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**The Density Matrix for the Ground State of 1-d Impenetrable Bosons in a Harmonic Trap**

We treat the problem of the density matrix $ \rho(x,y) $ of a system of $ N $ bosons interacting in one dimension with an infinite contact potential - the impenetrable Bose gas - in their ground state and confined in a harmonic trap. We do this by recasting the problem as an orthogonal polynomial system (OPS) on $ \R $ with a deformed Gaussian weight.

Employing tools from the theory of OPS with semi-classical weights we explicitly construct a number of ways to characterize $ \rho $ such as recurrence relations $ N \mapsto N+1 $ and partial differential equations with respect to $ x,y $. In the process we identify an integrable system here, which turns out to be a specialisation of the degenerate, two-variable Garnier system $ L(1,1,3;2) $. With no additional effort we investigate this system in full generality with parameters $ \mu, \nu, N $ deriving the Lax pairs of spectral and deformation derivatives, and the degenerate Schlesinger transformations $ N \mapsto N\pm 1 $, $ \mu \mapsto \mu\pm 1 $ and $ \nu \mapsto \nu\pm 1 $.