The set of real numbers x such that $(4-x)^2 > 9$ consists of two open intervals, one of the form $(-\infty, a)$ and one of the form (b, ∞) where $a < b$. Express a and b as integers.

The inequality $(4-x)^2 > 9$ can be written as $|4-x| > 3$, which implies that either $4-x < -3$ or $4-x > 3$. This imples that either $x > 7$ or $x < 1$. Therefore, the solution is $a = 1$ and $b = 7$.

Another way to solve the inequality is to graph by hand the parabola $f(x) = (4-x)^2$ and the line $g(x) = 9$. As the figure below shows, the parabola is above the line when x is in the interval $(-\infty, 1)$ or $(7, \infty)$.

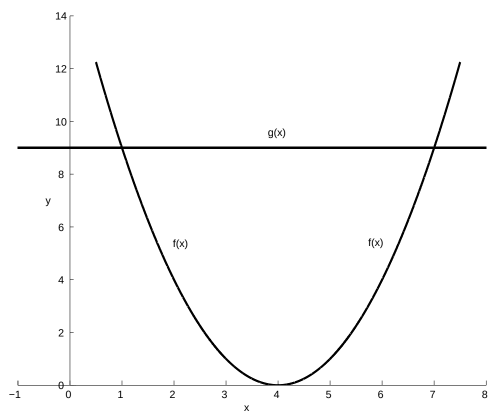

Figure 7.1: Two Curves

More help on graphing inequalities can be found at
http://www.sosmath.com/algebra/inequalities/ineq04/ineq04.html and
http://www.purplemath.com/modules/ineqgrph.htm

Example 7.3.4. The set of real numbers x that satisfy (both) $|x-1| < 2$ and $|x+1| < 2$ is the empty set. True or False?

By solving both inequalities, all values can be found. The first inequality is equivalent to $-1 < x < 3$ and the second inequality is equivalent to $-3 < x < 1$. Therefore, the numbers that satisfy both inequalities are in the open interval $(-1, 1)$. Therefore, the statement is False.

More help on inequalities can be found at
http://www.sosmath.com/algebra/inequalities/ineq03/ineq03.html

7.4 Trigonometric Functions

Identities:
For real numbers A and B it follows that

(i) $\sin^2(A) + \cos^2(A) = 1$

(ii) $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

(iii) $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

(iv) $\sin(2A) = 2\sin(A)\cos(A)$

(v) $\cos(2A) = \cos^2(A) - \sin^2(A) = 2\cos^2(A) - 1 = 1 - 2\sin^2(A)$

(vi) $\sin^2(A) = \frac{1}{2}(1 - \cos(2A))$

(vii) $\cos^2(A) = \frac{1}{2}(1 + \cos(2A))$

(viii) $\sin(A + 2\pi) = \sin(A)$ and $\sin(-A) = -\sin(A)$

(ix) $\cos(A + 2\pi) = \cos(A)$ and $\cos(-A) = \cos(A)$

Common Mistakes:

(i) $\sin^{-1} x$ is not equivalent to $\frac{1}{\sin x}$ (See example 7.4.4)

Example 7.4.1 The value of $\csc\theta$ is $-\frac{13}{12}$ and the value of $\cot\theta$ is positive. Express the value of $\cos\theta$ as an irreducible quotient of two integers.

> If $\csc\theta$ is negative, then θ must lie in quadrant III or IV. If $\cot\theta$ is positive, then θ must lie in quadrant I or III. It is given that $\csc\theta$ is negative and $\cot\theta$ is positive, so θ must lie in quadrant III, where $\cos\theta$ is negative. Using identity (i), the value of $\cos\theta$ is $-\frac{5}{13}$.
>
> More help on trigonometric functions can be found at
> http://mathmistakes.info/facts/TrigFacts/index.html
> More help on reference angles and radians can be found at
> http://mathmistakes.info/facts/TrigFacts/learn/uc/uc.html
> More help on evaluating trigonometric functions can be found at
> http://www.themathpage.com/aTrig/unit-circle.htm and
> http://mathmistakes.info/facts/TrigFacts/learn/vals/sum.html

Example 7.4.2 For some value of θ in the interval $[0, \pi]$, the value of $\cot \theta$ is 1. The value of θ can be written in the form $\alpha\pi$. Express the value of α as an irreducible quotient of two integers.

If $\cot \theta$ is positive, then θ must lie in quadrant I or III. It is given that θ lies in quadrants I or II, and θ must lie in quadrant I when $\cot \theta$ is 1. So θ must be $\frac{\pi}{4}$. Therefore, the value of α is $\frac{1}{4}$.

