

Title: Painlevé equations and orthogonal polynomials

Abstract: The past few decades it has become clear that Painlevé equations show up at various places in the theory of orthogonal polynomials. I will explain some of these occurrences. For semiclassical weights the recurrence coefficients in the three term recurrence formula for orthogonal polynomials often satisfy a discrete Painlevé equation. Furthermore, these recurrence coefficients also satisfy a Painlevé differential equation. The discrete Painlevé equation and the differential equation are connected through Toda type equations and Bäcklund transformations. Rational solutions of Painlevé equations are usually in terms of determinants (Wronskians) of orthogonal polynomials and the asymptotic distribution of the zeros and poles can be found by analyzing a Riemann-Hilbert problem for orthogonal polynomials. Many of the special function solutions of Painlevé equations are connected to the moment matrices (Hankel or Toeplitz) for semi-classical weights. Finally, the asymptotic behavior of orthogonal polynomials near critical points (points where the zero density vanishes or becomes unbounded) can be obtained using Painlevé transcendents and Riemann-Hilbert analysis.