Winter 2021 Graduate Course Descriptions

MATH 501 **AIM Student Seminar** **Alben** **Fri 1:00 PM – 2:00 PM**
Fri 2:00 PM – 3:00 PM

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

No book for this course.

MATH 506/IOE **Stochastic Analysis for Finance** **Bayraktar, E.** **TR 10:00 AM - 11:30 AM**

Math 526. Graduate students or permission of instructor. (3). (BS). May not be repeated for credit.

The aim of this course is to teach the probabilistic techniques and concepts from the theory of stochastic processes required to understand the widely used financial models. In particular concepts such as martingales, stochastic integration/calculus, which are essential in computing the prices of derivative contracts, will be discussed. The specific topics include: Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales), martingales in continuous time, stochastic integration (including It\^{o}'s formula), stochastic differential equations (including Feynman-Kac formula), change of measure (including Girsanov's theorem and change of numeraire), and, time permitting, stochastic control (including Merton problem). Applications from various areas of Finance (including, pricing of derivatives, risk management, etc) are used to illustrate the theory.
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MATH 521 **Life Contingencies II** **Natarajan, R.** **TR 10:00 AM - 11:30 AM**
TR 11:30 AM – 1:00 PM

MATH 520 with a grade of C- or higher. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

This course extends the single decrement and single life ideas of MATH 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520-521 covers the Part 4A examination of the Casualty Actuarial Society and covers the syllabus of the Course 150 examination of the Society of Actuaries. Concepts and Calculation are emphasized over proof.
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MATH 524 **Loss Models II** **Young, J.** **TR 8:30 AM - 10:30 AM**
Moore, K. **TR 1:00 PM - 2:30 PM**

STATS 426 and MATH 523. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management is of major concern to all financial institutions, especially casualty insurance companies. This course is relevant for students in insurance and provides background for the professional examination in Short-Term Actuarial Modeling offered by the Society of Actuaries (Exam STAM). Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc.) and have at least Junior standing.

Content: Frequentist and Bayesian estimation of probability distributions, model selection, credibility, simulation, and other topics in casualty insurance.

MATH 525/STATS **Probability Theory** **TBA** **TR 10:00 AM - 11:30 AM**
TBA **TR 1:00 PM - 2:30 PM**

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

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MATH 526/STATS **Stochastic Processes with Discrete State Spaces** **Wang, Z.** **TR 8:30 AM - 10:00 AM**
Wang, Z. **TR 10:00 AM - 11:30 AM**
Chakraborty, P. **TR 1:00 PM - 2:30 PM**

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

Textbook:

Durrett, Richard. (2016). Essentials of Stochastic Processes, 3rd Ed. Springer.

MATH 547/BIOINF/STATS Probabilistic Modeling in Bioinformatics Rajapakse, I. TR 4:00 PM - 5:30 PM

MATH, Flexible, due to diverse backgrounds of intended audience. Basic probability (level of MATH/STATS 425), or molecular biology (level of BIOLOGY 427), or biochemistry (level of CHEM/BIOLCHEM 451), or basic programming skills desirable or permission. (3). (BS). May not be repeated for credit.

This course is open to graduate students and upper-level undergraduates in applied mathematics, bioinformatics, statistics, and engineering, who are interested in learning from data. Students with other backgrounds such as life sciences are also welcome, provided they have maturity in mathematics. The mathematical content in this course will be linear algebra, multilinear algebra, dynamical systems, and information theory. This content is required to understand some common algorithms in data science. I will start with a very basic introduction to data representation as vectors, matrices, and tensors. Then I will teach geometric methods for dimension reduction, also known as manifold learning (e.g. diffusion maps, t-distributed stochastic neighbor embedding (t-SNE), etc.), and topological data reduction (introduction to computational homology groups, etc.). I will bring an application-based approach to spectral graph theory, addressing the combinatorial meaning of eigenvalues and eigenvectors of their associated graph matrices and extensions to hypergraphs via tensors. I will also provide an introduction to the application of dynamical systems theory to data including dynamic mode decomposition. Real data examples will be given where possible and I will work with you write code implementing these algorithms to solve these problems. The methods discussed in this class are shown primarily for biological data, but are useful in handling data across many fields. A course features several guest lectures from industry and government.

