501AIM Student SeminarAlben12:00-1:00pmPrerequisites: You must be a graduate student in the AIM program to register for this course.

**Course Description:** The Applied and Interdisciplinary Mathematics (AIM) student seminar is an introductory and survey course in the methods and applications of modern mathematics in the natural, social, and engineering sciences. Students will attend the weekly AIM Research Seminar where topics of current interest are presented by active researchers (both from U-M and from elsewhere). The other central aspect of the course will be a seminar to prepare students with appropriate introductory background material. The seminar will also focus on effective communication methods for interdisciplinary research. MATH 501 is primarily intended for graduate students in the Applied & Interdisciplinary Mathematics M.S. and Ph.D. programs. It is also intended for mathematically curious graduate students from other areas. Qualified undergraduates are welcome to elect the course with the instructor's permission. Student attendance and participation at all seminar sessions is required. Students will develop and make a short presentation on some aspect of applied and interdisciplinary mathematics.

Text: None

520 Life Contingencies 1 Marker TTh 8:30-10:00am & 10:00 - 11:30am *Prerequisites:* Math 424 and 425 or permission from the instructor.

**Course Description:** Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of this course is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are (1) developing probability distributions for the future lifetime random variable, and (2) using those distributions to price life insurance and annuities.

I require that you have taken Math 425, Introduction to Probability, or an equivalent calculusbased probability course. I also require that you have taken Math 424, Interest Theory. In fact, I assume that you did quite well in those courses, with grades of B+ or better.

**Text:** (Required) Actuarial Mathematics, second edition, by Bowers, Gerber, Hickman, Jones, and Nesbitt, published by the Society of Actuaries in 1997.

523	Loss Models I	Young	TTh 11:30am-1:00pm
		Herrmann	

Prerequisites: Math 425 with a grade of C- or better

**Course Description:** The goals of this course are to understand parametric distributions for the purpose of (1) modeling frequency, severity, and aggregate insurance losses, (2) analyzing the effects of insurance coverage modifications, and (3) estimating future insurance losses via credibility theory.

**Text:** (Required) Loss Models: From Data to Decisions, Klugman, Panjer, and Willmot, (4th Edition),

 525
 Probability Theory
 Rudelson
 TTh 8:30-10:00a & TTh 10:00-11:30am

 Stoev
 MWF 8:00-9:00a

Prerequisites: MATH 451; MATH 425/STATS 425 would be helpful

**Course Description:** This is a fairly rigorous introduction to probability theory with some emphasis given to both theory and applications, although a knowledge of measure theory is not assumed. Topics covered are: probability spaces, conditional probability, discrete and continuous random variables, generating functions, characteristic functions, random walks, limit theorems, and some more advanced topics (this may include Poisson processes, branching processes, etc.)

**Text:** (Optional) Knowing the odds: an introduction to probability, John B. Walsh. ISBN: 9780821885321

G. Grimmett and D. Stirzaker, Probability and Random Processes, Third edition, Oxford,2001 ISBN: 9780198572220 (required)

S.M. Ross, Introduction to Probability Models, Tenth Edition, Academic Press ISBN:9780123756862 (optional)

526 Discrete Stochastic Processes BayraktarTTH 10:00-11:30amStoevMWF 9:00-10:00amPrerequisites: Required: MATH 525 or STATS 525 or EECS 501

**Course Description:** Math 525 or basic probability theory including: probability measures, random variables, expectations, cumulative distribution and probability density functions, conditional probabilities and independence, law of large numbers. Good understanding of advanced calculus covering limits, series, the notions of continuity, differentiation and integration.

**Text:** (Required): Essentials of Stochastic Processes, 2nd ed. (Durrett). Optional: Introduction to Stochastic Processes (Cinlar), Stochastic Processes (Ross), Probability and Measure (Billingsley). ISBN : 9781461436140

550 Intro to Adaptive Dynamics Doering MW 8:30-10:00am *Prerequisites:* Equivalent of four semesters of calculus (especially familiarity with differential equations) and working knowledge of elementary linear algebra and probability.

