Winter 2016 Graduate Course Descriptions

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501 AIM Student Seminar Alben F 12:00-1pm & 3:00-4pm

Prerequisites: *You must be a graduate student in the AIM program to register for this course.*

Course Description: The Applied and Interdisciplinary Mathematics (AIM) student seminar is an introductory and survey course in the methods and applications of modern mathematics in the natural, social, and engineering sciences. Students will attend the weekly AIM Research Seminar where topics of current interest are presented by active researchers (both from U-M and from elsewhere). The other central aspect of the course will be a seminar to prepare students with appropriate introductory background material. The seminar will also focus on effective communication methods for interdisciplinary research. MATH 501 is primarily intended for graduate students in the Applied & Interdisciplinary Mathematics M.S. and Ph.D. programs. It is also intended for mathematically curious graduate students from other areas. Qualified undergraduates are welcome to elect the course with the instructor's permission.

Student attendance and participation at all seminar sessions is required. Students will develop and make a short presentation on some aspect of applied and interdisciplinary mathematics.

Text: None

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506 Analysis for Finance Nadtochiy TTh 8:30-10:00am.

Course Description: Stochastic Analysis for Finance --- The aim of this course is to teach the probabilistic techniques and concepts from the theory of continuous time stochastic processes required to understand the widely used financial models. In particular, such concepts as Brownian motion, martingales, stochastic integration/calculus, stochastic differential equations, as well as the associated mathematical tools (e.g the Feynman-Kac formula), are discussed in this course. The theory is illustrated with examples from asset pricing, optimal investment and risk management.

Text: Stochastic Calculus for Finance II, by Steven Shreve, First Edition, 9781441923110(Required)

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521 Life Contingencies II Moore TTh 8:30-10am & 10-11:30am

Prerequisites: *Math 520 or permission of the instructor.*

Course Description: Background and Goals: Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. This course extends the single decrement and single life ideas of Math 520 to multi-decrement and multiple-life applications related to life insurance.

We build on the pricing material from Life Contingencies I by (1) determining on-going reserves (money that the insurance company sets aside to pay future claims), (2) modeling insurance products with payment depending on the mode of decrement (how the individual died), (3) pricing products with payment depending on the life status of more than one person, and (4) including expenses and other business considerations in the price and reserves. This corresponds to chapters 7-11 and 15 of Bowers et al.

Text: Actuarial Mathematics, Second Edition, Bowers et al. Publisher: Society of Actuaries; 2nd edition (May 1997) ISBN-10: 0938959468, ISBN-13: 978-0938959465

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524 Loss Models II Marker TTh 11:30am-1pm & 1-2:30pm

Prerequisites: *Math 523 and Stats 426, each with a grade of C- or better.*

Course Description: Risk management and modeling of financial losses. Frequentist and Bayesian estimation of probability distributions, model selection, credibility, and other topics in casualty insurance.

Text: "Loss Models: From Data to Decisions" by Stuart A. Klugman, Harry H. Panjer, and Gordon E. Willmot, fourth edition, published by John Wiley in 2012.

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525 Probability Theory Nishry TTh 10:00-11:30a TTh 1:00-2:30pm

Prerequisites: *MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful*

Course Description: The aim of this course is to provide the mathematical basics of probability theory and some useful results on the subject. Some applications will be given through examples and exercises. There is a substantial overlap with MATH 425 (Intro. to Probability), but here more sophisticated mathematical tools are used and there is greater emphasis on proofs of major results. Topics covered include probability spaces, discrete and continuous random variables, joint distributions and conditional expectations, characteristic functions, limit theorems, and some more advanced topics (this might include branching processes, Markov processes or other topics).

Text: G. Grimmett and D. Stirzaker, Probability and Random Processes, Third edition, Oxford,2001 ISBN: 9780198572220 (required)

S.M. Ross, Introduction to Probability Models, Tenth Edition, Academic Press ISBN:9780123756862 (optional)

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526 Discrete Stochastic Processes

Angoshtari TTh 8:30-10am
TTH 10-11:30am
Blass TTH 11:30-1pm

Prerequisites: *Required: MATH 525 or STATS 525 or EECS 501*

Course Description: The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales). Significant applications will be an important feature of the course.

