Euclidean Geometry & Education

Nina White

Project Description: This project is intended for students interested in teaching high school geometry one day. We will start with a 3-week long mini-course introduction to Euclidean Geometry. Then students on the project will choose (with the help of the faculty mentor) a sub-topic of Euclidean Geometry to focus on more deeply.

The students on this project will collaboratively create a virtual, interactive mini-textbook (using R, geogebra, or some other medium) on the focus topic. Examples of such topics could be: definitions of and theorems about congruency, straight-edge and compass constructions (or a particular subset of interest thereof), comparison of different axiomatic systems in Euclidean and/or non-Euclidean Geometry, definitions and theorems on area measurement, similarity and trigonometry, geometry of circles, triangle centers, wall-paper groups, a classification of planar isometries, etc.

The skills developed in this project will include proving and problem-solving, writing expository mathematics, finding and using mathematical resources, and creating interactive virtual materials.

Prerequisites: Math 217 or similar proof-writing experience, a declared major in education or other demonstrable interest in education.

Unraveling the patterns of Painlevé zeros

Andrei Prokhorov & Jörn Zimmerling

Project Description

In this project we plan to study zeros of solutions to Painlevé ODEs. There are six Painlevé ODEs which are second order nonlinear equations with applications in other fields of mathematics and physics. They admit families of rational solutions. As the degree of these solutions gets large, their complex valued zeros fill out certain shapes in a complex plane, for example triangles for Painlevé III and combinations of triangles and rectangles for Panlevé IV. It can happen that the solutions depend on extra parameters and it is intriguing to observe how zeros move as parameter changes.

We want to focus on the less studied family of rational solutions to the Painlevé VI equation. We expect that it depends on parameters. Our first goal is to compute and visualize its fascinating zeroes pattern.

Prerequisites:
Some experience in a programming language or Mathematica.
Math 216 or equivalent
An understanding of what a complex number is. MATH 555 would be nice but is not required.

Orthogonal Polynomials

Ahmad Barhoumi

Project Description

Asymptotic analysis of orthogonal polynomials crops up in plenty of fields, and there is no shortage of their applications in approximation theory, mathematical physics, and random matrix theory among other fields. However, unlike classical orthogonal polynomials, polynomials orthogonal with respect to complex-valued weights may "degenerate". More precisely, for a complex weight $\rho(z)$, a polynomial $P_n(z)$ satisfying n orthogonality conditions

$$\int_\gamma z^k P_n(z)\rho(z)\mathrm{d}z = 0 \quad \text{for} \quad k = 0, 1, \cdots, n-1,$$

may have degree less than n. This degeneration can be a serious obstacle that often presents itself in subtle ways in the analysis.

Guided by the theory available, the objective of this project is to explore some specific families of orthogonal polynomials and compute examples of families exhibiting interesting degeneration patterns. Roughly speaking, the plan is to

1. learn what non-Hermitian orthogonal polynomials (OPs) are and how to compute them and their zeros,
2. begin computing OPs in symmetric situations where degeneration may occur while exploring how to do so most effectively,
3. compute OPs dependent on auxiliary parameters and consider how a polynomial may experience degeneration with choice of parameter(s),
4. explore ways to visualize this information in a reasonable manner.

Prerequisites

Math 217 and Math 351/451 or equivalents. A course in complex analysis would be nice, but not required.

Explicit computation of simplicial and cosimplicial algebras and perhaps more

Shizhang Li

Description of the project. The primary goal of this project is to make some explicit computations of the symmetric simplicial and cosimplicial algebra generated freely by a free module sitting in one degree. You probably don't understand these words, but don't worry, you will learn what these mean within two weeks. The procedure of making these computations are straightforward and concrete. In any case, you will be trying to find the dimension of a concretely defined matrix (albeit a complicated definition). For small degrees, one can do this computation by hand; for large degrees (say up to three hundred), one can use computers to make a program and patiently wait for the answer.

If some students are interested, we can learn together how this problem shows up in mathematical research. The story goes back to a classical paper of H. Cartan in which he computed the homology groups of some Eilenberg–MacLane spaces, and it is a fact that his computation is essentially about the homology of the simplicial algebra referred to in the previous paragraph. We can try to understand what statement his paper is proving, and see if the computation either by hand or by computer program matches with Cartan's prediction in the paper. There are also more recent research papers about this question, from both theoretical and computational angle, searching for these papers and write a survey can also be a bonus project.

Out of this, I think students will learn some theory of simplicial and cosimplicial algebras, and perhaps even some algebraic topology. On the other hand, they shall acquaint themselves with some programming.

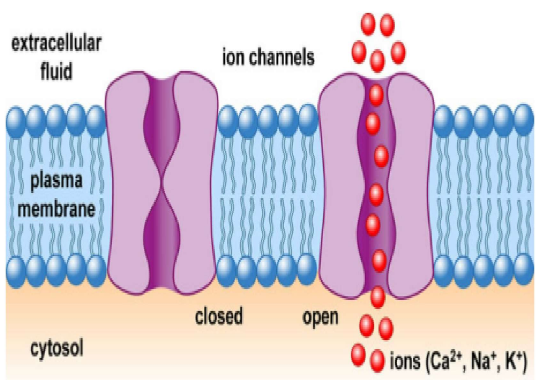
Prerequisites. Math 217. Some coding experience is strongly preferred. Having some knowledge of algebraic topology would be great, but is not necessary. Most importantly, students shall be willing to learn new concepts and struggle with it, as this is inevitable in any kind of research.