**Course Description:** This course is an introduction to applications and integration of dynamical systems and game theory to model population and ecological dynamics and evolutionary processes. Topics include Lotka-Volterra systems, non-cooperative games, replicator dynamics and genetic mechanisms of selection and mutation, and other adaptive systems.

Course Requirements: Attend lectures and complete the substantial homework assignments. There will also be a midterm test and the final exam.

Intended Audience: Applied math and other theoretically and mathematically minded natural science and engineering students. Students of biology, ecology, economics and other social sciences, natural resources, bioengineering, and bioinformatics—with suitable math backgrounds—are especially welcome.

**Text:** (Required) Evolutionary Games and Population Dynamics, Josef Hofbauer and Karl Sigmund, ISBN: 0-521-62570-X

555Intro to Complex VariablesLagariasTTh 1:00-2:30pmPrerequisites: Courses in elementary real analysis (e.g. Math 451) and multivariable calculus (eg., Math 215 or<br/>Math 255) are essential background.

**Course Description:** This course is an introduction to the analysis of complex valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply principles to problems are emphasized alongside rigorous proofs of the basic results in the subject.

Topics covered include the Cauchy-Riemann equations, Taylor series, Laurent expansions, Cauchy integral formula, residues, the argument principle, harmonic functions, maximum modulus theorem, conformal mappings and applications including evaluation of improper real integrals and fluid mechanics.

**Text**: R. A. Silverman, Complex Analysis with Applications, Dover Publications, 1984. ISBN 0-486-64762-5

556 Applied Functional AnalysisGilbertTTh 10:00 - 11:30amPrerequisites:Math 217, 419, or 420; Math 451 and Math 555.TTh 10:00 - 11:30am

**Course Description:** This is an introduction to methods of applied analysis with emphasis on Fourier analysis and partial differential equations. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, advanced calculus, and complex variables.

Content:

Topics in functional analysis that are used in the analysis of ordinary and partial differential equations. Metric and normed linear spaces, Banach spaces and the contraction mapping theorem, Hilbert spaces and spectral theory of compact operators, distributions and Fourier transforms, Sobolev spaces and applications to elliptic PDEs.

Alternatives: Math 454 (Bound Val. Probs. for Part. Diff. Eq.) is an undergraduate course on the same topics.

Subsequent Courses: Math 557 (Methods of Applied Math II), Math 558 (Ordinary Differential Equations), Math 656 (Partial Differential Equations), and Math 658 (Ordinary Differential Equations).

# 558 Advanced Ordinary Differential Simon Equations and Dynamical Systems

**Prerequisites:** Basic Linear Algebra, Ordinary Differential Equations (Math 216), Mul- tivariable Calculus (215). Some exposure to more advanced mathematics e.g. Advanced Calculus (Math 450/451) or Advanced Mathematical Methods (Math 454).

TTh 2:30-4:00pm

**Course Description:** This course surveys a broad range of differential equations topics, with a focus on techniques and results that are useful in applications. Topics covered include: dynamics in dimension 1 and 1.5, bifurcations, Poincaré map, existence, uniqueness, and perturbation theory, linear systems, spectral theorems, linearization at equilibria for nonlinear systems, phase plane solutions of linear and nonlinear systems, stable and unstable manifolds, Lyapunov functions, gradient flows and Hamiltonian systems, periodic solutions, stability, omega-limit set, Poincaré -Bendixson and Bendixson-duLac Theorems, bifurcation theory, and chaotic dynamics.

**Text:** (Required) Differential Equations, Dynamical Systems, and an Introduction to Chao, M. Hirsh, S. Smale, and R. Devaney, 3<sup>rd</sup> edition, ISBN: 978-0123820105.

559 Topic in Appiled Math: Booth MWF 8:00-9:00pm Computational and Mathematical Neuroscience

**Prerequisites**: Math 216 and 217 or 417, or permission of instructor. Math 463 recommended (required for undergraduate students).

