Syllabus for the AIM Preliminary Examination in Differential Equations & Linear Algebra

Ordinary Differential Equations

(Boyce and DiPrima)

1. First order scalar equations
   - Separable equations and separation of variables, exact equations, integrating factors, autonomous equations, homogeneous equations, Riccati equations.
   - Initial-value problems. Picard's method. The existence and uniqueness theorem.
2. Second order scalar linear equations
   - Homogeneous equations with constant coefficients.
   - The Wronskian, linear independence of solutions, and Abel's theorem.
   - Non-homogeneous equations, particular and complementary solutions, and the methods of undetermined coefficients and of variation of parameters.
   - Applications to mechanical and electrical vibrations, forced systems, steady-state and transient response, resonance.
3. Higher order linear equations
4. Series solution of second order linear equations
   - Ordinary points.
   - Singular points of regular and irregular type.
   - The method of Frobenius.
5. The Laplace transform
   - The domain of the Laplace transform.
   - Transforms of elementary functions.
   - Transform rules for derivatives and integrals.
   - Use of Laplace transforms to solve initial value problems for linear equations.
   - Impulse response, transfer function, and convolution.
6. Systems of first order linear equations
   - Homogeneous constant coefficient systems, and solutions generated from eigenvalues, eigenvectors, and generalized eigenvectors.
   - Matrix exponentials.
   - General homogeneous linear systems, fundamental solution matrices, Wronskian determinants, Abel's theorem for systems.
   - Non-homogeneous linear systems, particular solutions, and the methods of undetermined coefficients and of variation of parameters.
7. Nonlinear differential equations
   - Conversion of higher-order equations into equivalent first-order systems.
   - Phase space, and phase portraits for first-order nonlinear systems of two equations.

- Fixed points (equilibria) and periodic solutions.
- Stability of fixed points and periodic solutions. Local linearization and the Jacobian. Criteria for stability. Methods for assessing stability for indeterminate linearization including Liapunov functions and integration of trajectories (orbits). Limit cycles.
- Applications to population dynamics, predator-prey equations.

8. Two-point boundary value problems for second-order linear equations
   - Sturm-Liouville problems.
   - Orthogonal functions.
   - Eigenfunction expansions.
   - Green's function.

Partial Differential Equations

(Haberman, chapters 1-4)

1. Fourier series
   - Odd, even, and periodic extensions of functions on intervals.
   - Expansion of a function in Fourier series. Formula for coefficients.
   - Special forms. Exponential series, sine series, and cosine series.
   - Convergence properties of Fourier series. Gibbs' phenomenon.
2. The superposition principle for linear problems
   - Problem decomposition; isolation of influence of initial conditions, boundary conditions, and forcing terms.
   - Homogenization of boundary conditions.
3. The heat equation and the wave equation
   - Applications and interpretation of solutions.
   - Mixed initial/boundary value problems.
   - Lower order terms and source terms.
4. The Laplace equation
   - Homogeneous form. Harmonic functions.
   - Non-homogeneous form. Poisson equation.
   - Applications to equilibrium continuum mechanics.
   - Boundary conditions of Dirichlet and Neumann type.
5. The method of separation of variables
   - Product solutions, separation constants, and reduction to ordinary differential equations.
   - Role of homogeneous boundary conditions in selecting separation constants.
   - Superposition of product solutions. Series solutions on simple domains. The 1D rod, the 2D rectangle and circular disk, and the 3D rectangular box.
   - Separation of variables in different coordinate systems. Cartesian and polar coordinates.

Linear Algebra

(Strang)

1. Systems of linear algebraic equations
    - Solvability of linear systems.
    - Solution by Gaussian elimination.
    - Pivots and LU factorization.
2. Vector space principles
    - Vector space axioms.
    - Subspaces. The range and nullspace of a matrix.
    - Linear independence and rank. Equivalence of row and column rank.
    - Bases and dimension.
3. Linear transformations
    - Invertible matrices. Determinants and their properties.
    - Change of basis.
    - Similar matrices.
4. Inner products
    - Orthogonality of vectors.
    - Orthogonal projections.
    - Gram-Schmidt orthogonalization; QR factorization.
5. Overdetermined systems of linear equations
    - Approximation and least-squares solution.
    - Normal equations.
6. Eigenvalues and eigenvectors
    - Basic properties; characteristic polynomials, multiplicity of eigenvalues.
    - Extremal properties of eigenvalues; Rayleigh quotient, minimax principle.
    - Eigenvalues and eigenvectors of special types of matrices; real symmetric, real orthogonal, complex hermitian, complex unitary, normal, projection, and permutation matrices.
7. Canonical forms
    - Jordan canonical form of a matrix.
    - Polynomial and higher transcendental functions of a matrix.
    - Singular value decomposition.
8. Linear algebra and differential/difference equations
    - Linear structure of solution space of linear differential/difference equations. Particular and homogeneous (complementary) solutions of non-homogeneous equations.
    - Difference equations and computation of A^n.
    - Differential equations and computation of e^{At}.

9. Elements of numerical linear algebra
    - Matrix norms.
    - Spectral radius of a matrix.
    - The condition number of a matrix.