On the Tracy-Widom β - distribution for $\beta = 6$.

Alexander Its
Department of Mathematical Sciences
Indiana University Purdue University Indianapolis

1 Abstract

We study the Tracy-Widom distribution function for Dyson's β - ensemble with $\beta = 6$. The starting point of our analysis is the recent work of I. Rumanov where he produces a Lax-pair representation for the Bloemendal-Virag equation. The latter describes the Tracy-Widom functions corresponding to the general values of β. Using his Lax pair, Rumanov derives an explicit formula for the Tracy-Widom $\beta = 6$ function in terms of the second Painlevé transcendent and the solution of an auxiliary ODE. We show that Rumanov's Lax-pair can be interpreted as a certain gauge transformation of the standard Lax pair for the second Painlveé equation. The gauge-interpretation of Rumanov's Lax-pair allows us to highlight the steps of the original Rumanov's method which are needed rigorous justifications in order to make the method complete. We provide rigorous justification of some of this steps. The key issue which we also discuss and which is still open is the question of integrability of the auxiliary ODE in Rumanov's formula. We note that this question is crucial for the rigorous asymptotic analysis of the Tracy-Widom function. This is a joint work with T. Grava, A. Kapaev and F. Mezzadri.