**Course Description:** Computational neuroscience investigates the brain at many different levels, from single cell activity, to small local network computation, to the dynamics of large neuronal populations. As such, this course introduces students to modeling and quantitative techniques used to investigate neural activity at these different levels. Topics to be covered include: Passive membrane properties, the Nernst potential, derivation of the Hodgkin-Huxley model, action potential generation, action potential propagation in cable and multi-compartmental models, reductions of the Hodgkin-Huxley model, phase plane analysis, linear stability of equilibria, bifurcation analysis, synaptic currents, excitatory and inhibitory network dynamics, firing rate models, neural coding.

Text: None

565Combinatorics and Graph TheoryLamTTh 10:00 – 11:30amPrerequisites: Linear algebra, experience with proofs and abstract mathematics

**Course Description**: The goal of the course is to provide an introduction to some basic notions, techniques, and results in combinatorics. The first part of the course will be devoted to graph

theory. Topics we may discuss include graph colorings, Ramsey theory, extremal graph theory, and planar graphs. The second part of the course will cover a number of topics related to partially ordered sets, matroids, projective geometries and hyperplane arrangements.

**Text:** (Required) A course in combinatorics, J. H. van Lint and R. M. Wilson, 2<sup>nd</sup> edition, ISBN: 9780521567244

 571
 Numerical Linear Algebra
 LaRose
 MWF 11:00-12:00pm

 Prerequisites: Math 214, 217, 417, 419, or 420 and one of Math 450, 451, or 454.
 MWF 11:00-12:00pm

**Course Description:** This course is an introduction to numerical linear algebra, which is at the foundation of much of scientific computing. Numerical linear algebra deals with (1) the solution of linear systems of equations, (2) computation of eigenvalues and eigenvectors, and (3) least squares problems. We will study accurate, efficient, and stable algorithms for matrices that could be dense, or large and sparse, or even highly ill-conditioned. The course will emphasize both theory and practical implementation.

Text: (Required) Numerical Linear Algebra by Trefethen and Bau. ISBN: 9780898713619

573	Finacial Math I	Nadtochiy	TTH 1:00-2:30pm
Prerequ	<b>isites:</b> Math 526		

**Course Description:** This is an introductory course in Financial Mathematics. This course starts with the basic version of Mathematical Theory of Asset Pricing and Hedging (Fundamental Theorem of Asset Pricing in discrete time and discrete space). This theory is applied to problems of Pricing and Hedging of simple Financial Derivatives. Finally, the continuous time version of the proposed methods is presented, culminating with the BlackScholes model. A part of the course is devoted to the problems of Optimal Investment in discrete time (including Markowitz Theory and CAPM) and Risk Management (VaR and its extensions). This course shows how one can formulate and solve relevant problems of financial industry via mathematical (in particular, probabilistic) methods.

This is a core course for the Quantitative Finance and Risk Management Master's program and introduces students to the main concepts of Financial Mathematics. This course emphasizes the application of mathematical methods to the relevant problems of financial industry and focuses mainly on developing skills of model building.

**Text:** (Required) Stochastic Finance: An Introduction in Discrete Time, Follmer, Fallmer, & Schied, ISBN: 9783110183467 (Optional) Stochastic Calculus for Finance I, Shreve, ISBN: 9780387249681 (Optional) Introduction to the Economics and Mathematics of Financial Markets, Cvitanic, ISBN: 9780262033206 583 Probabilistic & Interactive Proof Systems Strauss MWF 100-2:00pm *Prerequisites:* Graduate students and advanced undergraduates in Math, Computer Science, and Philosophy. Students should have an interest in theoretical computer science, logic, randomized algorithms and graphs, and/or proofs and knowledge.

**Course Description:** Can we be convinced that a proof is correct, even if we only check it in three places? Can a proof convince us that a statement is true, while giving us no aid in convincing anyone else that the statement is true? The answer to both is affirmative. How? Using randomness and interaction, two elements missing from traditional deductive proofs. Why? Checking a proof in just a few places is useful for checking computer-generated proofs that are too long to read (an example bigdata algorithm); there are also surprising connections to showing that certain functions cannot be computed or even approximated efficiently. A "zero-knowledge proof" might be used, for example, for a customer to prove to a merchant that the customer is the rightful owner of a credit card, without giving the merchant any ability to prove (fraudulently) that the merchant is the owner of that credit card.

