Computational methods for the multivalued Painlevé transcendents and their application to the exploration of families of tronquée solutions to the third Painlevé equation

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The numerical pole field solver (B. Fornberg and J.A.C. Weideman, J. Comput. Phys. 230:5957–5973, 2011) is extended to enable the computation of the multivalued Painlevé transcendents, which are the solutions to the third, fifth and sixth Painlevé equations, on multiple sheets of their Riemann surfaces. Solutions to these equations are displayed and numerical evidence is provided for the existence of solutions to the sixth Painlevé equation that have pole-free sectors, known as tronquée solutions.

The computational method is used to explore families of tronquée solutions to the third Painlevé equation that were identified by McCoy, Tracy and Wu (J. Math. Phys. 18:1058–1092, 1977). Limiting cases, in which the solutions are singular functions of the parameters, are also investigated and it is shown that a particular set of limiting solutions is expressible in terms of special functions. Solutions that are single-valued, logarithmically branched and algebraically branched, with any number of distinct sheets, are encountered. The algebraically branched solutions have multiple pole-free sectors on their Riemann surfaces that are accounted for by using asymptotic formulae and Bäcklund transformations.