Syllabus for Math 596: Complex Analysis

1. **Holomorphic functions**. Complex differentiability and the Cauchy–Riemann equations. Holomorphic (=analytic) functions. Harmonic functions and harmonic conjugates. Elementary functions.

2. **Cauchy's formula and consequences**. Differential forms of degree 1 and 2. Real and complex line integrals. Stokes' theorem in the plane. Cauchy's theorem. The Cauchy integral formula. The Cauchy estimates. Liouville's theorem. The theorems of Morera and Goursat. The mean value property. The maximum principle. Power series and Laurent series expansions.

3. **Local properties**. Order of zeros and critical points. Open mapping theorem. Classification of isolated singularities. Characterization of removable singularities and poles. Casorati–Weierstrass theorem, Picard's theorem (statement only). Meromorphic functions.

4. **The Riemann sphere**. The Riemann sphere. Stereographic projection. Möbius transformations and their mapping properties. Holomorphic functions on domains in the Riemann sphere. Meromorphic functions as maps into the Riemann sphere.

5. **Schwarz's Lemma**. Schwarz's Lemma, Pick's Lemma. Automorphisms of the unit disc.

6. **Conformal mappings**. Definition of conformality. Basic examples of conformal mappings onto the unit disc. The Riemann mapping theorem.

7. **The argument principle and plane topology**. The argument principle. Rouché's theorem. Characterizations of simply connected domains. Winding numbers. The Jordan curve theorem (statement only).

8. **Residue calculus**. Residue of a function at an isolated singularity. The residue theorem. Evaluations of integrals using the residue theorem.

9. **Families of holomorphic functions**. Equicontinuity and the Arzelà–Ascoli theorem. Montel's theorem. Hurwitz's theorem.

10. **Other**. The Poisson integral formula. Schwarz reflection. Infinite products. Convergence tests (M-test) for infinite series and products.

References

- Gamelin; *Complex Analysis.*
- Ahlfors; *Complex Analysis.*