Other topics: We will also look at IP=PSPACE: converting a game, G, to another, G', such that if Peggy beats any expert in G, then she beats any expert in G'; if Peggy loses to an expert in G, she loses to a random (non-expert) player in G'. Thus the prover Peggy can convince a nonexpert verifier Victor that she wins G, even though the game tree is too big for Victor to read. We will also look at Multi-prover Interactive Proof systems and show that they are more powerful than single-prover systems, answering the question posed so eloquently by Professors Click and Clack: Do two people who don't know what they are talking about know more or less than one person who doesn't know what he's talking about?

Content: Probabilistically-checkable proofs, zero-knowledge proofs, and interactive proofs are studied and their computational, cryptographic, and other advantages discussed. The course will include, as modules, a presentation of the necessary background material, which (it turns out) are important tools for many other topics in discrete math and theoretical computer science. These modules include the Chernoff tail bound and other topics from probability theory, error-correcting codes, expander graphs, and randomized computation. Motivations and applications in other fields, such as secure computation and the philosophical nature of proof and knowledge, are briefly discussed.

Expected work: Students will transcribe lecture notes and present papers. Contact Martin Strauss, martinjs, with questions.

Text: None. Readings and handouts provided.

591General and Differential TopologyScottMWF10:00 - 11:00amPrerequisites: Math 451 and Math 590

**Course Description:** This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs. The first

2-3 weeks of the course will be devoted to general topology, and the remainder of the course will be devoted to differential topology. Note: this course has been recently restructured and is more advanced than it was in previous years.

Topics may include: Product and quotient topology, CW-complexes, group actions, topological groups, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, submersions, immersions and embeddings, smooth submanifolds, Sard's Theorem, the Whitney embedding theorem, transversality, Lie groups, vector fields, Lie brackets, Lie algebra, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, De Rham cohomology groups, homotopy invariance, degree theory.

**Text:** (Required) Introduction to Smooth Manifolds, Lee, John M., Second Edition, ISBN: 978-1-4899-9475-2. Topology Second edition Munkres, James R. 0-13-181629-2, Required Differential Topology, Victor Guillemin and Alan Pollack, Required

E02	Algebra I	Mustata	MWF 2:00 - 3:00pm
555	Algebrai	Iviustata	10100 - 2.00 - 2.00pm

**Course Description:** Math 593 is part of the year long (Math 593-594) introduction to algebra for incoming PhD students.

Topics to be covered in the course:

1) Introduction to categories and functors. Most of this will be done though in context.

2) Basic commutative algebra: localization, normalization, PIDs, UFDs, DVRs and valuations, the prime and maximal spectra of a ring.

3) The theory of modules over a ring, including tensor and exterior algebra, direct and inverse limits, the rudiments of homological algebra (projective and injective modules, resolutions, the derived functors Tor and Ext).

4). The structure theory of finitely generated modules over a PID with applications to classification theorems in linear algebra.

5). Bilinear algebra, including symmetric, hermitian and alternating maps.

The course assumes students have had (at least) a full year long sequence in algebra at the advanced undergraduate level. In particular, I will assume that students are already familiar with groups, rings, ideals, fields, homomorphisms and isomorphism theorems. In addition, I will assume students have had a serious abstract linear algebra course.

Recommended textbooks are Lang's ``Algebra" and Dummit and Foote's ``Abstract Algebra", but I do not plan to follow very closely either of them.

**Text:** (Optional) Algebra, Lang, ISBN: 978-0387953854 & Abstract Algebra, Dummit and Foote, ISBN: 978-8126532285

maotata

596	Analysis I (Complex)	Baik	MWF 9:00-10:00 am
550	Analysis i (Complex)	Daik	

**Course Description:** This is a theoretical and rigorous introductory course for complex analysis at the beginning graduate level. Topics to be discussed include holomorphic (analytic) functions, Cauchy's theorem, Cauchy's integral formula, power series, isolated singularities, meromorphic functions, Laurent series, conformal mappings, infinite product, and so on. Highly advanced undergraduate students may also take this course but they should also consider the option of taking the alternative course 555.

