Department of Mathematics

Syllabus for Math 593: Algebra I.

1. Basics about commutative rings with identity and ideals, prime and maximal ideals, polynomial rings, PIDs, UFDs and Euclidean rings. Fields of fractions. Mention that a polynomial ring in one and hence several variables over a UFD is a UFD. (The proof, including the Gauss Lemma, is an optional topic.) Basics about modules, linear maps, submodules, direct sums and free modules. (Optional topics: Noncommutative rings, chain conditions, Noetherian rings.)

2. Structure theory of finitely generated modules over a PID. Applications to classification of fintely generated abelian groups and to existence and uniqueness of rational and Jordan canonical forms. Students should know how to pass from a presentation of a finitely generated abelian group to its expression as a direct sum of cyclic groups by row and column operations on a matrix, and how to find the rational and Jordan forms of a matrix.

3. A real symmetric (resp. Hermitian) matrix is orthogonally (resp. unitarily) conjugate to a real diagonal matrix. Sylvester's Theorem on signature over the reals, structure theorems for alternating (i.e. skew-symmetric) matrices. (If there is a shortage of time, these topics may be left for the students to read on their own.) (Optional topics: Isometry groups for various bilinear forms. Greater detail on the above topics. Discussion of more general base fields. The Cartan-Dieudonné Theorem. Witt's Theorem on extending isometries. Normal matrices.)

4. Tensor products of modules over a commutative ring with strong emphasis on the field case. The exterior algebra, at least for vector spaces over a field. Applications to determinants. (Optional topic: The symmetric algebra.)

References:

Dummit, D., and Foote, R., *Abstract Algebra.*
Jacobson: *Basic Algebra.*
Lang: *Algebra.*
Hartley and Hawkes: *Rings, Modules & Linear Algebra*,
Hungerford: *Algebra.*
Zariski and Samuel: *Commutative Algebra.*

Topics listed as optional will not be included on the Qualifying Review Examinations. Other topics may be included in the Qualifying Review Examination even when they are not covered in a particular course.

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