551 INTRODUCTION TO REAL ANALYSIS Baik, J. M/W/F 12:00 PM – 1:00 PM

Advanced Calculus (MATH 295, 297, or 451) and Linear Algebra (MATH 217 or 296)

This is a new course that introduces the Lebesgue measure theory and a few other topics in real analysis for advanced math undergraduates, masters students, and AIM and non-math Ph.D. students.

We plan to cover (1) Lebesgue measure on Rn, (2) Lebsegue integral, (3) differentiation, (4) Lebesgue-Stieltjes measure, (5) product measures, (6) abstract metric spaces, and (7) Lp spaces.

We will cover about 2/3 of the book by Terry Tao, Introduction to Measure Theory (which is also available as an online version on the author's website), and a few sections of Royden's book, Real Analysis.

There are some overlaps with MATH 597, alpha course for MATH Ph.D. students, but this course will proceed at a gentler pace and emphasize measures on Rn instead of general spaces.

Required Textbooks:

1. Tao, Terry. (2011). **An Introduction to Measure Theory**. American Mathematical Society.
2. Royden & Fitzpatrick. (2017) **Real Analysis**, 4^{th} Ed. Pearson.

MATH 555 Introduction to Complex Variables Esedoglu, S. MW 8:30 AM - 10:00 AM

MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

This course is an introduction to the theory of complex-valued functions of a complex variable with substantial attention to applications in science and engineering. The prerequisite of a course in advanced calculus is essential. This is a core course for the AIM graduate program.

Required Textbook: **Complex Variables and Applications**, by James Ward Brown and Ruel V. Churchill, 9^{th} Ed., ISBN 978-0073383170

MATH 557 Applied Asymptotic Analysis Miller, P. TR 1:00 PM - 2:30 PM

MATH 217, 419, or 420; MATH 451; and MATH 555 or 596. (3). (BS). May not be repeated for credit.

Asymptotic analysis is the quantitative study of approximations that become increasingly accurate as a parameter tends to a limiting value. There are three aspects: (i) construction of approximations which is frequently based on heuristic reasoning, (ii) analysis of approximations to evaluate their accuracy, and (iii) use of approximations to solve important problems from diverse applications.

Topics include: asymptotic sequences and (divergent) series; asymptotic expansions of integrals and Laplace's method; methods of steepest descents and stationary phase; asymptotic evaluation of inverse Fourier and Laplace transforms; asymptotic solutions for linear (non-constant coefficient) differential equations; WKB expansions; singular perturbation theory including the method of multiple scales; and boundary, initial, and internal layers.

Applications include: the small-viscosity theory of shock waves, the theory of quantum mechanics in the semiclassical limit, aspects of the theory of special functions, vibrations in nonlinear lattices, and surface water waves.

Students will be evaluated on the basis of several homework sets, a term project involving some outside reading and culminating in a class presentation, and class participation.

Required Textbook: **Applied Asymptotic Analysis** by P.D. Miller, AMS, ISBN 0-8218-4078-9

MATH 564 Topics in Mathematical Biology Forger, D. TR 10:00 AM – 11:30 AM

Topic: The mathematics of wearables, mobile health, and physiological signals

One in five Americans uses a wearable. These wearables score sleep, predict alertness/stress/performance, and integrate into medical practice, for example, in COVID detection. This course will use sensor data measuring physiological signals to predict human performance and diagnose disease. We aim to teach students how to analyze real data from wearables or sensors, including data from students, athletes, travelers, or patents. Mathematical techniques introduced in this course include 1) Time Series Analysis, 2) Building Models with Differential Equations, 3) Parameter Estimation, 4) Uncertainty Analysis, 5)

Techniques from the Theory of Dynamical Systems. We will study potential applications, including Exercise and Heart Rate, Sleep, Circadian Rhythms, Mood, Weight, Music Performance, Infectious Disease, Addiction, … Course meetings will consist of prerecorded lectures, interactive lectures, and remote computer labs. Emphasis will be placed on the analysis of raw data. Consideration will be given in the problem sets and course project to interdisciplinary student backgrounds. Teamwork will be encouraged.