More help on evaluating trigonometric functions can be found at
http://www.themathpage.com/aTrig/functions-angle.htm and
http://www.sosmath.com/trig/Trig2/trig2/trig2.html

Example 7.4.3 For some real number x, $\cos(x) = -\frac{2}{3}$. Express the value of $\cos(-x)$ as an irreducible quotient of two integers.

The function $\cos(x)$ is an even function, so $\cos(-x) = \cos(x)$. Therefore, the value of $\cos(-x) = -\frac{2}{3}$.

More help on trigonometric identities can be found at
http://www.sosmath.com/trig/Trig5/trig5/trig5.html and
http://www.sosmath.com/trig/addform/addform.html

Example 7.4.4 The function $\cos^{-1}(x)$ is defined for all real values of x. True or False?

For all real values of θ, $-1 \leq \cos(\theta) \leq 1$. Therefore, $\cos^{-1}(x)$ is defined when $-1 \leq x \leq 1$, making this statement False.

More help on inverse trigonometric functions can be found at
http://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/inverse_trig_functions/v/inverse-trig-functions-arcsin and
http://mathmistakes.info/facts/TrigFacts/learn/vals/isum.html

Example 7.4.5 For all real values of θ, it follows that $\sin(6\theta) = 2\sin(3\theta)\cos(3\theta)$. True or False?

Identity (iv) states that $\sin(2A) = 2\sin(A)\cos(A)$. In this problem, $2A = 6\theta$, so $A = 3\theta$. Therefore, this statement is True.

More help on trigonometric identities can be found at
http://www.sosmath.com/trig/Trig5/trig5/trig5.html and
http://www.sosmath.com/trig/addform/addform.html

7.5 Logarithms/Exponentials

Facts:

- For real numbers x, $y > 0$, $b > 0$ and $b \neq 1$ it follows that if $y = b^x$, then $x = \log_b y$
- $\log_e x$ is denoted as $\ln x$

Properties:

(A) For real numbers x and y and $b > 0$ it follows that

(i) $b^{x+y} = b^x b^y$
(ii) $(b^x)^y = b^{xy}$
(iii) $b^{-x} = \frac{1}{b^x}$
(iv) $b^{x-y} = \frac{b^x}{b^y}$

(B) For real numbers $x > 0$ and $y > 0$, $b > 0$ and $b \neq 1$ it follows that

(i) $\log_b xy = \log_b x + \log_b y$

(ii) $\log_b \frac{x}{y} = \log_b x - \log_b y$

(iii) $y \log_b x = \log_b x^y$

(iv) $-\log_b x = \log_b \frac{1}{x}$

Common Mistakes:

(i) The the value of -2^2 is not 4, it is -4. (See example 7.5.1)

(ii) $(\ln x)^y$ is not equivalent to $y(\ln x)$ (See example 7.5.2)

Example 7.5.1. Express as an integer the value of $-8^{\frac{2}{3}}$.

The value of $-8^{\frac{2}{3}}$ can be read as $-(8^2)^{\frac{1}{3}}$. Working from the inside out, the value of $-(8^2)^{\frac{1}{3}}$ is $-(64)^{\frac{1}{3}}$ or -4.

More help on fractional exponents can be found at
http://www.purplemath.com/modules/exponent5.htm

Example 7.5.2. Express as an integer the value of $9^{2\log_3 2}$.

The value of $9^{2\log_3 2} = (3^2)^{2\log_3 2}$. Using property (ii) the value can be written as $3^{4\log_3 2}$ which by property (vii) is $3^{\log_3 2^4}$. By definition, $3^{\log_3 2^4}$ is equivalent to 2^4. Therefore, the solution is 16.

More help on logarithms and exponentials can be found at
http://www.purplemath.com/modules/logs.htm and
http://www.themathpage.com/aPreCalc/logarithmic-exponential-functions.htm
and
http://www.sosmath.com/algebra/logs/log4/log4.html#logarithm

Example 7.5.3. The value of $\log_\pi \pi$ is 1. True or False?

Let the value $\log_\pi \pi = x$. By definition, $\pi^x = \pi$, so x must be 1. Therefore, the statement is True.

More help on logarithms and exponentials can be found at
www.sosmath.com/algebra/logs/log4/log4.html#logarithm,
http://www.purplemath.com/modules/logs.htm and
http://www.themathpage.com/aPreCalc/logarithmic-exponential-functions.htm

Example 7.5.4. Let x denote a positive real number. If a and b denote real numbers, then $\frac{x^a}{x^b} = x^{\frac{a}{b}}$. True or False?

Using property (iv), the expression $\frac{x^a}{x^b}$ is equivalent to x^{a-b}. In general, $a - b$ is not equal to a/b. Therefore, the statement is False.

