**Components of a spin glass model**

- **Spin variable** \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_N) \) is a random vector in a high dimensional space.
- **Probability measure** \( p(\sigma) \) gives the distribution of the spins.
- **Disorder variables** are random parameters in the definition of \( p(\sigma) \) (for our model, they are \( M, g \) defined below). Because \( p(\sigma) \) has random parameters, it is a random measure.

**Key Questions:**

1. How does an external magnetic field affect spin distribution?
2. What happens in the transition between a model with an external field and one without?

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**Techniques**

Computing the overlaps involves three key steps:

1. **Use contour integral representation of** \( \mathcal{X}_N \).
   - The Gibbs measure involves partition function \( \mathcal{X}_N \). Due to specific properties of SSK, we can rewrite this as a contour integral:
     \[
     \mathcal{X}_N = C_N \int_{x \in \mathbb{R}^N} e^{\beta \mathcal{H}(x)} dx
     \]
   - where \( C_N \) is constant and \( \mathcal{G}(x) \) depends on \( M, g \). A contour integral is easier to compute than an \( N \)-dimensional surface integral.

2. **Write a moment generating function for each overlap.** These can be expressed as a ratio of contour integrals like the one above.

3. **Analyze using random matrix theory.** GOE matrices exhibit eigenvalue rigidity, meaning the eigenvalues are usually very close to their predicted locations. This and other properties help to analyze the random model.

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**Further Research**

- **Free energy of SSK** has transitional \( h \) regimes at all temperatures (unlike overlaps, where transition is only at low temperature).
- **Susceptibility** or “magnetization per external field strength,” is an application of our results.
- **Precise fluctuation terms** are included in our paper, in addition to leading order terms.
- **Rigorous proofs** are omitted in some places in our paper [1]. Some proofs were obtained in separate papers by Landon & Sosoe [3] and by Collins-Woodfin [2].

**Open Questions:**

- **Spin glass models**: Do they exhibit similar transitions?

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**References**

