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501 AIM Student Seminar TBD F 12:00-1:00pm & 3:00-4:00pm

Prerequisites: *You must be a graduate student in the AIM program to register for this course.*

Math 501 is a required course for all students enrolled in the Applied and Interdisciplinary Mathematics (AIM) MS and PhD graduate programs. In the Winter term, all first-year AIM students from both programs must sign up for this course. Due to the highly specialized content of the course, enrollment is available only for students in an AIM degree program.

The purpose of Math 501 is to address specific issues related to the process of studying applied mathematics in the AIM program and becoming an active member of the research community. The weekly meetings of the class will be divided among three types of sessions:

1. "Focus on. . . " presentations. These are presentations on various topics, some of immediate practical significance for students and others of a further-reaching nature. These discussions will include aspects of scholarly writing, research, and career development.

2. AIM Faculty Portraits. These are short presentations by faculty members in the Mathematics Department and other partner disciplines who are potential advisors or committee members for AIM students. The AIM faculty portraits provide a direct channel for students to discover what research is being done in various areas by current faculty, and to see what kind of preparation is required for participating in such research.

3. AIM Research Seminar Warm-up talks. One of the course requirements for Math 501 is weekly attendance of the AIM Research Seminar that takes place from 3-4 PM each Friday. The warm-up talks are presentations during the regular course meeting time by particularly dynamic speakers slated to speak in the AIM Research Seminar later the same day as a way to provide background material with the goal of making the AIM Research Seminar lecture more valuable for students.

Weekly attendance both of the course meeting and also of the AIM Research Seminar is required for Math 501. If you are registering for Math 501 you must be available both during the regular class time of 12-1pm on Fridays as well as during the AIM Research Seminar which runs 3-4pm on Fridays. If you are teaching, you should keep both of these obligations in mind when you submit your class/seminar schedule prior to obtaining a teaching assignment. Other requirements, including possible assignments related to topics discussed in the lectures, will be announced by the instructor in class.

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520 Life Contingencies 1 Marker TTh 8:30-10:00am & 10:00 - 11:30am

Prerequisites: *Math 424 and 425 or permission from the instructor.*

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of this course is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. In addition to actuarial students, this course is appropriate for anyone interested in mathematical modeling outside of the physical sciences.

The main topics are the development of (1) probability distributions for the future lifetime random variable; (2) probabilistic methods for financial payments on death or survival; 6and (3) mathematical models of actuarial reserving.

TEXT: Actuarial Mathematics (Second Edition) 2nd Bowers, Gerber, Hickman, Jones and Nesbitt (Society of Actuaries), Required

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523 Risk Theory Young TTh 11:30am-1:00pm

Prerequisites: *Math 425 with a grade of C- or better*

The goals of this course are to understand parametric distributions for the purpose of (1) modeling frequency, severity, and aggregate insurance losses, (2) analyzing the effects of insurance coverage modifications, and (3) estimating future insurance losses via credibility theory.

TEXT: Loss Models: From Data to Decisions, Klugman, Panjer, and Willmot, (4th Edition), Required

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525 Probability Theory Wang TTh 8:30 - 10:00am & TTh 10:00-11:30am

Zieve TTh 1:00 – 2:30pm

This is a fairly rigorous introduction to probability theory with some emphasis given to both the ory and applications, although a knowledge of measure theory is not assumed. Topics covered are: probability spaces, discrete and continuous random variables, conditional probability, gen erating functions, Markov chains, limit theorems.

TEXT: Probability and Random Processes 3rd Geoffrey R. Grimmett and David R. Stirzaker, Required
Introduction to probability models 10th Sheldon M. Ross, Optional

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526 Stochastic Processes TBD TTh 10:00-11:30am

Prerequisites: *Required: Math 525 or basic probability theory including: probability measures, Random variables, expectations, conditional probabilities and independence.*
Recommended: Good understanding of advanced calculus covering limits, series, the notions of continuity, differentiation and integration; interchanging the limit and integration/expectation (monotone and dominated convergence theorems); linear algebra, including matrices, eigenvalues and eigenfunctions.

The covers both discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrency and transiency, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales); time-permitting, introduction to stochastic integration and Ito's formula. Significant applications will be an important feature of the course.