**Text:** (Required) Complex Analysis (Princeton Lectures in Analysis No. 2), Elias Stein and Rami Shakarchi, 978-0691113852

(Optional) Complex Analysis: an introduction to the theory on analytic functions of one complex variable by Lars Ahlfors, 978-0070006577.

602	Real Analysis II:	Wu	TTh 2:30-4:00pm
	Functional Analysis		

Prerequisites: Linear Algebra, Point Set Topology, Real and Complex analysis.

**Course Description:** Functional analysis is a core subject in mathematics. It has connections to probability and

geometry, and is of fundamental importance to the development of analysis, differential equations,

quantum mechanics and many other branches in mathematics, physics, engineering and theoretical computer science. The goal of this course is to introduce students to the basic concepts, methods and results in functional analysis. Topics to be covered include linear spaces, normed linear spaces, Banach spaces, Hilbert spaces, linear operators, dual operators, the Riesz representation theorem, the Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem, compact operators, Fredholm Theory, reflexive Banach spaces, weak and weak\* topologies, spectral theory, and applications to classical analysis and partial differential equations.

**Text:** (Required) Functional Analysis (Pure and Applied Mathematics: A Wiley Series of Texts, Monographs and Tracts), 1st ed., Peter Lax, 0-471-55604-1.

(Required) Functional Analysis, Sobolev Spaces and Partial Differential Equations (Universitext) 2011th Edition by Haim Brezis ISBN-13: 978-0387709130

#### 612 Lie Algebras

Derksen

MWF 1:00-2:00pm

This course is an introduction to the theory of Lie algebras and their representations. Lie algebras appear naturally in the study of algebraic/Lie groups. They are fundemental in Algebra and in Geometry. The study of finite dimensional Lie algebras leads to beautiful combinatorial structures such as root systems and Weyl/Coxeter groups. **Text:** TBD 614 Topics in Commutative Algebra Bass MW 2:30-4:00pm *Prerequisites: Math 593* 

**Course Description:** Review of commutative rings and modules. Local rings and localization. Noetherian and Artinian rings. Integral independence. Valuation rings, Dedekind domains, completions, graded rings. Dimension theory.

**Text:** There is no textbook. Materials will be provided online.

623 Computational Finance Muhle-Karbe TuTh 10:00-11:30am *Prerequisites*: Differential equations (e.g. Math 316); probability theory (e.g. Math 525/526, Stat 515); numerical analysis (Math 471 and Math 472); mathematical finance (Math 423 and Math 542/IOE 552, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).numerical analysis (Math 471 or Math 472); mathematical finance (Math 423 and Math 542/IOE 552, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).numerical analysis (Math 471 or Math 472); mathematical finance (Math 423 and Math 542/IOE 552, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).

**Course Description:** Computational Finance --- This is a course in computational methods in finance and financial modeling. Particular emphasis will be put on interest rate models and interest rate derivatives. The specific topics include; Black-Scholes theory, no arbitrage and complete markets theory, term structure models: Hull and White models and Heath Jarrow Morton models, the stochastic differential equations and martingale approach: multinomial tree and Monte Carlo methods, the partial differential equations approach: finite difference methods.

Text: None

625ProbabilityBarvinokTTH 10:00-11:30amPrerequisites: Aptitude for analysis

**Course Description:** This is a fairly rigorous course in probability, covering independence, conditional expectation, martingales, laws of large numbers, large deviation inequalities, characteristic functions and the central limit theorem in some depth. Time permitting, we will discuss the concept of the Brownian motion.

**Text:** (Optional) Probability and Random Processes, G.R. Grimmett and D.R. Stirzaker, 3<sup>rd</sup> edition, ISBN: 0198572220

631 Algebraic Geometry I Ho MWF 11:00am-12:00pm *Prerequisites:* Math 594 or permission of instructor. Graduate Standing.

**Course Description:** Theory of algebraic varieties: affine and projective varieties, dimension of varieties and subvarieties, singular points, divisors, differentials, intersections. Schemes, cohomology, curves and surfaces, varieties over the complex numbers.