More help on exponential rules can be found at
http://www.sosmath.com/algebra/logs/log3/log3.html

7.6 Graph Recognition

To solve many calculus problems, there are several functions whose graphs you may need to be able to recognize and sketch **without** the use of a calculator. Some of the basic functions are:

(i) Linear Functions (lines): $f(x) = mx + b$ (See example 7.6.1)

(ii) Quadratic Functions (parabolas): $f(x) = a(x - h)^2 + k$ (See example 7.6.2)

(iii) Trigonometric Functions: $f(x) = \sin x, f(x) = \cos x$, etc. (See example 7.6.3)

(iv) Exponential/Logarithm Functions: $f(x) = b^x$ and $f(x) = \log_b x$ where $b > 0$ and $b \neq 1$ (See example 7.6.4)

Example 7.6.1. Sketch the line $4x + 2y = 6$.

One way to sketch a line one way is to first write its equation in *slope-intercept form*, or $y = mx + b$. Then it is easy to see the slope m and the y-intercept b. To write an equation in this form, solve the equation for y. The equation $4x + 2y = 6$ can be written as $y = -2x + 3$, and this line has slope -2 and a y-intercept 3. The line $4x + 2y = 6$ is sketched below.

Alternatively, one can find two points on the line, one by setting $x = 0$ and solving for y, and the other by setting $y = 0$ and solving for x.

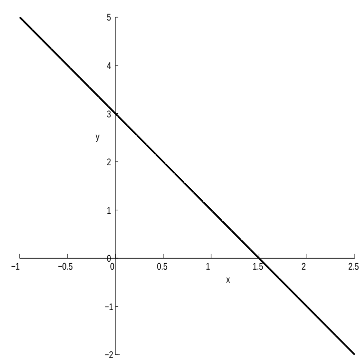

Figure 7.2: Line

More help on sketching lines can be found at http://www.purplemath.com/modules/graphlin.htm.

Example 7.6.2. Sketch the graph of $f(x) = x^2 - 2x + 3$ for $-1 \leq x \leq 3$.

The parabola can be sketched by first writing its equation in the *standard form* $y = a(x-h)^2 + k$, where the vertex is (h, k). The value of a determines how wide the parabola is and whether it opens up or down. By completing the square, the equation of the parabola $y = x^2 - 2x + 3$ can be written in standard form. To complete the square, $y = x^2 - 2x + 3$ can be written as $y = x^2 - 2x + 1 + 2$ which factors into $y = (x-1)^2 + 2$. The parabola's vertex is at the point (1,2) and opens upward. The parabola is sketched below.

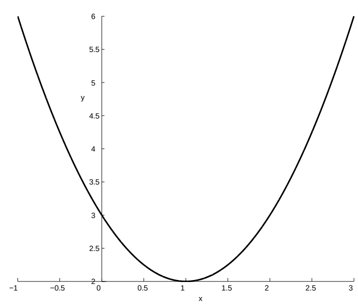

Figure 7.3: Quadratic

More help on quadratic graphs can be found at
http://www.purplemath.com/modules/grphquad.htm.
Help on completing the square can be found at
http://www.purplemath.com/modules/sqrvertx.htm and
http://www.themathpage.com/aPreCalc/complete-the-square.htm.

Example 7.6.3. Sketch the graphs of $\cos x$ and $\sin x$ for $-2\pi \leq x \leq 2\pi$.

These two trigonometric functions have the same basic shape, but $\sin(x)$ intersects the y-axis at the origin and $\cos(0) = 1$. To get the general shape of $y = \cos x$, use the values $\cos(\pi/2) = 0$, $\cos(\pi) = -1$, $\cos(3\pi/2) = 0$ and $\cos(2\pi) = 1$. To get the general shape of $y = \sin x$ use the values $\sin(\pi/2) = 1$, $\sin(\pi) = 0$, $\sin(3\pi/2) = -1$ and $\sin(2\pi) = 0$. The graphs of $\cos x$ (graph A) and $\sin x$ (graph B) are sketched below.

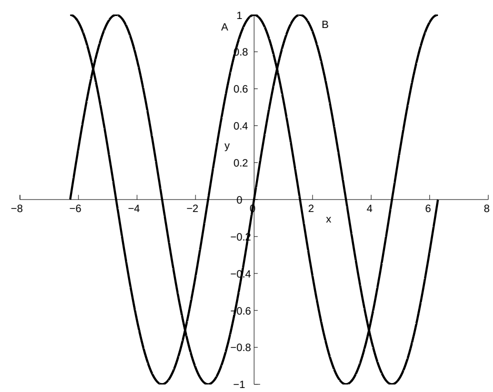

Figure 7.4: Two Trig Functions

More help on sketching trig functions can be found at
http://www.themathpage.com/aTrig/graphs-trig.htm.