Text: Required: Essentials of Stochastic Processes, 2nd ed. (Durrett). Optional: Introduction to Stochastic Processes (Cinlar), Stochastic Processes (Ross), Probability and Measure (Billingsley).
ISBN : 9781461436140

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537 Introduction to Differentiable Manifolds Uribe TTh 2:30-4:00pm

Course Description: The syllabus of 537 is brand new this term. The main part of the course will cover (finite dimensional) Morse theory. A Morse function is one whose critical points are non-degenerate. Morse theory is about the interplay between the critical points of Morse functions on a manifold and the topology of the manifold. Thanks to the existence of a normal form for such functions near a critical point, one can understand the change in the topology of the sub-level sets {f <c} as the level c crosses a critical value. (The topology changes by attaching a handle). This leads to the Morse inequalities, that relate the critical point data of a Morse function and the Betti numbers of the manifold. After developing the basic theory, we will spend some time studying applications (the Lefschetz hyperplane theorem, and perhaps the Morse theory of hamiltonian circle actions on symplectic manifolds). Time permitting we will explore another topic: charateristic classes of vector bundles. These are classes in the cohomology of the base of vector bundles, and are topological invariants that arise in many settings that involve questions of existence or classification of vector bundles. The Chern-Weil approach, using de Rham cohomology, will be emphasized. A theme unifying both subjects is their local-to-global aspect: Local, continuous data have global implications.

There will be homework assignments and students will be asked to give one or two presentations during the semester. The listed textbook reference is available online through the UM Science Library. The course should be accessible to students completing 591 this fall; other background material will be provided as needed.

Text: (Optional) An Invitation to Morse Theory, Liviu, Nicolauscu.

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555 Applied Complex Analysis Viswanath MW 8:30-10:00am

Prerequisites: *Math 451 or equivalent (Advisory)*

Course Description: The following topics will be introduced: complex numbers, elementary functions (fractional linear transformations, exponential and trigonometric functions) in the complex plane, Cauchy-Riemann equations, complex integration, power series, Laurent expansion, residue integration, argument principle, Schwarz reflection principle, and conformal maps. Applications will include aerofoil theory and basic properties of Chebyshev polynomials.

There is no textbook for the class. However, students will receive a list of half a dozen books, any of which they may use, during the first lecture.

Text: None

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557 Applied Asymptotic Analysis Borcea TTh 1:00-2:30pm

Prerequisites: *This course will assume a strong background in differential equations, linear algebra, and advanced calculus or real analysis. Even more important is technical skill in complex variables and analysis at the level of Math 555.*

Course Description: Applied Asymptotic Analysis --- Topics include: asymptotic sequences and (divergent) series; asymptotic expansions of integrals and Laplace's method; methods of steepest descents and stationary phase; asymptotic evaluation of inverse Fourier and Laplace transforms; asymptotic solutions for linear (non-constant coefficient) differential equations; WBK expansions; singular perturbation theory; and boundary, initial, and internal layers.

Text: P. D. Miller, Applied Asymptotic Analysis, Graduate Studies in Mathematics, volume 75, American Mathematical Society, Providence, RI, 2006. ISBN 0-8218-4078-9.

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563 Advanced Mathematical Biology: Math,Music & the Brain — Forger — MWF 12-1:00pm

***Prerequisites**: Basic knowledge of partial differential equations and linear algebra. Some familiarity with computer programming (e.g. the basics of matlab) and/or music notation is helpful.*

Course Description: How can our appreciation and performance of music be enhanced by understanding mathematics and basic principles of how the brain works? By studying music, can we learn about new mathematics and principles of signal processing in the brain? Can principles of music theory be deduced by analysis of the works by master composers? The connections between mathematics and music have been known for thousands of years. Yet, recent advances in technology, computational neuroscience and "big data" have provided new answers to these questions.

In this course, we study mathematical models for how the brain processes sound, and mathematical techniques for analyzing music performance. Although examples will be presented from ancient to contemporary music, we will focus on analyzing Bach's Trio Sonatas throughout the semester. Group work and original research will be encouraged. The class will meet alternately between a computer lab and a performance venue where live performances will regularly be included in class lectures.

Text: None

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566 Combinatorial Theory: Algebraic Combinatorics — Fomin — TTh 1:00-2:30pm

***Prerequisites:** Math 217 or above*

Course Description: This course is an introduction to algebraic and enumerative combinatorics at the beginning graduate level. Topics include: fundamentals of algebraic graph theory; applications of linear algebra to enumeration of matchings, tilings, and spanning trees; combinatorics of electric networks; partially ordered sets; integer partitions and Young tableaux.

