Winter 2022 Graduate Course Descriptions

****MATH 501 AIM Student Seminar Alben Fri 1:00 PM – 2:00 PM**
Fri 3:00 PM – 4:00 PM

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

No book for this course.

****MATH 506/IOE Stochastic Analysis for Finance Bayraktar, E. TR 10:00 AM - 11:30 AM**

Math 526. Graduate students or permission of instructor. (3). (BS). May not be repeated for credit.

The aim of this course is to teach the probabilistic techniques and concepts from the theory of stochastic processes required to understand the widely used financial models. In particular concepts such as martingales, stochastic integration/calculus, which are essential in computing the prices of derivative contracts, will be discussed. The specific topics include: Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales), martingales in continuous time, stochastic integration (including It\^{o}'s formula), stochastic differential equations (including Feynman-Kac formula), change of measure (including Girsanov's theorem and change of numeraire), and, time permitting, stochastic control (including Merton problem). Applications from various areas of Finance (including, pricing of derivatives, risk management, etc) are used to illustrate the theory.

****MATH 521 Life Contingencies II Natarajan, R. TR 10:00 AM - 11:30 AM**

MATH 520 with a grade of C- or higher. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

This course extends the single decrement and single life ideas of MATH 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520-521 covers the Part 4A examination of the Casualty Actuarial Society and covers the syllabus of the Course 150 examination of the Society of Actuaries. Concepts and Calculation are emphasized over proof.

****MATH 524 Loss Models II Young, J. TR 8:30 AM - 10:30 AM**

STATS 426 and MATH 523. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management is of major concern to all financial institutions, especially casualty insurance companies. This course is relevant for students in insurance and provides background for the professional examination in Short-Term Actuarial Modeling offered by the Society of Actuaries (Exam STAM). Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc.) and have at least Junior standing.

Content: Frequentist and Bayesian estimation of probability distributions, model selection, credibility, simulation, and other topics in casualty insurance.

****MATH 525/STATS Probability Theory Koziol, K. TR 10:00 AM - 11:30 AM**
Koziol, K. TR 1:00 PM - 2:30 PM

MATH 451 (strongly recommended). MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

****MATH 526/STATS Stochastic Processes with Discrete State Spaces Wang, Z. TR 8:30 AM - 10:00 AM**
Wang, Z. TR 10:00 AM - 11:30 AM
Chakraborty, P. TR 1:00 PM - 2:30 PM

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

Textbook: Durrett, Richard. (2016). Essentials of Stochastic Processes, 3rd Ed. Springer.

MATH 547/BIOINF/STATS Probabilistic Modeling in Bioinformatics Rajapakse, I. TR 4:00 PM - 5:30 PM

MATH, Flexible, due to diverse backgrounds of intended audience. Basic probability (level of MATH/STATS 425), or molecular biology (level of BIOLOGY 427), or biochemistry (level of CHEM/BIOLCHEM 451), or basic programming skills desirable or permission. (3). (BS). May not be repeated for credit.

This course is open to graduate students and upper-level undergraduates in applied mathematics, bioinformatics, statistics, and engineering, who are interested in learning from data. Students with other backgrounds such as life sciences are also welcome, provided they have maturity in mathematics. The mathematical content in this course will be linear algebra, multilinear algebra, dynamical systems, and information theory. This content is required to understand some common algorithms in data science. I will start with a very basic introduction to data representation as vectors, matrices, and tensors. Then I will teach geometric methods for dimension reduction, also known as manifold learning (e.g. diffusion maps, t-distributed stochastic neighbor embedding (t-SNE), etc.), and topological data reduction (introduction to computational homology groups, etc.). I will bring an application-based approach to spectral graph theory, addressing the combinatorial meaning of eigenvalues and eigenvectors of their associated graph matrices and extensions to hypergraphs via tensors. I will also provide an introduction to the application of dynamical systems theory to data including dynamic mode decomposition. Real data examples will be given where possible and I will work with you write code implementing these algorithms to solve these problems. The methods discussed in this class are shown primarily for biological data, but are useful in handling data across many fields. A course features several guest lectures from industry and government.

