# Topology QR Exam, Jan 2016, AM

1. Let $\Sigma_g$ be a compact connected surface of genus $g$. Let $f : \Sigma_g \to \Sigma_3$ be a covering space. Show that $g$ must be odd.

2. Let $K$ be the complete graph on 4 letters, i.e., $K$ has 4 vertices, and there is a unique edge connecting each pair of distinct vertices.

   (a) Calculate $\pi_1(K)$.

   (b) Show that $\Sigma_2$ is not a deformation retract of any space homotopy equivalent to $K$.

3. For each of the following, write "True" or "False", and give a proof or example justifying your answer.

   (a) Every map $S^2 \to S^1 \times S^1$ is null-homotopic.

   (b) Every map $S^1 \times S^1 \to S^2$ is null-homotopic.

4. Give the definition of a Lie group and prove that the matrix group $\mathrm{SL}(2, \mathbb{R})$ is a Lie group. Is it compact?

5. Either prove or give a counterexample for the following statement:

   For any complex vector bundle $E$ over a finite CW complex $X$ with zero section $o(X) \subset E$, the induced map $H_2(E - o(X)) \to H_2(X)$ is surjective.

# Topology QR Exam, Jan 2016, PM

1. Let $U, V \subset S^n$, $n \geq 2$, be two non-empty connected open subsets such that $S^n = U \cup V$. Show that $U \cap V$ is connected

2. Fix a prime number $p$. Let $X$ be a finite CW complex with an action of $G = \mathbf{Z}/p$.

   (a) If $\chi(X)$ is not divisible by $p$, show that the $G$ action on $X$ has a fixed point.

   (b) Give an example of such an action that is fixed point free with $\chi(X) = 0$.

3. If $M$ is a compact smooth manifold, show that its fundamental group $\pi_1(M)$ is finitely generated, and furthermore, it is also finitely presented.

4. Show that the sphere $S^2$ is not a Lie group, but $S^3$ is a Lie group.

5. Construct infinitely many nonisomorphic compactifications of the open interval $(0, 1)$. (Recall that two compactifications $\overline{X}^1$ and $\overline{X}^2$ of a topological space $X$ are isomorphic if the identity map on $X$ extends to a homeomorphism $\overline{X}^1 \to \overline{X}^2$.)