TEXT: Essentials of Stochastic Processes, 2nd ed. (R. Durrett) 2 R. Durrett Required
Introduction to Stochastic Processes (E. Cinlar) E. Cinlar Optional

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555 Intro to Complex Variables Miller TTh 1:00-2:30pm

Prerequisites: *Multivariable calculus (eg., Math 215 or Math 452), especially partial differentiation, path integration, and Green's Theorem.*

This course is an introduction to the theory of complex valued functions of a complex variable with the aim of preparing students to use the theory in applications in science and engineering. Concepts, calculations, and the ability to apply principles are emphasized over proofs, but arguments are rigorous. This course is a core course for the graduate program in Applied and Interdisciplinary Mathematics (AIM).

Topics covered include: differentiation and integration of complex valued functions of a complex variable, series, mappings, residues, and applications. The applications include the evaluation of improper real integrals, analysis of zeros of polynomials, ideal flow in fluid dynamics, and problems of electrostatics.

TEXT: Complex Analysis with Applications by Richard A. Silverman (First Edition, Dover Publications), ISBN-13: 978-0486647623, Required

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556 Applied Functional Analysis Schotland TTh 2:30 - 4:00pm

Prerequisites: *undergraduate analysis, linear algebra and complex variables.*

Introduction to topics in functional analysis that are used in the analysis of ordinary and partial differential equations. Metric and normed linear spaces, Banach spaces and the contraction mapping theorem, Hilbert spaces and spectral theory of compact operators, distributions and Fourier transforms, Sobolev spaces and applications to elliptic PDEs.

TEXT: Applied Analysis by Hunter and Nachtergaele, Required

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558 Applied Nonlinear Dynamics Rauch TTh 10:00-11:30am

Prerequisites: *Basic Linear Algebra, Ordinary Differential Equations (Math 216), Mul- tivariable Calculus (215). Some exposure to more advanced mathematics e.g. Advanced Calculus (Math 450/451) or Advanced Mathematical Methods (Math 454).*

Differential equations model systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the post Poincaŕ e era. The course surveys a broad range of topics with em- phasis on techniques, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program. Proofs are given. Homeworks and exam concentrate on using rather than proving.

Outline. Roughly Chapters 1-10 + 15 of Hirsh-Smale-Devaney. Plus online materials prepared to complement the text. There are more complements than we will treat.

- Phase line. Dynamics in dimension 1 and 1.5. Bifurcations. Poincaŕ e map.
- Existence, uniqueness, perturbation theory.
- Linearization at equilibria. Theory of constant coefficient systems. Spectral theorems.
- The geometry of phase plane of linear systems..
- Stable and unstable manifolds.
- Conjugation of sinks/sources.
- Lyapunov's method. LaSalle's invariance principal.
- Gradient flows and hamiltonian systems.
- Periodic solutions, stability, Poincaŕ e map, ω-limit set, Poincaŕ e-Bendixson (time per- mitting).
- Bifurcation theory of equilibria. Pitchfork and Hopf.
- Introduction to chaotic dynamics. Definitions and first examples

TEXT: Differential Equations, Dynamical Systems, and an Introduction to Chaos (3rd Edition) by M. Hirsh, S. Smale, and R. Devaney, Required

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559 Computational and Mathematical Neuroscience Booth MW 8:30- 10:00am

Prerequisites: *Math 216 and 217, or permission of instructor. Math 463 recommended.*

Computational neuroscience investigates the brain at many different levels, from single cell activity, to small local network computation, to the dynamics of large neuronal populations. As such, this course introduces students to modeling and quantitative techniques used to investigate neural activity at these different levels.

Topics to be covered include:

Passive membrane properties, the Nernst potential, derivation of the Hodgkin-Huxley model, action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, phase plane analysis, linear stability of equilibria, bifurcation analysis, synaptic currents, excitatory and inhibitory network dynamics, firing rate models, neural coding.

Readings and homework problems will be selected from a number of different texts including:

1. Foundations of Cellular Neurophysiology by D. Johnston and S.M. Wu (MIT Press, 1999).

2. Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems by P. Dayan and L. Abbott (MIT Press, 2005).

3. Biophysics of Computation by C. Koch (Oxford University Press, 1999).

4. Mathematical Foundations of Neuroscience by G.B. Ermentrout and D. Terman (Springer, 2010).

Numerical implementation and analysis of the models presented in the lectures will be an integral part of the course. MATLAB experience recommended but not required. Course requirements will include homework assignments containing a combination of analytical and numerical-based problems, a longer-term modeling project and an oral presentation of the project to the class at the end of the semester.

TEXT: No required textbook.

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565 Combinatorics and Graph Theory Blass MWF 11:00am – 12:00 PM

Prerequisites: *Linear algebra, and comfort with abstract mathematics.*

I plan to discuss a large variety of combinatorial topics, roughly grouped under three headings.