**551 INTRODUCTION TO REAL ANALYSIS Baik, J. M/W/F 2:00 PM – 3:00 PM

Advanced Calculus(295/297/451) and Linear Algebra(217/296)

This is a course that introduces the Lebesgue measure theory and a few other topics in real analysis for advanced math undergraduates, masters students, and AIM and non-math Ph.D. students. We plan to cover (1) Lebesgue measure on Rn, (2) Lebesgue integral, (3) differentiation, (4) Lebesgue-Stieltjes measure, (5) product measures, and (6) abstract metric spaces. If time permits, Lp spaces will also be covered. This course has some overlaps with MATH 597, but this course will proceed at a gentler pace and emphasize measures on Rn instead of general spaces.

Textbooks: Royden & Fitzpatrick. (2017) **Real Analysis**, 4th Ed. Pearson. (Required)
Tao, Terry. (2011). **An Introduction to Measure Theory**. American Mathematical Society. (Optional)

MATH 555 Introduction to Complex Variables Vig, A. MW 8:30 AM - 10:00 AM

MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

This course is an introduction to the theory of complex-valued functions of a complex variable with substantial attention to applications in science and engineering. The prerequisite of a course in advanced calculus is essential. This is a core course for the AIM graduate program.

Required Textbook: **Complex Variables and Applications**, by James Ward Brown and Ruel V. Churchill, 9th Ed., ISBN 978-0073383170

**MATH 557 Applied Asymptotic Analysis Borcea, L. TR 1:00 PM - 2:30 PM

MATH 217, 419, or 420; MATH 451; and MATH 555 or 596. (3). (BS). May not be repeated for credit.

The course introduces and analyzes mathematical methods for obtaining approximate solutions to integrals and differential equations that do not have closed form solutions and are also difficult to solve numerically. The problems are known collectively as asymptotic or perturbative, because they involve small or large parameters that are derived in practice by scaling arguments. The goal is to approximate their solutions in asymptotic limits of the parameters tending to zero or infinity.

Covered topics: Asymptotic expansions and series; Asymptotic expansions of integrals (Laplace type integrals, stationary phase, steepest descent); Asymptotic analysis of differential equations: regular and singular perturbation problems, method of multiple scales, asymptotics of oscillatry phenomena, boundary and inner layers.

Examples of applications discussed are: high frequency wave propagation and imaging with waves.

Required Textbook: **Applied Asymptotic Analysis** by P.D. Miller, AMS, ISBN 0-8218-4078-9

**MATH 566 Combinatorial Theory Fomin, S. TR 11:30 AM - 1:00 PM

MATH 465 group theory and abstract linear algebra. (3). (BS). May not be repeated for credit.

This course is an introduction to algebraic and enumerative combinatorics at the beginning graduate level. Topics include: fundamentals of algebraic graph theory; applications of linear algebra to enumeration of matchings, tilings, and spanning trees; combinatorics of electric networks; partially ordered sets; integer partitions and Young tableaux.

Optional Textbook**: Algebraic Combinatorics: Walks, Trees, Tableaux, and More**, R. P. Stanley, ISBN 978-1-4899-9285-7

**MATH 567 Introduction to Coding Theory Ho, W. TR 2:30 PM - 4:00 PM

One of MATH 217, 419, 420. (3). (BS). May not be repeated for credit.

Introduction to Coding Theory --- Introduction to coding theory focusing on the mathematical background for error-correcting codes. Topic include: Shannon's Theorem and channel capacity; review of tools from linear algebra and an introduction to abstract algebra and finite fields; basic examples of codes such and Hamming, BCH, cyclic, Melas, Reed-Muller, and Reed-Solomon; introduction to decoding starting with syndrome decoding and covering weight enumerator polynomials and the Mac-Williams Sloane identity

Required Textbook: **Introduction to Coding and Information Theory**, by S. Roman

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**MATH 571 Numerical Linear Algebra Viswanath, D. TR 10:00 AM - 11:30 AM

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454 or permission from the instructor.. (3). (BS). May not be repeated for credit.

This class is about solving linear systems numerically, finding eigenvalues and singular values, and solving linear least squares problems. We will discuss condition numbers, numerical stability, QR factorization, Cholesky, SVD, and the QR algorithm as well as iterative methods (GMRES, Arnoldi, Conjugate Gradients, Lanczos). The following applications are included: KKT conditions, convergence of the perceptron, and back propagation networks. The homework assignments will use either Python or Matlab, with the choice left to the student.

