RESEARCH PROJECTS

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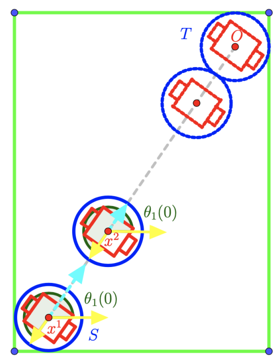
This research topic involves developing the method of *finite difference (discrete) approximations* for optimization problems, implementing numerical algorithms, and applying them, especially, to the calculations in machine learning, deep reinforcement learning, artificial intelligence, and their applications. Below, I briefly discuss some of the main results and sketch some ideas for the future research related to mobile robot models.

1 Optimization of Controlled Mobile Robot Model with Obstacles

Our approach is based on developing the method of discrete approximations. We derive the necessary optimality conditions of the discrete Euler-Lagrange type and the *numerical algorithms* to compute the optimal solution to the optimization and optimal control problems in robotics. Our further research goals concerning this model include developing efficient numerical algorithms (and coding in Python and Matlab) to solve the optimal control problems for them with large numbers of robotics in the corresponding models. It could be done, in particular, by using an appropriate discretization and employing numerical algorithms of finite-dimensional optimization to the discrete-time problems obtained in this way.



In this project we formulate and investigate an *optimal control* version of the *mobile robot model* with obstacles which dynamics is described as a sweeping process. We formulate the *sweeping optimal control problem* of type (P) that can be treated as a continuous-time counterpart of the discrete algorithm of the controlled mobile robot model by taking into account the model goal stated above. Consider the cost functional

$$\text{minimize} \quad J[x,u] := \frac{1}{2}\big\|x(T)\big\|^2, \tag{1.1}$$

which reflects model goal to *minimize the distance* of the robot from the admissible configuration set to the target. We describe the continuous-time dynamics by the controlled sweeping process and the dynamic noncollision condition $\|x^i(t) - x^j(t)\| \geq 2R$ amounts to the pointwise state constraints $x(t) \in C \iff \langle x^j_*, x(t)\rangle \leq c_j$ for all $t \in [0,T]$ and $j = 1,\ldots,n-1$, due to the construction of C and the normal cone definition. Next, applying the necessary optimality conditions for the sweeping controlled robot model allows us to obtain the optimal solution for this model.

We derive a new *discrete approximation method* to obtain the necessary optimality conditions for optimal control problem for sweeping processes and new *numerical algorithms* to find the optimal solution using the obtained conditions. We implemented the code for the robot model in the case n is a large number using the necessary optimality conditions that we obtained.

2 Undergraduate Research Projects

Students can implement the code for the robot model in the case n is a large number using the necessary optimality conditions that we obtained. We propose to use the *discrete approximation method* to approximate solution of the optimal control problems. Students can try to construct a numerical scheme for some well-known robotic models. It is also worth comparing the rate of convergence between the new method and the traditional one.

Such those projects can benefit the students in multiple ways. First, they are introduced new areas related to optimization and optimal control. Second, some well-known models help them to engage mathematics in real-world issues. Finally, it would also give the student an opportunity to gain a new skill by learning a programming language such as Matlab and Python to implement the numerical methods needed to verify the approximation's accuracy.

3 Prerequisites

- Coding skills in Matlab or Python.
- Courses or Tests:
  - Calculus classes: I, II, III, IV/ Minimum Grade of B/ May not be taken concurrently.
  - Linear Algebra/ Minimum Grade of B/ May not be taken concurrently.
  - Real Analysis/ Functional Analysis (Optional).