#### 636 Topics in Differential Geometry: Dynamics and Geometry

**Prerequisites:** Basic manifold theory and basic real analysis.

**Course Description:** Dynamical systems is the study of the long time behavior of diffeomorphims and flows. We will discuss ergodicity and mixing, and natural invariants such as entropy. This theory is particularly successful and interesting in the presence of a lot of hyperbolicity, i.e. when the flow or diffeomorphism expands and contracts tangent vectors exponentially fast. For these systems, we will then develop some of the crucial tools in Pesin theory such as unstable manifolds and Lyapunov exponents. We will illustrate the general theory by particular examples from geometry and dynamical systems on homogeneous spaces. I will apply these ideas to study some more general group actions, e.g. actions of higher rank abelian and semisimple groups. As was discovered in the last decade, these actions show remarkable rigidity properties, and sometimes can even be classified. These results have had important applications in other areas, e.g. geometry and number theory. While the material in the first part of the course is fundamental for many investigations in dynamics, geometry, several complex variables. the second half of the semester will bring us

right to the forefront of current research.

**Text:** (Optional) Introduction to the Modern Theory of Dynamical Systems, Hasselblad and Katok, Revised Edition, ISBN: 978-0521575577 (Optional) Ergodic Theory, Einsiedler, Ward, Graduate Texts in Mathematics, ISBN: 978-0-85729-021-2

637	Lie Groups	Prasad	TTH 1:00-2:30pm	
Prerequisites: linear algebra and differential topology				

**Course Description:** This is a basic introduction to Lie groups. We will begin with the definition of Lie groups, and give many examples. We will define the Lie algebra of a Lie group and connect them via left-invariant vector fields and the exponential map (prior knowledge of Lie algebras is not assumed). We will establish some structure theorems in Lie theory. Toward the end of the course we will discuss representation theory, with an emphasis towards representations of compact groups.

Text: None

651 Topics in applied math: Modeling and Mechanics *Prerequisites:* MATH 450 or 454

**Course Description:** The course will develop mathematical modeling methods that are useful for solving problems in continuum mechanics. We will discuss applications to fluid-structure interactions and the mechanics of organisms. Many of the modeling methods are useful in other areas of applied mathematics, physics, engineering, and biology. The course is aimed at graduate students in applied mathematics, engineering, and the sciences.

Alben

1:00-2:30pm

TTH

Topics: Dimensional analysis, Scaling, Tensors, Continuum, mechanics of solids and fluids, Elasticity, Potential theory, Fluid-structure interactions, Organismal locomotion.

Material will be drawn from a variety of sources including the following texts: Segel, Mathematics Applied to Continuum Mechanics Landau and Lifschitz, Elasticity Howison, Practical Applied Mathematics Fowler, Mathematical Models in the Applied Sciences.

Text: None

656 Partial Differential EquationsSchotlandMWF 10:00 - 11:00amPrerequisites:MATH 558, 596 and 597 or permission of instructor

**Course Description:** Characteristics, heat, wave and Laplace's equation, energy methods, maximum principles, distribution theory.

Text: Partial Differential Equations, Lawrence C. Evans, 2<sup>nd</sup> edition, ISBN: 978-0821807729

658 Topics in ODE: Bloch TTH 10:00-11:30am Nonlinear Dynamics and Geometic Mechanics *Prerequisites:* A first course in differential equations

**Course Description:** In this course we will discuss geometrical aspects of the modern theory of ordinary differential equations and dynamical systems, with applications to various mechanical and physical systems. Topics will include: the qualitative theory of ODE's on manifolds, symplectic and Poisson geometry, nonlinear stability theory, Lagrangian and Hamiltonian mechanics, integrable systems, reduction and symmetries, mechanical systems with constraints and controls, and optimal control.

Text: (Required) Nonholonomic Mechanics, Bloch, Anthony, 2<sup>nd</sup> edition. ISBN: 9781493930166

665	Combinatorial Theory:	Lam	TTH 11:30-1:00pm
	Schubert Calculus		

**Course Description:** This class is an introduction to combinatorial aspects of Schubert Calculus. A typical Schubert calculus problem is the following: given four lines in a complex threedimensional space, how many other lines intersect all four of these four lines? In modern language, problems of this kind translate into calculations in the cohomology ring of the Grassmannian. We will discuss the related combinatorics of Young tableaux, symmetric functions, Bruhat order, and Schubert polynomials.