Required: **Numerical Linear Algebra**, by Lloyd N. Trefethen and David Bau; ISBN-13: 978-0898713619

**MATH 572 Numerical Methods for Differential Equations Karni, S. TR 11:30 AM – 1:00 PM

MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Course Description: Math 572 is an introduction to numerical methods for differential equations, focusing on finite differences. This is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program, and should also appeal to graduate students from engineering and science departments, or anyone interested in scientific computing. It covers methods for ordinary and partial differential equations, including derivation of numerical schemes and systematic study of their accuracy, stability, and convergence. A solid background in advanced calculus and linear algebra, and proficiency in a computer language such as C, Fortran, or Matlab is a must.

Topics:
Finite differences, their derivation and truncation error. Two point boundary value problems, elliptic equations. Consistency, stability, and convergence. Efficient solution of resulting sparse linear systems (Jacobi, Gauss-Seidel, SOR, conjugate gradients, preconditioning). Multistep, Runge-Kutta methods for initial value problems. Absolute stability, stiff problems, and A-stability. Barrier theorems. Explicit and implicit finite difference schemes for parabolic equations. Stability and convergence analysis via the maximum principle, energy methods, and the Fourier transform. Operator splitting techniques, the alternating direction implicit method. Advection equation. Lax-Wendroff, upwind methods, the CFL condition. Hyperbolic systems, initial boundary value problems.

Textbook: (Required) **Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems** by R.J. LeVeque, ISBN: 978-0-898716-29-0

**MATH 574   Financial Mathematics II   Norgilas, D.   TR 1:00 PM – 2:30 PM

MATH 526 and MATH 573. (Prerequisites enforced at registration.) Although MATH 506 is not a prerequisite for MATH 574, it is strongly recommended that either these courses are taken in parallel, or MATH 506 precedes MATH 574. (3). (BS). May not be repeated for credit.

This is a continuation of Math 573. This course discusses Mathematical Theory of Continuous-time Finance. The course starts with the general Theory of Asset Pricing and Hedging in continuous time and then proceeds to specific problems of Mathematical Modeling in Continuous-time Finance. These problems include pricing and hedging of (basic and exotic) Derivatives in Equity, Foreign Exchange, Fixed Income and Credit Risk markets. In addition, this course discusses Optimal Investment in Continuous time (Merton's problem), High-frequency Trading (Optimal Execution), and Risk Management (e.g. Credit Value Adjustment).

Required Text: **Arbitrage Theory in Continuous Time**, by Tomas Björk, 3rd 978- 0199574742
Stochastic Calculus for Finance II: Continuous-Time Models, by Steven E. Shreve, (2004) Springer, ISBN: 978-0387401010

**MATH 575   Introduction to Theory of Numbers I   Prasanna, K.   TR 10:00 AM - 11:30 AM

MATH 451 and 420 or permission of instructor (Some background in abstract algebra – basics of groups, rings, fields – will be helpful). (1 - 3). (BS). May not be repeated for credit.

This course will be an introduction to number theory. Basic topics to be covered include factorization, congruences, Gauss and Jacobi sums, classical reciprocity laws such as quadratic and cubic reciprocity and some basic algebraic number theory.

No formal prerequisites, but some familiarity with abstract algebra, including the theory of groups, rings and fields will be assumed.

Required Textbook: **A classical introduction to modern number theory**,(Springer GTM 84), by Ireland and Rosen, 2nd edition, ISBN: 978-0387973296. (Note: This textbook is available free to UM users on SpringerLink - https://search.lib.umich.edu/catalog/record/016750641).

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*MATH 582   Introduction to Set Theory   Blass, Andreas   TR 1:00 PM - 2:30 PM

MATH 412 or 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

An introduction to axiomatic set theory, the foundations of mathematics, and the study of the infinite. We will cover topics including: the algebra of sets, the Zermelo- Fraenkel axioms of set theory, constructions of number systems, countable and uncountable sets, cardinals, ordinals, and the Axiom of Choice.