Text: (Optional) R. P. Stanley, Algebraic Combinatorics: Walks, Trees, Tableaux, and More, Springer, 2013. The text of this book (without exercises) is available online at <http://www-math.mit.edu/~rstan/algcomb/>.

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567 Introduction to Coding Theory Gelaki TTh 1:00-2:30pm

Prerequisites: *Math 217, 417, 419 or 513. Some knowledge about probability and abstract algebra is useful, but these things will be reviewed as they show up.*

Course Description: Coding Theory is an area of mathematics that has important applications to digital communication (like satellite communication, CD players). The goal of coding theory is to encode a digital message such that most of the lost information can be retrieved after the message has been sent over a noisy channel. Coding Theory was developed after Shannon proved some important results about digital communication.
Some of the topics that we will discuss are: entropy (a measure for information), Huffman codes (for data compression), channels and channel capacity, Shannon's theorem, error correcting block codes, fi- nite fields and constructions of various codes (Hamming codes, Golay codes, Reed-Muller codes, cyclic codes etc.), bounds for codes, weight distributions of codes and weight enumerators.

Text: (Optional) Introduction to Coding, Steven Roman. ISBN:0387947043

(Optional) Coding and Information Theory, Steven Roman. ISBN:3540978127

(Optional) Introduction to Coding Theory, J.H. van Lint, 3rd Edition. ISBN:3540641335

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571 Numerical Linear Algebra Krasny TTh 10:00-11:30am

Prerequisites: *Math 217, 417, 419, or equivalent.*

Course Description: Math 571 is an introduction to numerical linear algebra, a core subject in scientific computing. Three types of problems are considered: (1) linear systems, (2) eigenvalues and eigenvectors, and (3) least squares problems. These problems arise in many scientific applications and
we'll study the accuracy, efficiency, and stability of the methods that have been developed for their solution. As an application, we'll consider finite-difference schemes for boundary value problems in 1D and 2D. The course grade will be determined by homework sets and two exams.

Text: Numerical Linear Algebra by Trefethen and Bau. ISBN: 9780898713619

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572 Numerical Methods for Scientific Computing II Veerapaneni TTh 11:30-1pm

Prerequisites: *Solid background in advanced calculus, linear algebra and working knowledge of a computing programming language (such as C, C++ or Fortran) or a computing environment (such as Matlab or Python).*

Course Description: Math 572 is an introduction to numerical methods for boundary value and initial value problems. It covers methods for ordinary and linear elliptic, parabolic and hyperbolic partial differential equations. The course will focus on the derivation of methods, on their accuracy,

stability and convergence properties, as well as on practical aspects of their efficient implementation. This course should be very useful for students in applied and computational mathematics, and in any area of scientific computing and engineering. Students should have solid background in advanced calculus and linear algebra, and are expected to have experience with programming in Fortran/C/Matlab or others. Homework will be given regularly and will include programming assignments.

Text: (Optional) Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems by R.J. LeVeque, SIAM, 2007.

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574 Financial Math I Muhle-Karbe TTh 1:00-2:30pm

Prerequisites: *MATH 573*

Course Description: This is a continuation of Math 573. This course discusses Mathematical Theory of Continuous-time Finance. The course starts with the general Theory of Asset Pricing and Hedging in continuous time and then proceeds to specific problems of Mathematical Modeling in Continuous-time Finance. These problems include pricing and hedging of (basic and exotic) Derivatives in Equity, Foreign Exchange, Fixed Income and Credit Risk markets. In addition, this course discusses Optimal Investment in Continuous time (Merton's problem), High-frequency Trading (Optimal Execution), and Risk Management (e.g. Credit Value Adjustment).

Text: (Required) Arbitrage Theory in Continuous Time, Bjork, Thomas, 3rd edition.
ISBN: 9780199574742

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575 Theory of Numbers I Lagarias TTh 10:00-11:30pm

Prerequisites: *Some familiarity with writing proofs, at the level of Math 451, and some familiarity with groups, rings, fields to level of Math 412.*

Course Description: This is a first course in number theory-sometimes called the higher arithmetic. The theory of numbers
is unrivaled for the number and variety of its results and for the beauty
and wealth of its demonstrations. Topics covered will include
divisibility and prime numbers, factorization and primality testing, congruences, quadratic reciprocity, quadratic forms, arithmetic functions and distribution of primes. Other topics will be covered as time permits or by request, e.g. continued fractions, partitions.