Ion channel mesh generation project

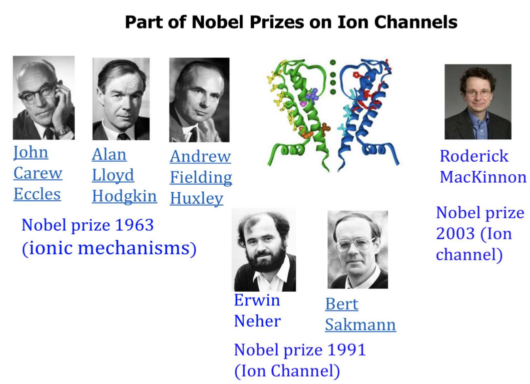
Zhen Chao
Department of Mathematics, University of Michigan

1 Motivation

We know that all living things are composed of cells, although cells have differences in structure and function, all of them have a cell membrane. As the outside of our skin separates our body from the environment, the biological cell membrane separates the contents of the cell from its exterior environment. It is the protective barrier that prevents unwanted material from passing in and has lots of functions, but one important function is to transport important materials into the cell to support necessary life functions. The cell membrane is made up of a double adjacent layer of lipids, which forms the basis of the cell membrane. There are many different proteins embedded within the membrane that have various functions and structures. One important type of these proteins is the pore-forming membrane protein, because these proteins are concerned with ion transport, they are referred to as ion channels, see Figure 2a.



(a) Ion channels are proteins with a hole down their middle, picture from Google.



(b) Nobel Prizes on Ion Channels

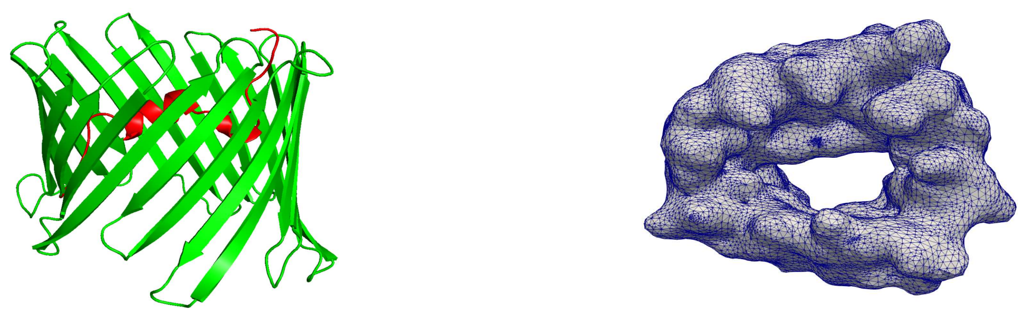
Figure 1: Ion Channels.

Ion channels play essential roles in the pathophysiology of various diseases. The study of ion channels has been for several decades and great progress has been made in understanding functions and structures of them, but there still exists lots of key open questions and basic issues. Over the past few decades, ion channel methodological progresses in X-ray crystallography and electron microscopy have led to tremendous progress in determining the structure of ion channels. This high resolution structural information can help us to better understand and capture some important functions. However, some microscopic activities that happen as ions pass through a channel – e.g., conformational changes in the protein, solvation/desolvation of ions along the journey – are almost impossible to obtain by experimental approaches. It is also impossible to detect by X-ray crystallography if the electron density is too small. Fortunately, they can be obtained by computational or theoretical approaches, which can help address some of the shortcomings of experimental methods.

2 Method

To solve an ion channel model by the finite element method, we need an ion channel mesh generation package to generate an interface fitted unstructured tetrahedral mesh for a box simulation domain. Actually, how to construct a high quality mesh for the ion channel simulation is one of the important and challenge topic in this field due to the irregular shape of proteins, see Figure 2. Due to different boundary conditions need to be enforced on different parts of the boundary, and different equations are defined on different regions. To handle them, we need to mark all the triangles of a triangular mesh on the interfaces/boundaries and all the tetrahedra of a mesh of the box domain in different labels. The low quality mesh may significantly affect the accuracy of a finite element solution. Thus how to develop an efficient and robust mesh generation packages and tools is very important for ion channel simulations. Up to now, there is only one ion channel mesh generation is available in open domain, but does not work well for complicated ion channels.

Our mesh generation algorithm primarily includes five parts: 1) A protein surface mesh generation; 2) a surface mesh of the box domain generation; 3) the corresponding volume mesh generation; 4) the membrane region construction



(a) mVDAC1 ion channel   (b) Surface mesh of mVDAC1

Figure 2: Ion Channels.

3 Goals

In this project, we will focus on the following topics:

(1) I have developed one package for on channel mesh generation but the efficient is not good, especially for large proteins, we need to improve the efficiency of it, this is main step of this project.

(2) How to improve the current protein surface mesh generation package, such as TMSmesh.

(3) If we have more time, we can compare our package with others, then apply our mesh to some ion channel model, such as Poisson-Nernst-Planck Equation.