Required Textbook: **Elements of Set Theory**, by Herbert Enderton, ISBN: 9780122384400

**MATH 590   Introduction to Topology   Truong, Linh.   MWF 12:00 PM - 1:00 PM

MATH 451. (3). (BS). May not be repeated for credit. Rackham credit requires additional work.

The purpose of this course is to introduce basic concepts of topology. Most of the course will be devoted to the fundamentals of general (point set) topology. Topics include metric spaces, topological spaces, continuous functions and homeomorphisms, separation axioms, quotient and product topology, compactness, and connectedness. We will also cover a bit of algebraic topology (e.g., fundamental groups) as time permits.

Required Textbook: **Topology**, by James Munkres, ISBN: 978-0134689517

MATH 592   Introduction to Algebraic Topology   Wilson, J.   MWF 10:00 AM - 11:00 AM

MATH 591. (3). (BS). May not be repeated for credit.

Algebraic topology studies topological invariants, i.e. algebraic structures constructed from topology which can help distinguish when two topological spaces are homeomorphic (i.e. "the same") or not. In the first part of the course, we study the fundamental group, its computation, and the theory of covering spaces. Some group theory is included, and some basic examples, such as compact surfaces. In the second part of the course, we introduce singular homology, as well as CW complexes and their homology, and examples of computation of homology. We also include geometric applications, such as Jordan's separation theorem in any dimension, and Invariance of domain.

No book for this course.

MATH 594   Algebra II   Speyer, D.   TR 10:00 AM – 11:30 AM

MATH 593. (3). (BS). May not be repeated for credit.

Topics include group theory, permutation representations, simplicity of alternating groups for n>4, Sylow theorems, series in groups, solvable and nilpotent groups, Jordan-Holder Theorem for groups with operators, free groups and presentations, fields and field extensions, norm and trace, algebraic closure, Galois theory, and transcendence degree.

**MATH 597   Analysis II   Baik, J.   MWF 11:00 AM - 12:00 PM

MATH 395/451 and 590. (3). (BS). May not be repeated for credit.

The topics include abstract measures, Lebesgue measure on R and R^n, measurable functions, integration, Fubini theorem, complex and signed measures, Lebesgue-Radon-Nikodim theorem, maximal function, differentiation of measures, and L_p spaces. If time permits, we will also cover introduction to Hilbert space and Fourier analysis.

Textbook: Real Analysis: Modern Techniques and Their Applications, Gerald B. Folland. ISBN: 978-047-1317-16 (Required)

__**MATH 604 Complex Analysis Barrett, D. MWF 1:00 PM - 2:00 PM__

Prerequisites: first-year graduate analysis

This is a second course in one-dimensional complex analysis, serving as a follow-up to a course such as Math 596.

The course will cover a number of fundamental topics in one-dimensional complex analysis, such as
(1) the Cauchy transform of a measure; use of the inhomogeneous Cauchy-Riemann equations to construct holomorphic and meromorphic functions;
(2) zeros and growth of holomorphic functions on the plane and on the unit disk; factorization for function spaces on the unit disk;
(3) construction of mappings with prescribed angular distortion (Beltrami's equation);
(4) geometric function theory: univalent functions, Schwarzian derivatives, conformal metrics;
(5) potential theory: harmonic and subharmonic functions, Perron's solution of the Dirichlet problem;
(6) Riemann surfaces: the Riemann surface of a holomorphic function, fractional power series, Newton polygons, algebraic functions; Riemann surfaces in general; uniformization theorem (every Riemann surface is covered by the disk, the plane, or the sphere); all Riemann surfaces admit non-constant meromorphic functions;
(7) complex structures on the torus, elliptic functions. Other topics may be added as time permits. (Choices here may reflect the background and interests of the enrolled students.)

MATH 612 Lie Algebras Chan, C. MW 11:30 AM - 1:00 PM

Math 593 and 594 (or equivalent)

Lie algebras arise naturally in many areas of mathematics and physics, and familiarity with these objects is fundamental for geometric and algebraic reasons. Lie algebras are fascinating in their own right, and the study of finite dimensional Lie algebras leads to interesting combinatorial structures, such as root systems, Dynkin diagrams, and Coxeter groups. This course should be valuable to those interested in representation theory and the study of algebraic and Lie groups, but should very likely also be useful to those whose interests lie in related areas such as combinatorics, geometry, and physics.