Text: (Required)**:** An Introduction to the Theory fo Numbers, Niven, Zuckerman, Montgomery, 5th Edition.

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582 Introduction to Set Theory Fernandez Breton MWF 12:00-1pm

Prerequisites: *The official prerequisite, "Math 412 or 451 or equivalent experience with abstract mathematics," means that students should be comfortable with writing mathematical proofs.*

Course Description: This is an introductory course in axiomatic set theory, whose main objective is to convince the student that most everyday mathematical objects can be conceived as sets of some sort.

Topics include:

* The intuitive concept of set; paradoxes.
* Type theory and the cumulative hierarchy of sets.
* The Zermelo-Fraenkel axioms for set theory.
* Set-theoretic representation of the fundamental concepts of mathematics (e.g., function, number) and proofs of basic properties of these concepts (e.g., mathematical induction).
* Infinite cardinal and ordinal numbers and their arithmetic.
* The axiom of choice and equivalent axioms (e.g., Zorn's Lemma).

Additional topics may be discussed if time permits.

No specific previous knowledge of set theory will be presupposed.

Text: "Elements of Set Theory" by H. B. Enderton ISBN: 9780122384400 (Required)

"Discovering modern set theory. I, The basics" by W. Just and M. Wiese, ISBN 9780821802663. There will be some assigned readings from this book, whose electronic version is available through the U of M library.

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590 Intro to Topology Vlamis MWF 12:00-1:00pm

Prerequisites: *Math 451*

Course Description: This is a course on point-set topology, which emphasizes the set-theoretic aspects of topology. The course will focus on the notions of continuity, connectedness, compactness, and other topics. The class is quite theoretical and requires extensive construction of proofs.

Text: (Required) Topology (2nd Edition), by James Munkres. ISBN:131816292

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592 An introduction to algebraic topology Spatzier MWF 10:00-11am

Prerequisites: *Previous exposure to point-set topology and familiarity with abstract algebra will be assumed.*

Course Description: This is a first course in algebraic topology. We will cover the fundamental group and covering spaces, singular homology theory and some additional topics (e.g. beginnings of cohomology and/or applications) as time permits.

Text: No text is required, but the following are recommended reading:

(1) "Algebraic topology" by Allen Hatcher. ISBN:9780521795401
(2) "A concise course in algebraic topology" by Peter May. ISBN:9780226511832
(3) "Elements of Algebraic Topology" by James R. Munkres. ISBN:9780201162782

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594 Algebra II: Groups and their Actions Zieve MW 2:30-4:00pm

Prerequisites: *Admission to the Math PhD program OR Math 493-494.*

Course Description: Graduate level algebra, focusing on group actions on sets, and including group theory, field theory, Galois theory, and representation theory.

Text: (Optional) Algebra, Serge Lang, 3rd edition.

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597 Analysis II (Real Analysis) Baik WF 1:00pm-2:30pm

Prerequisites: *Math 451*

Course Description: MATH 596 (Complex analysis) and MATH 597 (Real analysis) are two core courses on analysis for students beginning study towards the PhD degree is mathematics. These courses can be taken more or less independently of each other. The topics discussed in MATH 597 include: measure theory on the real line, abstract measure space, integration theory, Lp space, basics of Hilbert space and Banach space, and an introduction to Fourier analysis.

Text: T. Tao, An Introduction to Measure Theory, T. Tao. (Optional) ISBN:9780821869192

An Epsilon of Room, 1: Real Analysis, Terence Tao (Optional) ISBN:9780821852781

Real Ananlysis, Gerald Folland, ISBN:9780471314166

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615 Topics in Commutative Algebra Hochster MWF 2:00-3:00pm

Prerequisites: *Math 614 and Math 631, at least concurrently. Other prerequisites needed briefly for some segments will be presented seminar style. In class I will assume familiarity with Tor and Ext, but I will give some supplementary lectures in a time slot to be determined introducing these functors for those unfamiliar with them.*

Course Description: Topics will include the theory of Groebner bases, Cohen-Macaulay, Gorenstein, and regular local rings, Grothendieck groups, the use of Tor, Ext and other homological methods, and an introduction to the technique of reduction to characteristic p. Lecture notes will be provided.