The first broad topic is graph theory. Although the concept of graph is very simple --- just a finite set of points, some of which are joined by edges --- it leads to a surprisingly rich theory. Among the topics that I plan to cover are trees, Eulerian and Hamiltonian graphs, matchings, graph coloring and the five-color theorem, and Kuratowski's characterization of planar graphs.

The second broad topic concerns techniques of enumeration. Particular topics here include binomial coefficients and Catalan numbers. I'll also discuss general techniques, especially generating functions. This topic also connects with graph theory, for example in the matrix-tree theorem.

The third broad topic concerns partially ordered sets. Of particular importance here are Möbius functions and matroids. These also have strong connections with the previous two broad topics.

TEXT: A Course in Combinatorics by Van Lint & Wilson (2nd Edition) Required

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571 Numerical Linear Algebra Gilbert TTh 8:30-10:00am

Direct and iterative methods for solving systems of linear equations (Gaussian elimination, Cholesky decomposition, Jacobi and Gauss-Seidel iteration, SOR), methods for computing eigenvalues and eigenvectors, methods for solving least squares equations and simple optimization problems, applications to modern data applications and machine learning.

TEXT: Numerical Linear Algebra, L.N. Trefethen and D. Bau, Optional

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575 Introduction to Number Theory Lagarias MWF 10:00am-11:00am

Prerequisites: *Students should be familiar with groups, rings and fields to the level of Math 412. They should be comfortable with writing proofs, at the level of Math 451. Proofs are emphasized but are often pleasantly short.*

Number theory has long been admired for its beauty and elegance, and for its rich legacy of fundamental unsolved problems in mathematics. It has recently turned out to have many applications in coding theory and cryptography. This is a first course in number theory-also called the higher arithmetic. It is faster paced than Math 475.

Topics covered will include:
divisibility and prime numbers, factorization and primality testing, congruences, public key cryptography (RSA), p-adic numbers, arithmetic functions and distribution of prime, quadratic reciprocity and binary quadratic forms. It will include special topics as time permits. These could include: continued fractions Diophantine equations, solutions of equations over finite fields, partitions.

TEXT: An Introduction to the theory of numbers, Fifth Edition, 1991. I. Niven, H. S. Zuckerman, H. L. Montgomery, Required

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591 General and Differential Topology Scott MWF10:00 - 11:00am

Prerequisites: *Math 451*

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

The course will cover the following topics:

General topology: topological and metric spaces, continuity, subspaces, products and quotient topology, compactness and connectedness, extension theorems, topological groups, topological manifolds.

Differential topology: smooth manifolds, tangent spaces, vector fields, submanifolds, inverse function theorem, immersions, submersions, partitions of unity, Sard's theorem, embedding theorems, transversality, classification of surfaces.

TEXT:
Topology Second edition Munkres, James R. 0-13-181629-2, Required
Differential Topology, Victor Guillemin and Alan Pollack , Required

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593 Algebra I Smith MWF 2:00 - 3:00pm

Prerequisites: *Math 493/494 or permission of instructor.*

Math 593 is part of the year long (math 593-594) introduction to algebra for incoming PhD students.

The goal is to teach students abstract algebra with an emphasis on universal properties and structures, commonalities across different algebraic categories.

We will not include an exhaustive treatment of category theory, but the basic definitions and language of categories will discussed as they arise naturally in context.

Specific topics include: 1) Basic commutative algebra--localization, normalization, PIDs, UFDs, DVRs and valuations, the prime and maximal spectra of a ring; 2) The theory of modules over a ring, including tensor and exterior algebra, direct and inverse limits, the rudiments of homological algebra (presentations, free resolutions, projective and injective modules, the derived functors Tor and Ext); 3). The structure theory of finitely generated modules over a PID with applications to classification theorems in linear algebra (for example, to quick proofs of the Jordan and rational canonical form); 4). Bilinear algebra, including symmetric, hermitian and alternating maps. Later in Math 594, the emphasis will be on groups and their representations.

The course assumes students have had (at least) a full year long sequence in algebra at the advanced undergraduate level. In particular, I will assume that students are already familiar with groups, rings, ideals, fields (including algebraic closure and fields of fractions), homomorphism and isomorphism theorems. In addition, I will assume students have had a serious abstract linear algebra course. If this is not the case, Math 493-494 covers all the prerequisite abstract and linear algebra.

There will be weekly problem sets, regular quizzes and two exams.