In this course, we will study the basic theory of Lie algebras, with the majority of our focus on the complex semisimple case. We intend to cover most of the content of Humphrey's book (Introduction to Lie Algebras and Representation Theory), especially structure theorems for Lie algebras, classifications of root systems, universal enveloping algebras, the Poincaré-Birkhoff-Witt Theorem, and highest weight modules.

Required Text: **Lie Groups and Lie Algebras**(Chapters 4-6), by Bourbaki, 978-3-540-69171-6

__**MATH 615 Topics in Commutative Algebra Hochster, M. MWF 2:00 PM - 3:00 PM__

Topic: Tight Closure Theory

Math 614(631 desired)

This course will present the theory of tight closure, which gives a systematic method of proving results in positive characteristic p. We will also discuss positive characteristic methods more generally, and how one uses these results to get corresponding theorems for rings containing a field of characteristic zero, which is known as "reduction to characteristic p." Many deep theorems only have proofs by these methods. Properties described in terms of characteristic p phenomena lead to notions of various kinds of "good" singularity that are closely connected to properties defined in characteristic 0. A very large number of open questions will be raised. Applications of tight closure theory include the Cohen-Macaulay property for rings of invariants of certain actions of algebraic groups, the Briancon-Skoda theorem on integral closures of ideals, and comparison theorems for ordinary and symbolic powers of prime ideals. The material will be presented in such a way as to take account in of differences in the background levels of the students.

No Textbook Required

__**MATH 623 Computational Finance Feng, Q. TR 8:30 AM - 10:00 AM__

This is a course on computational methods in finance and financial modeling. Using financial mathematics (like many branches of applied mathematics) in practice involves three tasks. First, one needs to develop mathematical models that accurately describe the real-life phenomena that one wishes to study – in the present case, probabilistic models for the evolution of prices, interest rates, and other relevant quantities. Once a model is chosen, the second task is to derive theoretical equations, or formulas, which establish relations between various objects in the financial markets: for example, the prices of derivative securities (options, bonds, etc), and the risk profiles of investment portfolios, as functions of risk factors. Finally, one needs to design and implement numerical methods to perform computations based on these formulas and equations.

This course is concerned with the latter task, and it has three components. In the first part, we will study the lattice (or, tree) methods, which correspond to the models based on discrete time Markov chains (e.g. the binomial model). We will discuss the pricing and hedging of financial derivatives in such models, using the arbitrage theory, or, more specifically, the risk-neutral pricing. We will, then, proceed to analyze the diffusion-based models of financial mathematics (including, e.g., the Black-Scholes model) and the associated Partial Differential Equations (PDEs).

We will discuss the finite difference methods, which provide numerical approximations for solutions to these PDEs. Both explicit and implicit schemes will be studied, the concepts of stability and convergence will be introduced, and a connection between the finite difference schemes and lattice methods will be established. After that, we will turn to the Monte Carlo simulations – the most general computational method for probabilistic equations. This method is based on generating a large number of paths of the underlying stochastic processes, in order to approximate the expectations of certain functions of these paths (which, e.g., may determine prices, portfolio weights, default probabilities, etc.). In addition to the standard Monte Carlo algorithms, we will study the variance reduction techniques, which are often necessary to obtain accurate results. The computational methods presented in this course will be illustrated using the popular models of equity markets (e.g. Black-Scholes, Heston), fixed income (e.g. Vasicek, CIR, Hull-White, Heath-Jarrow-Morton) and credit risk (e.g. Merton, Black-Cox, reduced-form models).

__**MATH 626/ STATS High Dimensional Probability Rudelson, M. TR 11:30 AM – 1:00 PM__

MATH 625/STATS 625 and Instructor’s Approval. (3). (BS). May not be repeated for credit.

The course will focus on discrete time Markov chains and ergodic theory. After covering the basics of Markov chain theory, we will concentrate on mixing in finite chains. Mixing time characterizes how fast a Markov chain approaches the stationary distribution. This theory has seen rapid progress in the last 20 years. Mixing in Markov chains plays a key role in many sampling and approximate counting algorithms in computer science.