If time permits, we will also discuss a range of generalizations such as: Schubert Calculus in other Lie types; quantum Schubert calculus; K-theoretic Schubert calculus; affine Schubert calculus; and equivariant Schubert calculus.

Text: (Optional) Young tableaux, W. Fulton, ISBN: 9780521567244

### 671 Topics in Numerical Methods: Veerapaneni TTh 1:00-2:30pm Fast Algorithms

**Prerequisites:** Numerical analysis, basic theory of ordinary and partial differential equations.

**Course Description:** This course will cover selected topics in fast algorithms research. Emphasis will be given on techniques that can be used for discretization and computational solution of partial differential equations. Topics will include:

- Complexity Analysis, Review of linear & nonlinear solvers, Krylov subspace methods
- Cartesian grid methods, Multigrid and multiresolution methods
- Fourier spectral methods, Nonuniform FFTs, Butterfly Algorithms, Ewald summation
- Numerical Methods & fast algorithms for integral equations.

Fast Gauss Transform, Adaptive algorithms, Tree codes, Fast multipole methods **Text:** None

676 Theory of Algebraic Numbers Prasanna TTH 2:30-4:00pm *Prerequisites:* We will assume familiarity with basic abstract algebra (groups, rings, fields) and Galois theory.

**Course Description:** This course will cover the basics of algebraic number theory. Topics to be covered include rings of integers, prime splitting and ramification, ideal classes, units, classical reciprocity laws. The latter half of the course will deal with completions and the theory of local fields, in preparation for class field theory (which will be taught in the winter term).

Text: (Optional) References:

Number Fields, Daniel Marcus. ISBN-13: 978-0387902791 Algebraic Number Theory, Serge Lang. ISBN-13: 978-1461269229 Local Fields, J.P. Serre. ISBN-13: 978-0387904245

677 Diophantine Problems Montgomery TTH 8:30-10:00am

**Course Description:** The course is devoted to Diophantine approximations and the geometry of numbers, with applications to Diophantine equations. Specific topics

will be as follows: 1. Homogeneous approximation. 2. Continued fractions. 3. Lattices: bases, unimodular matrices, Smith normal form.

4. Lattice constants, reduction, successive minima, LLL theorem.

5. Inhomogeneous approximation, Kronecker's theorem. 6. Uniform distribution (mod 1). 7. Thue's theorem. 8. Discrepancies, uniform distribution.

Text: None: TeXed notes will be provided

696 Equivariant Algebraic Topology Kriz MWF 11:00-12:00pm *Prerequisites:* Math 695 or comparable intermediate course in algebraic topology, or prior discussion with the instructor

**Course Description:** Description: `Equivariant' means `involving a group action'. In algebraic topology, concepts arising from group actions are a major current area of interest. In this course, we will develop basic techniques of algebraic topology with group action, especially equivariant generalized (co)homology, cover basic results of this field and also talk about some interesting calculations. We will also delve into some further extensions of this subject, such as equivariant parametric (co)homology theories, and also touch on the subject of modern topological methods of algebraic geometry.

**Text:** (Optional) Equivariant Stable Homotopy Theory, Lewis, May, Steinberger. Lewis, May, Steinberger: Equivariant stable homotopy theory (available online free on the second author's web page), for reference purposes other texts referenced and/or distributed in class.