TEXT: The recommended textbook will most likely be Lang's ``Algebra", supplemented by Dummit and Foote's ``Abstract Algebra," although in most cases, it is still appropriate for students to take Math 593 even if they have had a year-long sequence from Dummit and Foote, since that book includes a large amount of lower-level material as well.

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596 Analysis I (Complex) Baik MWF 9:00-10:00 am

This is a theoretical and rigorous introductory course for complex analysis for beginning graduate students.

Advanced undergraduate students may also take this course.

We will discuss homomorphic functions, Cauchy's theorem, power series, meromorphic functions, analytic continuation, conformal mappings, and Gamma and zeta functions.

Complex Analysis (Princeton Lectures in Analysis No. 2), Elias Stein and Rami Shakarchi, 978-0691113852, Optional

Complex Analysis by Lars Ahlfors, 978-0070006577, Optional

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602 Real Analysis II: Functional Analysis — Smoller — TTh 8:30-10:00am

Prerequisites: *one grad course in mathematics, or sufficient mathematical maturity.*

This is a basic course in functional analysis for anyone who is interested in analysis, applied mathematics, and topology. It deals with fundamental notions needed in these disciplines. The following topics will be covered: contraction mapping theorem, category, Hahn Banach theorem, Banach spaces, dual spaces, Holder and Minkowski inequalities, Riesz representation theorem, closed graph and open mapping theorems, uniform boundedness principle, condensation of singularities, Hilbert space, orthonormal sets, Fourier expansions, completeness, bounded operators on Hilbert space, self-adjoint operators, compact operators, Schauder fixed point theorem, theory of distributions.

For these topics, I will usually give applications; for example: local existence theorem for ODE's, existence of a continuous nowhere differentiable function, closable operators, existence of a continuous periodic function with a divergent Fourier series at a point, von Neumann's mean ergodic theorem, isoperimetric theorem, Heisenberg's Uncertainty Principle, existence of fundamental solutions, etc.

TEXT:
Functional Analysis (Pure and Applied Mathematics: A Wiley Series of Texts, Monographs and Tracts), 1st ed., Peter Lax, 0-471-55604-1, Required

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612 Lie Algebras — Prasad — TTh 1130am-1:00pm

Prerequisites: *A course on Linear Algebra and matrix theory*

Math 612 is a detailed introduction to the theory of finite dimensional Lie algebras. The contents of the book on Lie Algebras by J.E. Humphreys will be covered. This course should be useful for students interested in learning Lie group theory, algebraic groups/algebraic geometry and combinatorics of root systems and Weyl groups.

TEXT: Lie Algebras Graduate Texts in Mathematics, Springer-Verlag J.E.Humphreys, Required

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614 Commutative Algebra TBD TTh 11:30am-1:00pm

This is an introduction to commutative algebra, a subject that plays a significant role in algebraic geometry, number theory, combinatorics, and several complex variables.

The emphasis will be on the theory of commutative Noetherian rings. Topics will include the technique of localization, the theory of discrete valuation rings and Dedekind domains, primary decomposition, theorems on the behavior of prime ideals under integral extensions, the Noether normalization theorem, Hilbert's Nullstellensatz and basic affine algebraic geometry, dimension theory, the structure of Artinian rings, and the technique of completion. Grades will be based on problem sets.

TEXT: Atiyah and Macdonald, Introduction to Commutative Algebra

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623 Computational Finance Conlon TuTh 10:00-11:30am, 1:00-2:30pm

Prerequisites: *Differential equations (e.g. Math 316); basic probability theory (e.g. Math 425, Stat 515); numerical analysis (Math 471); mathematical finance (Math 423 and Math 542/IOE 552 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).*

This class is a course on mathematical finance with an emphasis on numerical and statistical methods. It is assumed that the student is familiar with basic theory of arbitrage pricing of equity and fixed income (interest rate) derivatives in discrete and continuous time. The course will focus on numerical implementations of these models as well as statistical methods for calibration, i.e. obtaining the parameters of the models. Specific topics include finite-difference methods, trees and lattices and Monte Carlo simulations with extensions.

TEXT:
The Mathematics of Financial Derivatives Cambridge, 1995 Wilmott, Howison, Dewynne 0-521-49789, Optional
Monte Carlo Methods in Finance Wiley, 2002 Jaeckel 0, Optional

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625 Probability with Martingales Bayraktar TTh 10:00-11:30am

A graduate level introduction to probability theory and the theory of martingales in discrete time. Topics include measure theory and integration; characteristic functions; convergence concepts; limit theorems; conditional expectation; martingales/supermartingales (uniform integrability, martingale convergence theorems, optional sampling theorem, optimal stopping); convex analysis on L^0, some applications to mathematical finance.