Recommended Textbooks: R. Durrett, Probability: theory and examples, fifth edition. Cambridge University Press, 2019. ISBN 978-1-108-47368-2

MATH 628 Machine Learning for Finance I Nazari, A. F 12:00 PM - 1:30 PM

****MATH 632 Algebraic Geometry II Stapleton, D. TR 11:30 AM - 1:00 PM**

MATH 631 and Graduate standing. (3). (BS). May not be repeated for credit.

This is a continuation of Math 631. Topics will include sheaf cohomology, algebraic curves, differentials, and the Riemann-Roch theorem.

Optional Texts: **Algebraic Geometry**, by Robin Hartshorne
Foundations of Algebraic Geometry, by Ravi Vakil (unpublished and available online)

****MATH 635 Differential Geometry Bieri, L. MWF 1:00 PM - 2:00 PM**

591 or equivalent. Consent of instructor required. (3). (BS). May not be repeated for credit.

In this course we will discuss important local and global aspects of differential geometry as well as the relation with the underlying topology. We will start with manifolds, connections, Riemannian metrics, curvature and the basic tools such as variational methods, Jacobi fields, and comparison theorems. Then we will continue to study sphere theorems, rigidity theorems and related topics. If time permits we will consider more advanced topics.

Required Textbook: **Riemannian Geometry**, by Manfredo Perdigão do Carmo, 2nd Edition, ISBN: 0817634908

****MATH 636 Topics in Differential Geometry Burns, D. TR 10:00 AM - 11:30 AM**
Geometric Quantization

MATH 635 and Graduate standing. (3). (BS). May not be repeated for credit.

Geometric quantization seeks to give a more or less canonical way to associate a Hilbert space and operators to a classical mechanical system, using the underlying geometry of the classical system. Often this requires one to look at a classical phase space (a symplectic manifold) and choose a way to split the variables in half, usually by the choice of a polarization. This can be real or complex, and it is a basic question as to whether all choices lead to the same physics. We will concentrate on the geometry here.

More specifically, we will discuss the following: symplectic mechanics, (real) integrable systems, complex manifolds and complex polarizations, relation to Bohr-Sommerfeld conditions (the original quantization of energy levels) and cases where the geometric method and quantum representation give sharp correspondence, e.g., toric varieties. The method is less clear for integrable systems on K3 surfaces. This brings us to the relation to the Strominger-Yau-Zaslow Ansatz for Mirror Symmetry, which leads to a PDE approach involving curvature equations. The course will end with a discussion of either these equations or the approach of K. Zhang using quantization methods to prove the Yau-Tian-Donaldson conjecture on the existence of Kaehler-Einstein metrics (some complex analysis will have to be assumed if we follow this second option), either of which gives an application of the geometric quantization picture to complex differential geometry.

Recommended resources will be available online.

****MATH 657 Nonlinear Partial Differential Equations Wu, S. TR 2:30 PM - 4:00 PM**

MATH 656. (3). (BS). May not be repeated for credit.

Partial Differential Equations are mathematical structures for models in science and technology. It is of fundamental importance in physics, biology and engineering design with connections to analysis, geometry, probability and many other subjects. The goal of this course is to introduce students (both pure and applied) to the basic concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations.

Topics to be covered will include Sobolev spaces, second order elliptic equations, parabolic and hyperbolic equations, shock waves, and nonlinear wave equations. Course material will be taken from Chapters 5, 6, 7 and 12 of the text. Grading: Grades will be based on a few sets of homework and attendance and participation.

Required Textbook: **Partial Differential Equations**, by Lawrence C. Evans, 2nd. ISBN-13: 978-0821849743

****MATH 669 Topics in Combinatorial Theory Lam, T. TR 1:00 PM - 2:30 PM**
Topic: Combinatorics and Geometry of Amplitudes

Good knowledge of linear algebra (3). (BS). May not be repeated for credit.

In recent years, geometric combinatorics has been connected to the physics of scattering amplitudes, functions that predict the interaction of elementary particles (say, photons and electrons).

We will explain this story and discuss some of the novel mathematics being developed, with a focus on the relations to polytopal geometry, Schubert calculus, total positivity, and cluster algebras. No familiarity with these topics, or with physics will be assumed.

