Explicit computation of simplicial and cosimplicial algebras and perhaps more

Shizhang Li

Description of the project. The primary goal of this project is to make some explicit computations of the symmetric simplicial and cosimplicial algebra generated freely by a free module sitting in one degree. You probably don't understand these words, but don't worry, you will learn what these mean within two weeks. The procedure of making these computations are straightforward and concrete. In any case, you will be trying to find the dimension of a concretely defined matrix (albeit a complicated definition). For small degrees, one can do this computation by hand; for large degrees (say up to three hundred), one can use computers to make a program and patiently wait for the answer.

If some students are interested, we can learn together how this problem shows up in mathematical research. The story goes back to a classical paper of H. Cartan in which he computed the homology groups of some Eilenberg–MacLane spaces, and it is a fact that his computation is essentially about the homology of the simplicial algebra referred to in the previous paragraph. We can try to understand what statement his paper is proving, and see if the computation either by hand or by computer program matches with Cartan's prediction in the paper. There are also more recent research papers about this question, from both theoretical and computational angle, searching for these papers and write a survey can also be a bonus project.

Out of this, I think students will learn some theory of simplicial and cosimplicial algebras, and perhaps even some algebraic topology. On the other hand, they shall acquaint themselves with some programming.

Prerequisites. Math 217. Some coding experience is strongly preferred. Having some knowledge of algebraic topology would be great, but is not necessary. Most importantly, students shall be willing to learn new concepts and struggle with it, as this is inevitable in any kind of research.