TEXT: Probability Theory with Martingales, David Williams, 521406056, Required

Probability and Stochastics by Erhan Cinlar, 387878580, Required

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631 Algebraic Geometry I Speyer MWF 11:00am-12:00pm

Prerequisites: *591, 594 and 614. 614 may be taken concurrently. Undergraduates and graduates with weak algebra backgrounds, please speak with the professor before enrolling.*

We will develop the theory of affine and projective varieties over the complex numbers, focusing on developing the dictionary between commutative algebra and geometric intuition. Topics: Affine varieties, localization and the Zariski topology, projective varieties, elimination theory, Bezout's theorem, dimension theory, discrete valuation rings, derivations and differentials, smoothness, normality and normalization, blowing up. Algebraic curves: definitions of genus, Riemann-Hurwitz, Riemann-Roch and Serre duality.

TEXT: Basic Algebraic Geometry (2nd Edition) by Shafarevich, 387548122, Required

Introduction to Commutative Algebra by Atiyah and Macdonald, 978-0201407518, Required

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636 Topics in Differential Geometry: Dynamics and Geometry Spatzier MWF 12:00-1:00pm

Prerequisites: *Basic manifold theory and basic real analysis.*

I will start with an introduction to dynamical systems, i.e. the study of the long time behavior of diffeomorphims and flows, discuss ergodicity and mixing, and natural invariants such as entropy. We will then develop some of the crucial tools in Pesin theory such as unstable manifolds and Lyapunov exponents. We will illustrate the general theory by particular examples from geometry and dynamical systems on homogeneous spaces.

I will apply these ideas to study some more general group actions, e.g. actions of higher rank abelian and semisimple groups. As was discovered in the last decade, these actions show remarkable rigidity properties, and sometimes can even be classified. These results have had important applications in other areas, e.g. geometry and number theory.

While the material in the first part of the course is fundamental for many investigations in dynamics, geometry, several complex variables. the second half of the semester will bring us right to the forefront of current research.

TEXT: No textbook required.

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656 Introduction to Partial Differential Equations Wu TTh 1:00-2:30pm

Prerequisites: *Math 451, 452, Math 556 or Math 597.*

Partial Differential Equations are mathematical structures for models in science and technology. It is of fundamental importance in physics, biology and engineering design with connections to analysis, geometry, probability and many other subjects. The goal of this course is to introduce students (both pure and applied) to the basic concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics to be covered will include the first order equations; elliptic, parabolic and hyperbolic equations. The method of characteristics, energy methods, distributions, Fourier transform, and if time permits, Sobolev spaces will be introduced.

Grading: Grades will be based on homework.

Subsequent Courses: Math 657 Nonlinear Partial Differential Equations

TEXT: Partial Differential Equations by John Fritz (Fourth Edition) 978-0387906096, Required

Partial Differential Equations by Lawerence C. Evans (Second Edition) 978-0821849743, Optional

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658 Topics in Ordinary Differential Equations: Nonlinear Dynamics and Geometric Mechanics — Bloch — TTh 10:00-11:30am

Prerequisites: *Some background in differential equations and some mathematical sophistication.*

This course will discuss geometric aspects of the modern theory of ordinary differential equations and dynamical systems, with applications to various mechanical and physical systems. Topics will include: the qualitative theory of ODE's on manifolds, symplectic and Poisson geometry, nonlinear stability theory, Lagrangian and Hamiltonian mechanics, integrable systems, reduction and symmetries, mechanical systems with constraints and controls, and optimal control.

TEXT: Nonholonomic Mechanics and Control by A.M. Bloch (First Edition) 0-387-95535-6, Required

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665 Combinatorial Theory: Cluster Algebras — Fomin — TTh 11:30am-1:00pm

Cluster algebras are a class of commutative rings constructed from a certain kind of combinatorial data via a recursive “mutation” procedure. They arise in a variety of algebraic and geometric contexts including representation theory of Lie groups, Teichmueller theory, discrete integrable systems, classical invariant theory, and quiver representations. This course will provide an elementary introduction to the fundamental notions and results of the theory of cluster algebras, and its most basic applications. Combinatorial aspects will be emphasized throughout.

No special background in commutative algebra, representation theory, or combinatorics will be required.

TEXT: No textbook required.

