Modes of infinitely long strings

Jörn Zimmerling

Project Description: The oscillations of a string with variable density that is clamped on two ends is described by a differential equation well described by regular Sturm-Liouville theory. The differential operator of such a problem has real eigenvalues and eigenfunctions that are orthogonal and complete.

An infinitely long string no longer fits this theory, however, in certain cases the differential operator has complex eigenvalues with eigenfunctions "orthogonal" in a (non-positive definite) bilinear form. In the literature such eigenfunctions are known as quasi-normal modes.

In this project we try to expand solutions to the infinite string equations using these quasi-normal modes. We will study the connections between differential operators and their discretizations. The main goal is to develop and implement an algorithm that computes the solution to an infinite string with variable density on a bounded interval excited by an external force.

Prerequisites:
- Some form of Differential equations and Linear algebra
- Knowing what a complex number is
- Some form of programming
- (numerical methods for ODEs) is a big plus but we will go through everything in this project.