Text: There is no textbook. I will provide written materials on the course Web site.

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626 High Dimensional Probability Vershynin TTh 10am-11:30am

Prerequisites: *A graduate probablitity course*

Course Description: This is a course in high-dimensional probability theory and its applications. The goal is to expose graduate students to basic probabilistic methods and ideas that go beyond standard probability curriculum and are essential for research in modern mathematics and other quantitative areas.

The course will cover basic concentration inequalities (for sums of independent random variables, vectors, matrices, Lipschitz functions, and martingales), geometric and combinatorial theory of stochastic processes (Gaussian processes via metric entropy: Sleipan, Dudley, Sudakov inequalities; empirical processes via Vapnik-Chervonenkis dimension), and various applications in high-dimensional settings (examples include dimension reduction, compressed sensing, random matrices, networks, covariance estimation, regression).

Text: None

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632 Algebraic Geometry II: Shemes, Coherent sheaves, and their Cohomology

Foster MWF 3:00-4:00pm

Prerequisites: *MATH 614 (Commutative algebra), MATH 631 (Algebraic geometry I)*

Course Description: This course will cover the foundations of modern algebraic geometry, with a focus on schemes and their morphisms, relative properties of schemes, and coherent sheaves and their cohomology.

Text: (Required) The Rising Sea: Foundations of Algebraic Geometry, online notes at http://math.stanford.edu/~vakil/216blog/

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635 Differential Geometry Van Limbeek MWF 12:00-1:00pm

Prerequisites: *Differential topology (basic theory of manifolds): Math 537 or 591 or equivalent*

Course Description: This course is an introduction to differential geometry with emphasis on Riemannian geometry. We will discuss the basic ideas of connections, Riemannian metrics, curvature and the basic tools in the subject, especially variational methods, Jacobi fields, and comparison theorems. After those more local ideas, we will turn to global differential geometry which relates geometric ideas to the underlying topology. Examples are the Cartan-Hadamard Theorem, sphere theorems, structural results both in positive and negative curvature, and rigidity theorems. If time permits we will pursue more advanced topics.

Text: Do Carmo, Riemannian Geometry, Birkhauser. ISBN: 817634908, The book is optional but highly recommended.

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636 Topics in Differential Geometry: Sympletic Geometry & Integrable Systems Burns TTh 1pm-2:30pm

Prerequisites: *Basic Manifolds*

Course Description: In (conservative) classical mechanics, a distinguished role is played by the Hamiltonian systems. The current geometric formulation of the geometry underlying such systems is symplectic geometry. The most symmetric and rigid of these dynamical systems are the completely integrable systems, ones for which one can solve the equations completely because of the existence of a maximal number of (Poisson) commuting integrals. Modern guises of this are toric manifolds (toric varieties) and their various generalizations. Special as

such systems may seem, they arise widely in diverse areas of geometry, especially in algebraic geometry. We will study the basics of symplectic geometry here, and lead up to open questions relating integrable systems to “quantum” integrable systems of flag manifolds.
The general breakdown of the course will be as fiollows:
1.) Basics: definitions, Darboux’s theorem, almost complex structures, Hamiltonian fields, integrability and toric manifolds, moment map and convexity, Morse theory and the moment map. Action angle coordinates; the base as a tropical manifold.
2.) Delzant theory: Delzant polytopes and Kaehler realizations. Finiteness theorem of McDuff.
3.) Examples of integrable systems: toric manifolds, singular integrable systems, “quantum integrability” versus classical integrability, examples from moduli problems, flag varieties and affine homogeneous spaces.
4.) Special bases for homology groups. Bohr-Sommerfeld correspondence. Interpretation in classical cases, especially geometric ones.

Text: Suggested books:

Introduction to Symplectic Geometry, D.McDuff, D. Salamon.

Moment Maps and Combinatorial Invariants of Hamiltonian T^n-Spaces, V.Guillemin.

Funtion Theory on Symplectic Manifolds, L. Polterovich, D. Rosen.