No textbook required

MATH 566 Combinatorial Theory Lam, T. MWF 11:00 AM - 12:00 PM

MATH 465 group theory and abstract linear algebra. (3). (BS). May not be repeated for credit.

This course is an introduction to algebraic and enumerative combinatorics at the beginning graduate level. Topics include: fundamentals of algebraic graph theory; applications of linear algebra to enumeration of matchings, tilings, and spanning trees; combinatorics of electric networks; partially ordered sets; integer partitions and Young tableaux.

Optional Textbook**: Algebraic Combinatorics: Walks, Trees, Tableaux, and More**, R. P. Stanley, ISBN 978-1-4899-9285-7

MATH 567 Introduction to Coding Theory Ryan, T. TR 2:30 PM - 4:00 PM

One of MATH 217, 419, 420. (3). (BS). May not be repeated for credit.

Introduction to Coding Theory --- Introduction to coding theory focusing on the mathematical background for error-correcting codes. Topic include: Shannon's Theorem and channel capacity; review of tools from linear algebra and an introduction to abstract algebra and finite fields; basic examples of codes such and Hamming, BCH, cyclic, Melas, Reed-Muller, and Reed-Solomon; introduction to decoding starting with syndrome decoding and covering weight enumerator polynomials and the Mac-Williams Sloane identity

Required Textbook: **Introduction to Coding and Information Theory**, by S. Roman

MATH 571 Numerical Linear Algebra Krasny, R. TR 10:00 AM - 11:30 AM

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454 or permission from the instructor.. (3). (BS). May not be repeated for credit.

Direct and iterative methods for solving systems of linear equations (Gaussian elimination, Cholesky decomposition, Jacobi and Gauss-Seidel iteration, SOR, introduction to multi-grid methods, steepest descent, conjugate gradients), introduction to discretization methods for elliptic partial differential equations, methods for computing eigenvalues and eigenvectors.

Required: **Numerical Linear Algebra**, by Lloyd N. Trefethen and David Bau; ISBN-13: 978-0898713619

MATH 572 Numerical Methods for Differential Equations Karni, S. TR 10:00 AM - 11:30 AM

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Course Description: Math 572 is an introduction to numerical methods for differential equations, focusing on finite differences. This is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program, and should also appeal to graduate students from engineering and science departments, or anyone interested in scientific computing. It covers methods for ordinary and partial differential equations, including derivation of numerical schemes and systematic study of their accuracy, stability, and convergence. A solid background in advanced calculus and linear algebra, and proficiency in a computer language such as C, Fortran, or Matlab is a must.

Topics:

Finite differences, their derivation and truncation error. Two point boundary value problems, elliptic equations. Consistency, stability, and convergence. Efficient solution of resulting sparse linear systems (Jacobi, Gauss-Seidel, SOR, conjugate gradients, preconditioning). Multistep, Runge-Kutta methods for initial value problems. Absolute stability, stiff problems, and A-stability. Barrier theorems. Explicit and implicit finite difference schemes for parabolic equations. Stability and convergence analysis via the maximum principle, energy methods, and the Fourier transform. Operator splitting techniques, the alternating direction implicit method. Advection equation. Lax-Wendroff, upwind methods, the CFL condition. Hyperbolic systems, initial boundary value problems.

