LAB OF GEOMETRY AT MICHIGAN

PROJECT DESCRIPTIONS

Contents

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Curves in surfaces and mapping class group.

Faculty advisors: Jun Li and Becca Winarski

Grad student mentor: Bradley Zykoski

Project description. The main goal of the project is to understand the construction of mapping classes of surfaces coming from curves, provide visualization, and produce new constructions.

We will begin with reviewing the geometry and topology of surfaces, and learn some basic properties (include the important Nielson-Thurston classification and pseudo-Anosov maps) of mapping class group, which describes the symmetry of the surfaces. We will also study how to construct the elements of the latter from the former. Questions we are going to explore are:

(1) What kinds of important structures (well focus on lamination and train tracks) about curves are used in the geometry/topology of surface, and why are they interesting? Well learn some of Thurstons seminal work.
(2) Whats interesting and significant about pseudo-Anosov maps?
(3) What are the existing constructions using structures in Question 1 to construct pseudo-Anosov maps in Question 2, and how to visualize them? In particular, well focus on Penners construction.
(4) It was recently shown that Penners construction does not cover all pseudo-Anosov maps. We will learn about whats known about pseudo-Anosov maps that are not coming from Penners construction. An interesting open problem is to find new constructions for these maps using laminations and train tracks.

Prerequisites. Math 351, 451 or equivalent is required. Recommended background includes either Math 433 (Intro to Differential Geometry) or Math 490/590 or equivalent (Intro to Topology) or programming experience with one of python/C/C++/Java/Matlab.

Growth rates of tent maps.

Faculty advisor: Harry Bray

Grad student mentor: Yiwang Chen

Project description. A tent map is a piecewise linear function from the unit interval to itself with slope of $+k$ on the interval $[0, 1/k]$ and then $-k$ on the interval $[1/k, 1]$ for some k between 1 and 2, so that the graph of the function looks like a tent. Naming this function f_k, we get a family of continuous maps from the interval to itself.

We study the dynamics of such a map by iterating, or applying it to the unit interval repeatedly. Then a tent map f_k is called superattracting if there is some iterate of f_k which fixes the point 1. It is a theorem that the slope of a superattracting tent map is the root of a polynomial with integral coefficients, and plotting the roots in the complex plane of (the minimal degree factor) of these polynomials over all f_k produces a mysterious, fractal-like figure.

In this figure, it is clear that roots are not distributed uniformly. One goal of this project is to better understand how slopes k distribute on the unit interval as the number of iterates of f_k needed to fix 1 increases. For instance, are the lower density points in the 3D figure of roots coming from a lower density of slopes? With this exploration, can we determine a more efficient way to draw these pictures?

Another possible exploration is about slopes which do not determine superattracting tent maps. For such slopes, what is the dynamics of the orbit of 1? Does it always have dense orbit, or can it have an orbit which accumulates in a certain region? Can we describe a set of such slopes that are characterized by interesting dynamical behavior?

Prerequisites. Math 217 or equivalent proof-writing experience.

The secrets of the Tracy-Widom distribution.

Faculty advisors: Guilherme Silva and Joern Zimmerling

Grad student mentor: Yuchen Liao

Project description. You and your friend are sightseeing in New York, walking from Columbus circle to the Brooklyn bridge. Your friend has a route planned that takes them along all interesting sights. You, on the other hand, dont want to be seen near a map and instead take arbitrary turns leading south and east, trying to see as many sights as possible. You dont see all the sights but arrive earlier to the Brooklyn Bridge. What is the chance that you wait for your friend for a short time? The Tracy-Widom (TW) distribution will tell you.

This naive example is one of many modern problems in mathematical physics, number theory, statistics, random networks, data science, which are deeply connected to the TW distribution. However, unlike in the Gaussian case that gives rise to the bell-shaped curve, visualizing the graph of the TW distribution is a formidable task. Luckily enough, there are now many different ways to express the TW distribution in mathematical terms. Perhaps the most prominent ones are 1. expressing the TW in terms of a solution to a nonlinear differential equation, 2. calculating the TW using a large determinant expression and 3. extracting the TW from a random model.

The goal of this project is to explore these three different mathematical venues from the numerical perspective. With numerical experiments we want to explore several aspects of the TW, compare the different methods of computing a TW and visualize them to gain intuition. Time permitting, novel related distributions will also be explored under the same perspective.

Prerequisites. linear algebra and ODEs (Math 217 and Math 216/316 or equivalent), and coding skills (EECS 280 or knowledge in a high level programming language). Knowledge in probability gives a better intuition on the models, but is not required, and will be acquired along the way. This is a programming-oriented project, but some proof-based mathematical maturity is a plus, as the concepts involved in the computations are described in mathematical terms.