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671 Topics in Numerical Methods: Numerical Methods for Conservative Laws — Karni — TTh 1:00-2:30pm

Prerequisites: *572 or permission, computer programing language.*

This course discusses the theory and numerical solution of nonlinear hyperbolic systems of conservation laws. It covers (i) the basic mathematical theory of the equations: the notion of weak solutions, entropy conditions, and the wave structure of solutions to the Riemann problem; and (ii) it discusses high resolution shock-capturing methods, including the theory of total variation diminishing (TVD) methods and the use of limiter functions.

TEXT: Numerical Methods for Conservation Laws by Randall J. LeVeque (2nd Edition) 978-3764327231, Required

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676 Theory of Algebraic Numbers — Zieve — TTh 10:00-11:30am

This course will develop the theory of algebraic number fields and their subrings, especially the subring of algebraic integers in the number field. This includes the splitting of prime ideals in extensions of such rings, the inertia group and decomposition group and their connection with such splitting, the structure and uses of the ideal class group and the unit group of such subrings, and a study of the localizations and completions of these rings. Applications will be made to diophantine equations, recursive sequences, and other topics.

TEXT: Algebraic Theory of Numbers by Pierre Samuel, Optional

Local Fields by J.W.S. Cassels, Optional

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678 Modular Forms — Lagarias — MWF 2:00-3:00pm

Prerequisites: *Some prior number theory on the level of Math 575. Some experience with functions of a complex variable is recommended.*

Modular forms involve a wonderful overlap of arithmetic, algebra, analysis and geometry. This is a basic course on modular forms, expected to take an analytic viewpoint, but covering algebraic aspects. It will cover the modular group, classical modular forms, (holomorphic and non-holomorphic) Eisenstein series, and related spectral theory for SL(2, R). This will include Hecke operators, and the connection to Dirichlet series with Euler products. There will be a discussion of the adelic viewpoint and the connection with representation theory.

Applications may include theory of partitions, representations of quadratic forms, connections to elliptic curves, with and without complex multiplication. Some other possible subjects: mock theta functions, mock modular forms, weakly holomorphic Maass forms

The textbooks cover more material than the course can cover.

TEXT: Automorphic Forms and Representations by Daniel Bump

A First Course in Modular Forms by F. Diamond and J. Shurman, 978-00387-27226-9

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682 Combinatorial Set Theory Blass MWF 1:00 –2:00pm

This will be a course in combinatorial set theory, also known as infinitary combinatorics. Its central theme is that rather elementary structures on infinite sets can have surprisingly rich properties. An easily stated illustration of the sort of thing I have in mind is (a special case of) Ramsey's Theorem: If X is an infinite set and if every two elements of X are joined by a red or green thread, then there is an infinite subset Y of X such that all threads joining its elements are the same color.

In the first part of the course, I'll develop several combinatorial results that essentially involve only the smallest infinite sets, the countably infinite ones. I'll also discuss the so-called compactness phenomenon, which relates the behavior of infinite sets and large finite sets. For example, the infinite Ramsey theorem quoted above implies various finite versions, including some that cannot be proved without a detour into the infinite.

In the second part of the course, I'll describe some of the new phenomena that occur in uncountable sets. Here is one example: If X is an uncountable, well-ordered set and if a function f assigns to each member x of X except the first some earlier member f(x), then some single member must be f(x) for uncountably many distinct x. I'll also discuss uncountable analogs of some of the countable phenomena from the first part of the course. For example, in Ramsey's theorem quoted above, how large must X be if we want to guarantee an uncountable Y? Answer: The cardinality of the continuum is not large enough, but any larger cardinal is.
If time permits, I plan to discuss properties of the "exponential" function that maps the cardinality of a set X to the cardinality of the family of all subsets of X. In addition to the easily verified (weak) monotonicity, this function has some surprisingly subtle additional properties.
There will not be time in the course to treat independence results. Such results will be mentioned where appropriate but not proved. Thus, this course will be disjoint from the versions of 682 offered in some previous years. Students who have had one of these earlier

versions and wish to also take this version are welcome to do so, but will have to register under a reading-course number because 682 is not officially repeatable for credit.

The set-theoretic prerequisites for this course are minimal. Math 582 is more than enough. I'll briefly review the necessary material, basic cardinal and ordinal arithmetic, in class. So the only real prerequisite is the "mathematical maturity" ordinarily presupposed in graduate courses.

Grading will be based on several homework assignments.

TEXT: There will be no textbook. I plan to put on reserve in the library several books whose union includes most of the course material.