Textbook**:** (Required) **Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems** by R.J. LeVeque, ISBN: 978-0-898716-29-0

MATH 574 Financial Mathematics II Norgilas, D. TR 1:00 PM – 2:30 PM

MATH 526 and MATH 573. (Prerequisites enforced at registration.) Although MATH 506 is not a prerequisite for MATH 574, it is strongly recommended that either these courses are taken in parallel, or MATH 506 precedes MATH 574. (3). (BS). May not be repeated for credit.

This is a continuation of Math 573. This course discusses Mathematical Theory of Continuous-time Finance. The course starts with the general Theory of Asset Pricing and Hedging in continuous time and then proceeds to specific problems of Mathematical Modeling in Continuous-time Finance. These problems include pricing and hedging of (basic and exotic) Derivatives in Equity, Foreign Exchange, Fixed Income and Credit Risk markets. In addition, this course discusses Optimal Investment in Continuous time (Merton's problem), High-frequency Trading (Optimal Execution), and Risk Management (e.g. Credit Value Adjustment).

Required Text: **Arbitrage Theory in Continuous Time**, by Tomas Björk, 3rd 978- 0199574742

Stochastic Calculus for Finance II: Continuous-Time Models, by Steven E. Shreve, (2004) Springer, ISBN: 978-0387401010

MATH 575 Introduction to Theory of Numbers I Liu, Y. TR 10:00 AM - 11:30 AM

MATH 451 and 420 or permission of instructor. (1 - 3). (BS). May not be repeated for credit.

This is a first course in number theory. Topics covered include divisibility and prime numbers, congruences, quadratic reciprocity, quadratic forms, arithmetic functions, and Diophantine equations. Other topics may be covered as time permits or by request.

Optional Textbook: **A classical introduction to modern number theory**,(Springer GTM 84), by Ireland and Rosen, 2nd edition, ISBN: 978-0387973296, (Note: this textbook is available free to UM users on SpringerLink).

MATH 582 Introduction to Set Theory Smythe, Iian TR 1:00 PM - 2:30 PM

MATH 412 or 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

An introduction to axiomatic set theory, the foundations of mathematics, and the study of the infinite. We will cover topics including: the algebra of sets, the Zermelo- Fraenkel axioms of set theory, constructions of number systems, countable and uncountable sets, cardinals, ordinals, and the Axiom of Choice.

Required Textbook: **Elements of Set Theory**, by Herbert Enderton, ISBN: 9780122384400

MATH 590 Introduction to Topology Martone, G. MWF 12:00 PM - 1:00 PM

MATH 451. (3). (BS). May not be repeated for credit. Rackham credit requires additional work.

Topics include metric spaces, topological spaces, continuous functions and homeomorphisms, separation axioms, quotient and product topology, compactness, and connectedness. We will also cover a bit of algebraic topology (e.g., fundamental groups) as time permits.

Required Textbook: **Topology**, by James Munkres, ISBN: 978-0134689517

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MATH 592 Introduction to Algebraic Topology Wilson, J. MWF 10:00 AM - 11:00 AM

MATH 591. (3). (BS). May not be repeated for credit.

Algebraic topology studies topological invariants, i.e. algebraic structures constructed from topology which can help distinguish when two topological spaces are homeomorphic (i.e. "the same") or not. In the first part of the course, we study the fundamental group, its computation, and the theory of covering spaces. Some group theory is included, and some basic examples, such as compact surfaces. In the second part of the course, we introduce singular homology, as well as CW complexes and their homology, and examples of computation of homology. We also include geometric applications, such as Jordan's separation theorem in any dimension, and Invariance of domain.

No book for this course.

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MATH 594 Algebra II Snowden, A. TR 10:00 AM – 11:30 AM

MATH 593. (3). (BS). May not be repeated for credit.

Topics include group theory, permutation representations, simplicity of alternating groups for n>4, Sylow theorems, series in groups, solvable and nilpotent groups, Jordan-Holder Theorem for groups with operators, free groups and presentations, fields and field extensions, norm and trace, algebraic closure, Galois theory, and transcendence degree.