Example 7.6.4. Sketch the graph of $f(x) = e^{x+1}$ and $f(x) = \ln(x-1)$.

The graphs of $y = e^x$ and its inverse $y = \ln x$ are used to graph the two functions. The first, $y = e^x$, intersects the y-axis at the point $(0, 1)$ and grows exponentially. The second, $y = \ln x$, is the reflection of $y = e^x$ across the line $y = x$. The graph of the exponential $y = e^{x+1}$ is the graph of $y(x) = e^x$ shifted one unit to the left. The graph of $y = \ln(x-1)$ is the graph of $f(x) = \ln(x)$ shifted one unit to the right.

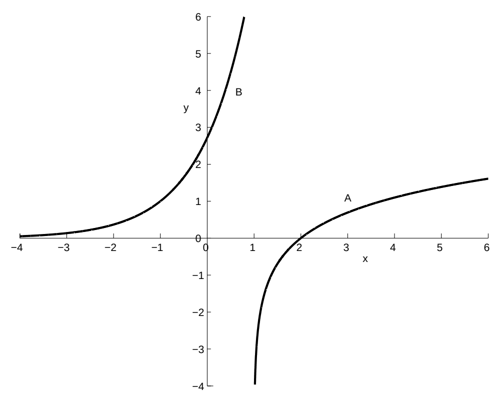

Figure 7.5: Exponential and Logarithm

More help on exponential and logarithmic graph can be found at
http://www.themathpage.com/aPreCalc/logarithmic-exponential-functions.htm.
More help on exponential graphs can be found at
http://www.purplemath.com/modules/graphexp.htm.
More help on logarithmic graphs can be found at
http://www.purplemath.com/modules/graphlog.htm.

Chapter 8

ANSWERS

Chapter 2 Arithmetic/Algebraic Operations

Hand Calculations without Calculators

1. 1
2. $\frac{1}{4}$
3. $\frac{50}{27}$
4. $\frac{23}{30}$
5. 100
6. 8
7. 1
8. $a = 7$; $b = -5$ (or *vice-versa*)
9. $a = 1$; $b = -1$; $c = -5$; $d = -3$
10. 17
11. $a = 1$; $b = -3$; $c = 2$; $d = 0$; $e = -6$
12. $a = 3$; $b = -5$
13. -3, $\frac{1}{2}$
14. $a = 1$; $b = 2$

True/False

1. T
2. T
3. T
4. F Recall that $\sqrt{x^2} = |x|$, so $\sqrt{x^2} = x$ or $\sqrt{x^2} = -x$.
5. F
6. F

7. T
8. F
9. T
10. F

Chapter 3 Inequalities

Hand Calculations without Calculators

1. $a = -1;\ \ b = 1$
2. $a = -4$
3. $a = -2;\ \ b = 2$ Recall that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
4. $a = 2$
5. $a = 0,\ \ b = 3$
6. $a = -1;\ \ b = 7$
7. $a = -4;\ \ b = 0$
8. $a = 0;\ \ b = 2$
9. $a = 0;\ \ b = 2$
10. $a = -2;\ \ b = 3$
11. $a = -2;\ \ b = -1$
12. $a = 1;\ \ b = 2$
13. $a = -2;\ \ b = 1$
14. $a = 0$

True/False

1. T
2. F
3. T
4. T
5. F
6. T
7. F
8. T
9. F
10. F
11. T
12. F
13. T

14. F
15. T
16. T
17. T

Chapter 4 Trigonometric Functions

Hand Calculations without Calculators

1. 5
2. $\frac{3}{5}$
3. $-\frac{4}{5}$
4. $\frac{4}{25}$
5. $\frac{2}{3}$
6. 2
7. 1
8. 3
9. 4
10. 4
11. $\frac{2}{3}$
12. 15
13. 8
14. 3
15. $-\frac{4}{5}$

True/False

1. T
2. F
3. T
4. T
5. T
6. F
7. T
8. F
9. F
10. F
11. T
12. T

13. T
14. T
15. F
16. T
17. T
18. T
19. F

Chapter 5 Logarithms/Exponentials

Hand Calculations without Calculators

1. 16 Recall that for $a > 0$ one has $a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^p = (a^p)^{\frac{1}{q}}$.
2. 3
3. 0
4. 2
5. 3
6. 9
7. 6
8. 1
9. 15
10. 2
11. 1
12. 8
13. 2
14. 2
15. -1
16. 9
17. 0

True/False

1. T
2. T
3. F
4. F
5. T
6. T
7. T

8. T
9. F
10. T
11. T
12. T
13. F
14. T
15. T
16. F
17. F

Chapter 6 Graph Recognition

1. A
2. A
3. B
4. A
5. B
6. B
7. A
8. B