697	Topic in Topology:	Ji	MWF 12:00-1:00pm
	Introduction to Rier	nann Surfaces	

**Course Description:** Riemann surfaces have played a fundamental role in mathematics since they were introduced by Riemann in his thesis in 1851. They are 1-dimensional complex manifolds and algebraic curves. Besides their own interest, they are basic ingredients in Teichmuller theory and the theory of moduli spaces. They have also motivated a lot of problems and results on higher dimensional complex manifolds and algebraic varieties. As Donadlson wrote in the preface to his recent book on Riemann surfaces: ``The theory of Riemann surfaces occupies a very special place in mathematics. It is a culmination of much of traditional calculus, making surprising connections with geometry and arithmetic. It is an extremely useful part of mathematics, knowledge of which is needed by specialists in many other fields. It provides a model for a large number of more recent developments in areas including manifold topology, global analysis, algebraic geometry, Riemannian geometry and diverse topics in mathematical physics." In this course, we will start from basics and give an introduction to important results on Riemann surfaces such as the Riemann-Roch Theorem, the uniformization theorem, and some basic results on deformation and moduli spaces of Riemann surfaces, and the geometry of algebraic curves. We will indicate directions of how results on Riemann surfaces can be generalized. The book by Donaldson will be one of the main references. We will use other books and papers.

Text: (Optional) Riemann Surfaces , Simon Donaldson, ISBN: 199606749

Additional Resources: 2. H. Farkas, I. Kra, Riemann surfaces. Second edition. Graduate Texts in Mathematics, 71. Springer-Verlag, New York, 1992. xvi+363 pp.

3. Phillips Griffiths, Introduction to algebraic curves. Translated from the Chinese by Kuniko Weltin.

Translations of Mathematical Monographs, 76. American Mathematical Society, Providence, RI, 1989. x+221 pp.

4. R. Miranda, Algebraic curves and Riemann surfaces. Graduate Studies in Mathematics, 5. American Mathematical Society, Providence, RI, 1995. xxii+390 pp.

# 709Modern Analysis I:BaikMWF 11:00-12:00pmExactly Solvabel Models in Probability & Statistical Physics

**Prerequisites:** We assume that the students have strong complex analysis background (MATH 555 or 596), and some background on functional analysis (MATH 602 or 556) and probability (MATH 525 or 625).

**Course Description:** We will consider a variety of models in probability and statistical physics which can be solved "exactly" in the sense that one can evaluate their distribution functions or moment generating functions or some other "observables" in explicit form, which then can be evaluated in the large time and size limit using various asymptotic method. Concretely, we plan to study random matrix theory, determinantal point processes, random tilings and non-intersecting random processes, directed polymers, and other topics depending on time and interest. Some of the topics are of currently active area of interest and research. Along the way, we discuss analytic topics such as Fredholm determinants, the method of steepest-descent, Riemann-Hilbert problems, integrable differential equations, and so on. The emphasis is on the analytic method instead of probabilistic method. We assume that the students have strong complex analysis background (MATH 555 or 596), and some background on functional analysis (MATH 602 or 556) and probability (MATH 525 or 625).

# 731 Topics in Algebraic Geometry I: Jonsson TTh 11:30- 1:00pm Berkovich Spaces

**Prerequisities:** While Berkovich spaces are to be viewed as analytic spaces, many of the tools used for studying them tend to be algebraic. For this reason, some familiarity with commutative algebra and algebraic geometry is necessary (the first three chapters of Hartshorne, excluding sheaf cohomology, is more than enough.), as is some basic functional analysis. Prior knowledge of other topics (complex manifolds, Banach algebras, p-adic numbers, rigid geometry...) may prove useful, but are by no means indispensable.

**Course Description:** Berkovich spaces are analogues of complex manifolds that appear when replacing complex numbers by the elements of a general valued field, e.g. p-adic numbers or formal Laurent series. They were introduced in the late 1980's by Vladimir Berkovich as a more honestly geometric alternative to the rigid spaces earlier conceived by Tate. In recent years, Berkovich spaces have seen a large and growing range of applications to complex analysis, tropical geometry, complex and arithmetic dynamics, the local Langlands program, Arakelov geometry,...

The first part of the course will be devoted to the basic theory of Berkovich spaces (affinoids, gluing, analytifications). In the second part, we will discuss various applications or specialized topics, partly depending on the interest of the audience.

**Text:** (Required) Spectral Theory and Analytic Geometry over non-Archimedean Fields, V. Berkovich, SBN: 978-0-8218-9020-2 (Optional) S. Bosch, U. Guentzer and R. Remmert.: Non-Archimedean Analysis. ISBN 978-3-540-12546-4