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695 Algebraic Topology Kriz MWF 10:00-11:00am

Prerequisites: *Some familiarity with fundamental group and homology, such as covered in Math 592 or a comparable course, is helpful.*

The main goal of this course is to develop basic techniques of algebraic topology which are useful throughout mathematics, but are beyond the scope of a 1st year graduate course. This includes homology with coefficients, cohomology, basic homological algebra, products, duality in manifolds and beyond, and basic homotopy theory. The deeper goal of the course is to learn how to think in derived categories, focusing on the examples of spaces and modules over a ring. Some more special topics may also be covered if there is time, such as cohomology of groups, cohomology of spaces with local coefficients, Eilenberg-MacLane spaces and Hurewicz's theorem.

There are 3-6 homework problems collected weekly, no exams.

TEXT: A Consise Course in Algebraic Topology by J. Peter May, 226511839, Optional

Elements of Algebraic Topology by J.R. Munkres, 201627280, Optional

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697 An Introduction to Geometric Group Theory Canary MWF 11:00am-12:00pm

Prerequisites: *The only prerequisite for this course is the basic material on covering spaces and fundamental groups, which occurs in Math 592.*

In this course we focus on the geometric viewpoint on the study of groups. The basic idea here is that it is often easier to study a group by studying its geometric action on some space. Hyperbolic groups are those groups which act properly discontinuously and cocompactly (i.e. with compact quotient) on a space which ``coarsely'' has negative curvature. It is one of Gromov's fundamental discoveries that it suffices to work with a very naive notion of coarse negative curvature, e.g. that all geodesic triangles are ``thin'', and that one can still obtain powerful consequences, e.g. the solvability of the word problem for hyperbolic groups. Therefore, proofs of seemingly complicated result can be reduced to their simple essence and become quite accessible. Examples of hyperbolic groups include free groups, fundamental groups of surfaces of genus at least 2, and fundamental groups of negatively curved manifolds. Gromov's concise and elegant notion of coarse negative curvature has been tremendously influential in a wide swath of mathematics where geometric techniques or ideas are used.

We will begin with a brief introduction to the topological viewpoint on the the study of groups and their decompositions. One beautiful example you have probably already seen is the topological proof that every subgroup of a free group is itself a free group. This brief initial segment will culminate in the Bass-Serre theory of graphs of groups. The largest segment of the course will involve the study of hyperbolic groups. We will also study CAT(0)-groups which are groups which act properly discontinuously and cocompactly on CAT(0)-spaces. CAT(0) spaces have a local non-positive curvature requirement which is often easily checked. CAT(0)-groups have recently become quite prominent through the work of Agol and Wise. Agol recently used them in his proof that every closed hyperbolic 3-manifold has a finite cover which fibers over the circle. In the process, he resolved many of the most important problems in the understanding of closed 3-manifolds and their fundamental groups.

TEXT: No textbook required.

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703 Topics in Complex Function Theory: Holomorphic Dynamics in Several Complex Variables Sibony TTh 1:00-2:30pm

Prerequisites: *Math 695 or Math 592, or some knowledge of beginning to intermediate*

I will give an introductory course on recent developments in holomorphic dynamics in several variables. The emphasis will be on pluripotential methods (positive closed or ddc-closed currents). For simplicity, I will first focus first on the dimension 2-case: Rigidity results for polynomial automorphisms of C^2 and automorphisms of positive entropy of compact Kähler surfaces. I will then discuss the notion of entropy for meromorphic maps on compact Kähler manifolds and it's relation with dynamical degrees. Finally, I will introduce the theory of super-potentials which permits to develop a calculus on positive closed currents of bi-degree (p,p). This is an essential tool for concrete equidistribution problems in holomorphic dynamics. If time permits, I will discuss some analogies with the dynamics of (singular) foliations by Riemann-surfaces and with Nevanlinna's theory of value distribution.

TEXT: No text required.

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709 Modern Analysis I: Brownian Motion — Rudelson — TTh 10:00-11:30am

Prerequisites: *Math 625.*

The course will discuss the Brownian motion, which is one of the most fundamental objects in probability. Brownian motion enjoys the universality property similar to that of a normal random variable. The Central Limit Theorem asserts that a normal random variable is the limit of a scaled of sums of independent random variables. If we consider the sequence of such sums, we would obtain a random walk, and the scaled limit of such random walk will be a Brownian motion.

We will start with constructing a one-dimensional Brownian motion and then discuss both its probabilistic and analytic properties. This includes the Markov property and Donsker's theorem, which plays the role of the Central Limit Theorem, as well as the smoothness properties of the sample paths. We will also discuss how Brownian motion was used in a recent solution of a classical Hardy-Littlewwod problem is analysis. At a later stage of the course we will consider a multidimensional Brownian motion and discuss fractal structures associated with it.