****MATH 671 Analysis of Numerical Methods I Krasny, R. TR 10:00 AM - 11:30 AM**
Topics in Scientific Computing: Particle Methods

Some familiarity with differential equations, linear algebra, numerical methods, complex variables, Fourier series, and Matlab or a similar tool

This course surveys topics related to particle methods in scientific computing. Particles interact with each other through fields and we'll consider boundary value problems for fields. Examples of particle systems will be given from fluid dynamics (point vortices) and electrostatics (point charges). Topics include discrete Fourier transform, fast Fourier transform, finite-difference schemes for boundary value problems, spectral method, Green's function method, particle-in-cell method (PIC) for electrostatic plasmas described by the Vlasov-Poisson equations, vortex method for incompressible fluids described by the Euler equations, spherical harmonics, Barnes-Hut treecode, Greengard-Rokhlin fast multipole method, Ewald summation for triply periodic charged particle systems, kernel-independent methods based on barycentric Lagrange interpolation, possible additional topics: multigrid method, single and double layer potentials, boundary element methods. Several homework sets will be assigned involving some analysis and computing. No required textbook, lecture notes will be posted on course website.

**MATH 675 Analytic Theory of Numbers Rome, N. TR 10:00 AM - 11:30 AM

Math 475, 575, and 596

This is a first course in analytic number theory. Our aim is to study the distribution of the prime numbers, the most mysterious and interesting objects in all of mathematics. We will prove the prime number theorem, Dirichlet's theorem on primes in arithmetic progressions and, if time allows, we will discuss the recent theorem of Maynard and Tao that there are infinitely often bounded gaps between the primes.

Along the way, we will meet the Riemann zeta function (and the related Dirichlet L-functions), and learn what the Riemann Hypothesis is, and why it is a million dollar question.

Textbooks: **Multiplicative Number Theory,** by Harold Davenport, 3rd Ed., 0-387-95097-4 (Optional)
The Distribution of Prime Numbers, by Dimitris Koukoulopoulos, 978-1470447540 (Optional)

**MATH 676 Class Field Theory Snowden, A. MW 10:00 AM - 11:30 AM

Class field theory is the study of abelian extension of number fields. It is an extremely important topic in modern number theory, and has found a wide range of applications. This course will cover the most important aspects of the subject.

**MATH 681 Mathematical Logic Harrison-Trainer, M. TR 11:30 AM - 1:00 PM

Mathematical maturity appropriate for a 600-level MATH course. Graduate standing. (3). (BS). May not be repeated for credit.

Mathematical logic is the study of mathematics itself as a formal process where theorems, written down in a formal language, are proved from assumptions using formal deductive principles. The first part of this course will set up the basic ideas, defining the formal language of first-order logic in which we can write down mathematical statements and writing down a set of deductive rules which we can use to build valid proofs. These mathematical statements are supposed to describe mathematical structures, and so we will also define these and say what it means for a statement to be true of a particular model. We will show that if we prove something, then it must be true (soundness); that is, if from A we can prove B, then any model which makes A true will also make B true. This means that our rules of deductive reasoning are all correct. On the other hand, we might wonder whether we might be missing any deductive rules; we will show that everything true is provable (completeness), which means that our deductive rules are sufficient for all reasoning.

In the rest of the course, we will learn many of the central tools of logic which are not only useful in all areas of logic, but also give a useful perspective on other areas of mathematics such as algebraic geometry. The main phenomenon here is that if we have any non-trivial model, there will be other non-isomorphic models which satisfy all of the same first-order sentences as our given model. By studying these non-standard models, we can learn something about our original model. The main theorem here is the compactness theorem, which says that if we have an infinite set of sentences, and any finite set is consistent (has a model), then the whole set is consistent.

While this is not precisely a course in model theory, many of these tools are model-theoretic in nature. However we will focus on tools which are applicable in all areas of logic, and we will have more "wild" examples such as Peano arithmetic and non-standard models of arithmetic than a course in model theory normally would. Other topics we will cover may include: quantifier elimination, which is a generalization of Chevalley's theorem on constructible sets; decision problems and interpretations; back-and-forth arguments, or why the rationals are the only countable dense linear order without endpoints; ultraproducts, which let us build copies of the reals with infinitesimal elements; types and the topology of the type space; interpolation theorems; indiscernibles and Ramsey's theorem; and amalgamations of structures and Fraisse Limits.