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MATH 597 Analysis II Rudelson, M. MWF 11:00 AM - 12:00 PM

MATH 451 and 420; or MATH 395. (3). (BS). May not be repeated for credit.

This is one of the basic courses for students beginning the study towards a Ph. D. degree in mathematics. The topics include general construction of a measure, Lebesgue measure on R and R^n, measurable functions, integration, Fubini theorem, complex and signed measures, Lebesgue-Radon-Nikodim theorem, maximal function, differentiation of measures, L_p spaces, introduction to Hilbert space and Fourier analysis.

Grades will be based on homeworks a midterm, and a final exam.
Textbook: (Optional) Real Analysis: Modern Techniques and Their Applications, Gerald B. Folland. ISBN: 978-047-1317-16

MATH 605 Several Complex Variables Jonsson, M. MWF 11:00 AM -12:00 PM

MATH 596 and 597

Analysis in several complex variables is formally just the extension of complex analysis in one variables to the higher-dimensional case. However, it has a much more geometric flavor, and the analytic techniques it provides can be quite powerful in fields such as complex algebraic geometry.

The course will start out with basic properties of holomorphic functions in several variables and some surprising phenomena, such as the Hartogs extension theorem. We will also study local properties of analytic sets, that is, zero loci of holomorphic functions.

After that we will focus on L^2 methods, one of the main techniques for constructing holomorphic functions. Along the way, we will study pseudoconvex sets and plurisubharmonic functions, the complex cousins of convex sets and functions in real Euclidean space.

Finally we will turn to geometric applications. We will study basic properties of complex manifolds and extend some of the results obtained in C^n. Towards the end of the course, we will prove Kodaira's Embedding Theorem, which gives a criterion for a compact complex manifold to embed into some projective space, and hence be algebraic.

Problem sets will be distributed about every other week.

The main reference will be the book "An Introduction to Complex Analysis in Several Variables" by Lars Hormander. It will be complemented by various notes.
Optional Text: **An Introduction to Complex Analysis in Several Variables**, by Lars Hormander

MATH 612 **Lie Algebras** **Daniels, P.** **MWF 2:00 PM - 3:00 PM**
Math 593 and 594 (or equivalent)

This course is an introduction to Lie algebras and their representations. Lie algebras arise naturally in the study of algebraic and Lie groups, and familiarity with these objects is fundamental in geometry and algebra. Moreover, Lie algebras are fascinating in their own right, and the study of finite dimensional Lie algebras leads to interesting combinatorial structures, such as root systems, Dynkin diagrams, and Coxeter groups. This course should be most valuable to those interested in representation theory and the study of algebraic and Lie groups, but it will likely be useful to those whose interests lie in related areas as well.

In this course we will study the basic theory of Lie algebras, with the majority of our focus on the complex semisimple case. We intend to cover most of the content of Humphrey's book, with a focus on structure theorems for Lie algebras, classifications of root systems, universal enveloping algebras, the Poincaré-Birkhoff-Witt Theorem, and highest weight modules.
Required Text: **Introduction to Lie Algebras and Representation Theory**, by James E Humphreys, ISBN: 0-387-90052-7

MATH 626/ STATS **High Dimensional Probability** **Mark Rudelson** **TR 11:30 AM – 1:00 PM**
MATH 625/STATS 625 and Graduate standing. (3). (BS). May not be repeated for credit.

High dimensional probability studies random quantities depending on alarge but fixed number of parameters. Unlike the classical probability con cerned with limits of various stochastic objects as the dimension or time tend to infinity, this theory strives to derive precise properties of these objects valid with high probability in all dimensions. We will be interested in the typical behavior of functions depending on a large number of independent or weakly dependent random variables. The examples may include the number of triangles in a random graph or the norm of a random matrix.