TEXT: Brownian Motion by Morters, Peter, Peres, Yuval, 978-0-521-76018-8, Optional

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711 Topics in Birational Geometry — Mustata — MWF 1:00-2:00pm

Prerequisites: *Math 631, as well as familiarity with cohomology of coherent sheaves on algebraic varieties.*

Birational geometry is concerned with the birational classification of higher-dimensional algebraic varieties. This is a subject whose origins go back to the classification of surfaces, completed by the Italian school at the beginning of the 20th century. The modern pint of view on this area has been developed during the 80s, culminating with the main results for 3-folds and with the general framework set-up in arbitrary dimension. The last 10 years have brought major progress, with some of the most important questions settled in arbitrary dimension. The main goal of the course is to give an introduction to this circle of ideas.

Here is a rough outline of the course:
1. Basics about ample, nef, and big divisors.
2. Vanishing theorems.
3. A quick introduction to singularities in birational geometry.
4. Asymptotic invariants of line bundles.
5. Finite generation of the canonical ring after Cascini and Lazic.
6. The basic results in the Minimal Model Program (after Corti and Lazic).

Depending on the time left, we might discuss other topics, such as:

7. Birational rigidity.
8. The Kodaira canonical bundle formula and generalizations.
9. Mori Dream Spaces.

TEXT: No textbook required.

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731 Topics in Algebraic Geometry: Hodge Theory — Burns — TTh 11:30am-1:00pm

Prerequisites: *Basic complex variable and either an introduction to algebraic geometry or to smooth manifolds. See instructor if there is doubt.*

Hodge theory denotes initially the use of PDE methods to make fine decompositions of the cohomology of a smooth, complex algebraic variety. This decomposition is tied, conjecturally intimately, to the existence of algebraic sub-varieties on the ambient variety X by means of the Hodge conjecture and its generalizations. In this course we hope to introduce the basics of the subject and then begin the study of two important applications in algebraic geometry: the study of deformations of varieties (moduli problems) through the variation of Hodge structures, and the association to algebraic cycles. The main reference will be the book of Claire Voisin, "Hodge Theory and Complex Algebraic Geometry, I", and possibly parts of volume II, as well as papers from the original literature. Prerequisites would be basic complex analysis and either an introduction to algebraic geometry or to smooth manifolds. There will be regular problem sets, and group discussion of such problems.

The course will hopefully extend into the winter term, 2015, and cover recent developments in the geometry of period-matrix domains and cycles.

TEXT: Hodge Theory and Complex Algebraic Geometry by Claire Voisin, Optional

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756 Advanced Topics in Partial Differential Equations — Bieri — MW 8:30am-10:00am

Prerequisites: *Some knowledge in partial differential equations and differential geometry.*

Partial differential equations (PDE) on manifolds with rich geometrical features are studied in pure mathematics to unravel the structures of their solutions and the spaces they live in. PDE describe phenomena in the real world including physics, medicine, biology or economics. They have become essential to science, technology and to modern life. In general relativity (GR) the Einstein equations describe the laws of the Universe. GR unifies space, time and gravitation. A spacetime in GR is a Lorentzian manifold where the metric solves the Einstein equations. They can be written as a system of nonlinear, second-order, hyperbolic PDE. The unknown is the metric. Typical physical questions are formulated as initial value problems for the Einstein equations under specific conditions. The solution will lay open the geometry of the resulting spacetime. Today, the methods of geometric analysis have proven to be most effective to investigate these structures. In this course, we introduce some of these methods which are universal and can be applied to other PDE outside GR.

This course will be taught as a mixture of lectures and seminar-style student work. The students with guidance of the professor will explore some of the important topics in mathematical GR, and will also work through some of the latest results in research. First, we will introduce the spacetime as a solution of the Einstein equations. Then we discuss topics from linear and nonlinear wave equations on flat and on curved backgrounds. Along the way, the role of curvature in GR will be given special attention. We will study the initial value problem in GR. Finally, we will address questions in modern research on gravitational waves and their geometric-analytic structures. These are produced during extreme events in our Universe like supernovae and when binary black holes merge. These waves are expected to be seen in experiments in the near future. This course features an outreach component. Towards the end of the semester, students will be asked to present a topic they learnt to high school students. This will be optional, and the participating students will work in groups. It is important to communicate intricate developments in mathematics and science to the public. Therefore, the students will be asked: How do you explain this topic to a broad public? What would you like to learn about it in an exhibit? And how?

TEXT: The Cauchy Problem in General Relativity by Hans Ringstroem, 2009, Optional