The only official prerequisite will be mathematical maturity appropriate for a 600-level course, but we will frequently use examples coming from algebra and analysis (e.g., rings and fields). Students unsure whether they are adequately prepared for the course are encouraged to write to the instructor. No textbook is required. The book Fundamentals of Mathematical Logic by Hinman covers much of the material of the course, but we will not be following it closely.

Optional Text: **Fundamentals of Mathematical Logic**, by Hinman

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**MATH 697 Topics in Topology Kriz, I. MWF 2:00 PM - 3:00 PM

Topic: Introduction to Current Methods in Algebraic Topology

Graduate standing, basic knowledge of Homotopy Theory. (2 - 3). (BS). May not be repeated for credit.

The purpose of this course is to provide an introduction to the methods used in contemporary algebraic topology. Some of the topics discussed will include equivariant homology and cohomology, cobordism theory, structured homotopy theory, and topological Hochschild homology. We will introduce key aspects of foundations as well as techniques for calculation.

No Textbook required.

**MATH 710 Topics in Modern Analysis II Uribe, A. TR 1:00 PM - 2:30 PM

Topic: Introduction to semiclassical and microlocal analysis

MATH 597 and Graduate standing. (3). (BS). May not be repeated for credit.

This course is an introduction to microlocal and semiclassical methods in linear PDEs. These techniques are based on the classical-quantum (or particle-wave) correspondence. Mathematically, they are a blend of harmonic analysis and symplectic geometry. In semiclassical analysis one considers the asymptotic behavior of linear partial differential operators that include a small parameter, as the parameter tends to zero in a suitable regime. The small parameter is usually referred to as Planck’s constant, h, as the primary example where this theory applies is to the Schrödinger operator of quantum mechanics. The limit h –> 0 is known as the semi-classical limit (or WKB method in the physics literature), and it was investigated from the very beginning of the quantum theory. In this limit quantum mechanics is supposed to “converge” to classical mechanics. The asymptotics can be about the spectrum and eigenfunctions of the operator, or about the time-dependent evolution of suitable initial conditions under fairly general Schrödinger-type equations. The microlocal point of view (which is due to Kohn, Nirenberg and Hörmander and others) looks somewhat different but is in fact more general. There is no small parameter; instead one deals with singularities of distributions. Both theories relate analytic objects (say in L^2(R^n)) with geometric objects in phase space, R^n\times R^n and its symplectic geometry.

The course will start with the theory of pseudodifferential operators in Euclidean space and their symbol calculus. We will then consider wave packets and their propagation, and other applications (e.g. to spectral theory) as time allows. We will skip details but will strive to state precise results. Most of the time we will work in Euclidean space, but towards the end of the course we may cover topics where symplectic geometry enters more heavily, depending on the audience. References include: Semiclassical analysis by M. Zworski; The Analysis of Linear Partial Differential Operators by L. Hörmander, and Harmonic Analysis in Phase Space by G. Folland.

The course grade will be based on homework assigned approximately every other week.

****MATH 732 Topics in Algebraic Geometry II Mustata, M. TR 1:00 PM - 2:30 PM**

Topic: Introduction to Singularities

MATH 631-632 or equivalent. (3). (BS). May not be repeated for credit.

The course will give an overview of several points of view on singularities of algebraic varieties. We will begin by discussing log canonical thresholds, multiplier ideals, and some applications, then move on to talk about similar invariants in positive characteristic. We will then turn to arc spaces and related invariants, such as the motivic zeta function of Denef and Loeser. We will end by discussing invariants of singularities that come from the theory of D-modules.

No Textbook Required

****MATH 797 Advanced Topics in Topology: Wright, A. MWF 3:00 PM – 4:00 PM**

Topic: Teichmuller Theory and Coarse Geometry

Math 695 or equivalent level of study. (3). (BS). May not be repeated for credit.

We will cover a selection of topics related to Teichmuller theory, mapping class groups, and coarse geometry. Possible topics may include Gromov hyperbolic spaces, curve complexes and subsurface projections, hierarchically hyperbolic spaces, Teichmüller geodesics, foliations and laminations, etc.