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654 Introduction to Fluid Dynamics Doering MW 8:30-10am

Prerequisites: *Math 451, 454, 555 (556 recommended), elementary physics, Newtonian mechanics, elementary dynamical systems theory, differential equations, and computer literacy as expected of mathematically minded science and engineering graduate students.*

Course Description: This course introduces and analyzes the Euler and Navier-Stokes (partial differential) equations, conservation laws for mass, momentum, and energy. Topics developed include vorticity & potential flow, viscous flow & hydrodynamic instabilities, turbulence theory & modeling, and challenges to proving existence & regularity of solutions of the incompressible 3D Navier-Stokes equations.

Text: (Required) Applied Analysis of the Navier-Strokes Equations, by Charles R. Doering & J.D. Gibbon, 1st Edition revised printing 2004, ISBN: 052144568X.

(Optional) A Mathematical Introduction to Fluid Mechanics, by A.J. Chorin & J.E. Marsden, Springer-Verlag 3rd Edition, ISBN:0387979182

(Optional) Introduction to Hydrodynamic Stability, by P.G. Drazin, Cambridge University Press, 1st Edition, ISBN:0521009650

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669 Topics in Combinatorial Theory: Combinatorial Representation Theory — Stembridge — MWF 11:00am-12:00pm

Prerequisites: *Math 594 or Equivalent*

Course Description: Our goal is to survey some of the many connections between representation theory and combinatorics. Some of the most interesting results in combinatorics have been derived by means of representation-theoretic tools. Enumeration of plane partitions, unimodality theorems, and the Rogers-Ramanujan identities are all examples of this. In the opposite direction, the symmetry groups that occur most frequently in nature (the symmetric groups, the classical groups) have representations with intrinsic combinatorial structure.

The course will be divided into three unequal parts.

Part 0 will consist of a self-contained development of the representation theory of finite groups in characteristic 0, and a less self-contained discussion of compact groups.

Part 1 will be a detailed study of the representation theory of the symmetric groups and closely related groups, and the applications thereof. If there is time, we hope to discuss the W-graphs of Kazhdan-Lusztig theory.

Part 2 will be concerned with a combinatorial approach to the representations of GL(n) and related groups (SL(n), U(n), etc).

We will not follow any text, but "Representation Theory: A First Course" by W. Fulton and J. Harris (Springer-Verlag, 1991) is a good companion for the representation theory side. The combinatorial aspects will be cobbled together from multiple sources.

Previous exposure to combinatorics or group representations will be advantageous but not necessary.

Text: None.

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678 Modular Forms: Iwasawa Theory Snowden TTh 2:30-4pm

Prerequisites: *Algebaic Number Theory*

Course Description: Iwasawa theory studies the behavior of objects of arithmetic interest in towers of number fields. The simplest case, and the focus of this course, is the study of ideal class groups in Z_p-extensions (especially the cyclotomic Z_p-extension). We will develop the basic theory and prove the so-called Main Conjecture of Iwasawa theory. Nothing beyond basic algebra and algebraic number theory will be assumed.

Text: None

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684 Recursion Theory Blass TTh 10:00-11:30am

Course Description: Recursion theory, also called computability theory, is the study of the extent to which functions F and relations R are computable. Is there an algorithm which, given as input any element x of the domain of F, produces after finitely many steps the output F(x)? In the case of relations R, is there an algorithm which, given input x, decides in finitely many steps whether or not R holds of x? In some important cases, we do not have computability in this sense but only semi-computability, meaning that there is an algorithm that will produce the correct result when R(x) holds but will compute forever with no result when R(x) fails. These and other variants of computability will be rigorously defined and studied in this course. A closely related topic is relative computability: Could F (or R) be computed by an algorithm that has black-box access to information about another function G? The study of such questions has led to deep combinatorial methods, including the so-called priority method, whose simplest incarnation (finite injury priority arguments) will be covered in this course.

In addition to the general study of computability, the course will treat some of the most important applications of computability in mathematical logic. In particular, I will cover the incompleteness theorems of Gödel, which imply in particular that one cannot deduce all mathematical truths, or even all truths of the arithmetic of natural numbers, from any computable set of true axioms. They also imply that, under reasonable assumptions, a consistent theory cannot prove its own consistency.

The course will not presuppose any prior knowledge of mathematical logic, provided you are willing to believe some facts that I'll state without proof. (Alternatively, those facts are covered in each of Math 581 and 681.) The only real prerequisite for Math 684 is mathematical maturity appropriate for a 600-level mathematics course.