The course will include several topics. We will discuss measure concen tration inequalities for martingales, random vectors and random matrices. Another direction is controlling the suprema of a Gaussian and sub-Gaussian random processes using chaining, and the relation between geometric and probabilistic properties of a random process. We will also consider combi natorial approach to stochastic processes based on VC dimension and com binatorial dimension of the parameter set. The results will be illustrated by examples from pure mathematics, as well as computer science such as spec trum of Hermitian random matrices, dimension reduction, and compressed sensing

Recommended Textbooks: R. Vershynin, **High dimensional probability**. An introduction with appli cations in Data Science. Cambridge University Press, 2018.
R. van Handel, **Probability in high dimension**, Princeton University, 2014.

MATH 632 **Algebraic Geometry II** **Pixton, A.** **TR 11:30 AM - 1:00 PM**
MATH 631 and Graduate standing. (3). (BS). May not be repeated for credit.

This is a continuation of Math 631. Topics covered will include sheaf cohomology, algebraic curves, differentials, and the Riemann-Roch theorem. We will mostly be following Ravi Vakil's book "Foundations of Algebraic Geometry".

MATH 635 **Differential Geometry** **Uribe, A.** **MWF 1:00 PM - 2:00 PM**
591 or equivalent. Consent of instructor required. (3). (BS). May not be repeated for credit.

This is an introduction to Riemannian geometry. We will study the notions of connections, Riemannian metrics, geodesics, curvature, and Jacobi fields. We will cover the Hopf-Rinow and the Bonnet-Myers theorems. Then we will turn to complex manifolds and we will discuss some basic ideas in Kähler geometry. The book by do Carmo is not required but is highly recommended.

Optional Textbook: **Riemannian Geometry**, by Manfredo Perdigão do Carmo, 2nd Edition, ISBN: 0817634908

MATH 636 **Topics in Differential Geometry** **Luo, Y.** **TR 11:30 AM - 1:00 PM**
MATH 635 and Graduate standing. (3). (BS). May not be repeated for credit.

Conformal mappings play an extremely important role in complex analysis. In many applications, conformality turns out to be too restrictive. Quasiconformal mappings, first introduced by Grötzsch in the 1920s and developed by Ahlfors in the 1930s, provide more flexible settings. The importance of such mappings was only fully realized after Teichmüller had published his groundbreaking work on the classical moduli problem for Riemann surfaces around 1940. Quasiconformality turns out to be the correct notion of regularities for many applications in geometry and dynamics. Nowadays, the quasiconformal techniques are recognized as a standard tool in various areas such as Teichmüller theory, Kleinian groups and complex dynamics.

We will start with a basic discussion on conformal invariants: extremal lengths, hyperbolic metrics, and introduce the notion of quasiconformal mappings. We will prove the compactness theorem for quasiconformal mappings and the measurable Riemann mapping theorem which are two most important tools for the applications. The goal of the course is to discuss various applications using quasiconformal mappings. We are aiming to broadly cover
1) Teichmüller existence and uniqueness theorem.
2) Teichmüller space for hyperbolic surfaces and for rational maps.
3) Ber's simultaneous uniformization theorem.
4) Ahlfor's finiteness theorem in Kleinian groups.
5) Sullivan's no wondering domain theorem in complex dynamics. If time permits, we will also discuss the roles of quasiconformal techniques in some more recent progress on
6) Symmetries of the Mandelbrot set.
7) Classification of various hyperbolic rational maps 8) Degenerations of length spectrum for rational maps.

MATH 654 **Intro to Math Fluid Dynamics** **Doering, C.** **MW 8:30 AM - 10:00 AM**
MATH 451, 454, 555. Math 556 is recommended. Graduate standing. (3). (BS). May not be repeated for credit.

The Euler and Navier-Stokes (partial differential) equations: conservation laws for mass, momentum and energy, vorticity & potential flow, viscous flow &

hydrodynamic instabilities, turbulence theory & modeling, and challenges to proving existence & regularity of solutions of the incompressible 3D Navier-Stokes equations.

Required: **Applied Analysis of the Navier-Stokes Equations**, by C.R. Doering & J.D. Gibbon 1st edition, revised printing 2004, ISBN: 052144568X
Optional: **A Mathematical Introduction to Fluid Mechanics,** by A.J. Chorin & J.E. Marsden, Springer-Verlag (3rd edition, corrected fourth printing, 2000), ISBN: 0387979182

MATH 657 Nonlinear Partial Differential Equations Bieri, L. TR 2:30 PM - 4:00 PM
MATH 656. (3). (BS). May not be repeated for credit.

Partial Differential Equations are mathematical structures for models in science and technology. It is of fundamental importance in physics, biology and engineering design with connections to analysis, geometry, probability and many other subjects. The goal of this course is to introduce students (both pure and applied) to the basic concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics to be covered will include Sobolev spaces, second order elliptic equations, parabolic and hyperbolic equations, shock waves, and nonlinear wave equations. Course material will be taken from Chapters 5, 6, 7 and 12 of the text. Grading: Grades will be based on a few sets of homework and attendance and participation.

Required Textbook: **Partial Differential Equations**, by Lawrence C. Evans, 2nd. ISBN-13: 978-0821849743

MATH 669 Topics in Combinatorial Theory Barvinok, A. TR 1:00 PM - 2:30 PM
Good knowledge of linear algebra (3). (BS). May not be repeated for credit.

Topics in Convexity

We will discuss various topics in convexity with a view towards applications in algebra, geometry and analysis: Theorems of Rado, Helly and Caratheodory, valuations and the Euler characteristic, the structural theory of polyhedra and polytopes and of some non-polyhedral convex bodies such as the cone of positive semidefinite quadratic forms with applications to "quadratic convexity", and the Brunn - Minkowski theory. Grading: we will have a number of homework problem sets.

Optional: **A Course in Convexity**, by Alexander Barvinok, ISBN: 978-0821829684

MATH 678 Arithmetic of Elliptic Curves Li, S. MW 11:30 AM - 1:00 PM
MATH 594 and Graduate standing. (3). (BS). May be repeated for credit.

Introduction to modular forms and modular curves.

Optional Texts: **A First Course in Modular Forms**, by Fred Diamond and Jerry Shurman, ISBN: 0-387-23229-X
Modular Functions in One-Variable

MATH 684 Recursion Theory Norwood, Z. TR 11:30 AM - 1:00 PM

One of the primary goals of this is to prove Gödel's First Incompleteness Theorem, which implies that one cannot deduce all truths of the arithmetic of natural numbers from any computable set of true axioms. To achieve this, we will need to make sense of what "computable" means in this context, and to do so we will develop some of the theory of computable (or recursive) relations and functions.

The course will also cover some model theory of first-order structures, enough to prove Rosser's form of Gödel's Theorem and to prove the Paris–Harrington Theorem, which gives a natural combinatorial statement that, though true, cannot be proved from Peano Arithmetic.

If time allows, our development of the theory of recursive functions and relations might culminate in a proof of the Friedberg–Muchnik Theorem using a finite-injury priority argument. Either Math 481 or Math 681 would provide adequate preparation for the course. The only official prerequisite will be mathematical maturity appropriate for a 600-level course. Students unsure whether they are adequately prepared for the course are encouraged to write to the instructor.

No textbook required.

MATH 697 Topics in Topology Truong, L. MWF 2:00 PM - 3:00 PM
Graduate standing. (2 - 3). (BS). May not be repeated for credit.

The heart of low-dimensional topology is the study of three-manifolds and four-manifolds. Constructions of three-manifolds include taking the complement of a knot, performing Dehn surgery along a knot, or forming a covering space branched along a knot, while four-manifolds can be described by Kirby diagrams involving knots. Thus knot theory is intimately connected to three-dimensional and four-dimensional topology. This course will present modern invariants of low-dimensional manifolds and knots that arise from symplectic geometry or combinatorial methods.