Text: (Optional) Fundamentals of Mathematical Logic, Peter G. Hinman**.**

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697 Topic in Topology: Teichmuller space vs. Symmetric Space Ji
MWF 11:00-12pm

Course Description: The upper halfplane H^2 is a basic space and can be interpreted and generalized
in different ways. For example, it can be written as the quotient space SL(2, R)/SO(2) and
is the simplest symmetric space of noncompact type. It is also the Teichmuller space
of Riemann surfaces of genus 1.

The modular group SL(2, Z) is a discrete subgroup of the Lie group SL(2, R)
and acts properly on H^2. The quotient space SL(2, Z) \ H^2 is a locally symmetric space
and is also the moduli space M_1 of Riemann surface of genus 1.

This action of SL(2, Z) can be generalized in two directions: (1) discrete subgroups of Lie groups such as SL(n, Z), n >2, and their actions on symmetric spaces,
(2) mapping class groups of surfaces and their actions on Teichmuller spaces.

In this course, we will start with basic properties of the action of SL(2, Z) on H^2,
study it in detail from the perspectives of geometry, geometric topology, algebraic topology and analysis, examine what can be generalized
in the above two situations, and then study and understand similarities and differences between three pairs:
(a) Teichmuller spaces vs. symmetric spaces,
(b) mapping class groups of surfaces vs. discrete subgroups of Lie groups,
(c) moduli spaces of Riemann surfaces vs. locally symmetric spaces.

We will use a combination of papers and books as references, which include the following:

1. B. Farb, D. Margalit, A primer on mapping class groups. Princeton Mathematical Series, 49. Princeton University Press, Princeton, NJ, 2012. xiv+472 pp.
2. N. Ivanov, Mapping class groups. Handbook of geometric topology, 523--633, North-Holland, Amsterdam, 2002.

3. J. Harer, The cohomology of the moduli space of curves. Theory of moduli (Montecatini Terme, 1985), 138--221, Lecture Notes in Math., 1337, Springer, Berlin, 1988.
4. D. W. Morris, Introduction to arithmetic groups. Deductive Press, 2015. xii+475 pp.
5. L. Ji, A tale of two groups: arithmetic groups and mapping class groups. Handbook of Teichmuller theory. Volume III,
157--295,
IRMA Lect. Math. Theor. Phys., 17, Eur. Math. Soc., Zurich, 2012.

Text: none

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732 Topics in Algebraic Geometry II: Fulton TTh 11:30am- 1pm

Prerequisites: *Math 631-2, and the alpha courses in topology and algebra, should suffice as prerequisites. My book Intersection Theory should contain most omitted details.*

Course Description: Intersection theory began with Bezout, finding and counting solutions to polynomial equations It developed in the 19th century with the rise of enumerative geometry, Schubert calculus, and formulas for degeneracy loci. Foundations came in the 20th century with tools from topology, algebra, and algebraic geometry. This course will trace this history, including the rigorous foundations, with emphasis on motivations and the main geometric constructions; some proofs will be left to participants. We will also discuss some more recent variations, such as equivariant theories and algebraic cobordism.

Text: none

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775 Topics in Analytic Number Theory: Montgomery TTh 8:30- 10am

Prerequisites: *Math 675 or Equivalent Background in algebraic number theory, at the level of Math 676.*

Course Description: The following topics will be covered: 1. Vinogradov's method of prime number sums; 2. The large sieve; 3. Prime numbers in arithmetic progressions (on average); 4. Additive prime number theory; 5. Maynard's theorem and its consequences

In 2013, Yitang Zhang showed that there is a constant C such that for arbitrarily large x the

interval $[x, x+C]$ contains at least two prime numbers. Within a few months, James Maynard showed that for
and positive integer k there is a constant $C(k)$ such that there exist
arbitrarily large x such that the interval $[x, x+C(k)]$ contains at least
k primes. This theorem of Maynard has many consequences, some
of which will be explored. If time permits, we will treat also the recent
work of Maksym Radziwill and Kaira Matom\"aki concerning mean values of multiplicative functions in short intervals, and the further
development of that work with Terry Tao.

Text: (required) Multiplicative Number Theory II: Modern Developments, H. L. Montgomery & R. C. Vaughan. The text is a prepublication coursepack which will be available from
Dollar Bill Copying.