Since its introduction in the late 1980s, Floer homology has become one of the most important tools in symplectic and low-dimensional topology. This course will introduce a version of Floer homology called Heegaard Floer homology, an invariant for knots, three- manifolds, and four-manifolds. The course will begin with background on Morse theory, symplectic geometry, and Heegaard diagrams. We will then define Heegaard Floer homology and compute examples. As applications we may discuss minimal genus problems, detecting exotic smooth structures on four-manifolds and finding topological properties of knots. On the combinatorial side, we will discuss Khovanov homology, a knot invariant with connections to contact and symplectic geometry, Heegaard Floer homology, and knot concordance. Topics will be chosen according to class interests.

This course is aimed at Ph.D. students in pure mathematics and will assume a basic understanding of smooth manifolds (smooth maps, derivatives, differential forms) and algebraic topology (homology, cohomology).

No Textbook required.

MATH 700 **TBD** **Nazari, A.** **TBD**

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MATH 715 **Advanced Topics in Algebra** **Kaletha, T.** **TR 10:00 AM – 11:30 AM**

Bruhat-Tits theory

We will discuss the structure theory due to Bruhat-Tits of reductive groups over Henselian discretely valued fields. This includes the construction and properties of a polysimplicial complex called the affine building, the affine root system, the integral models associated to each polysimplex, and the structure of the reductive quotient of the special fiber of such integral models, Bruhat, Cartan, and Iwasawa decompositions, the Kottwitz homomorphism and component groups of integral models, lattice chain models for classical groups, Moy-Prasad filtrations, unramified and tamely ramified descent.

MATH 732 **Topics in Algebraic Geometry II** **Perry, A.** **TR 1:00 PM - 2:30 PM**
MATH 631-632 or equivalent. (3). (BS). May not be repeated for credit.

Bridgeland stability

Bridgeland stability conditions are a powerful tool for extracting geometry from homological algebra. In particular, they give a framework for studying moduli spaces of objects in a triangulated category, such as the derived category of an algebraic variety. The subject was born as a mathematical interpretation of work in string theory, but has since impacted many areas, including classical algebraic geometry, derived categories of coherent sheaves, enumerative geometry, homological mirror symmetry, and symplectic geometry.

In this course, we will develop the general theory of Bridgeland stability conditions, study some of the known constructions of stability conditions, and discuss moduli spaces of Bridgeland stable objects and their applications in algebraic geometry.
Textbook: No book for this course

MATH 797 **Advanced Topics in Topology:** **Spatzier, R.** **MWF 3:00 PM – 4:00 PM**
Math 695 or equivalent level of study. (3). (BS). May not be repeated for credit.

GEOMETRIC STRUCTURES, DYNAMICS, RIGIDITY

Geometry is static, no fun, dynamics a bore.* But put the two together, and you have drama, depth, fantastic theorems, and fabulous examples. **

And there is beautiful duality: take a geometric structure with lots of automorphisms. When is it completely determined (RIGIDITY!!) Sometimes you can use dynamical features of the automorphisms to tame the beast.

Vice versa, take a group acting on a manifold. When do you know the action (RIGIDITY!!)? Sometimes you can build (or assume) geometric structures to tame the beast.

Some of this was realized in the geometric rigidity program started in the eighties, well really goes back further to Mostow and Margulis. Then the Zimmer program studies actions of complicated and LARGE groups, like SL(3,Z).

Classically there is the study of automorphism groups of certain geometric structures, that they are Lie groups (isometries, affine structures) - or not (volume preserving, symplectic structures). Then Gromov invented Gromov rigid structures - very useful in some dynamics.

The dynamicists found normal forms that tame maps, a kind of geometric structure.

Then there is rigidity, suddenly, out of the blue. So I hope to discuss some aspects of these ideas.

Of course, there is some background I will likely need to fill in - depending on the audience. Maybe you all know it better than me, and can teach me.

* I am lying! I am lying! *** Not a lie!!!
No book for this course