Syllabus for the AIM Preliminary Examination in Advanced Calculus & Complex Variables

Elementary Real Analysis

(Ross)

1. Natural, rational, and real numbers. The completeness axiom. The symbols $\pm\infty$.
2. Sequences
   - Limits of sequences.
   - Limit theorems for sequences.
   - Monotone sequences. Cauchy sequences.
   - Subsequences.
   - $\limsup$ and $\liminf$.
   - Basic topological concepts in metric spaces (open and closed sets, metrics, Cauchy sequences and completeness, Bolzano-Weierstraß theorem, interior and boundary points, open covers, subcovers, and compactness).
   - Infinite series.
   - Alternating series and integral tests.
3. Continuity
   - Continuous functions and their properties.
   - Uniform continuity.
   - Limits of functions.
4. Sequences and series of functions
   - Power series, intervals and radii of convergence, comparison tests and other tests for convergence.
   - Uniform and absolute convergence.
   - Differentiation and integration of power series.
   - Weierstraß approximation theorem.
5. Differentiation
   - Basic properties of the derivative.
   - The mean value theorem.
   - L'Hôpital's rule.
   - Taylor expansion and Taylor's theorem.
6. Integration
   - The Riemann integral and its properties.
   - The fundamental theorem of calculus.
   - Improper integrals.

Multivariable Calculus

(Stewart)

1. Parametric equations
    - Curves and surfaces defined by parametric representations.
    - Polar, cylindrical, and spherical coordinates.
    - Conic sections.
2. Vector algebra
    - Dot and cross products.
    - Equations of lines and planes.
    - Cylinders and quadric surfaces.
3. Vector functions
    - Space and plane curves as vector functions.
    - Tangent, normal, and binormal vectors of space curves.
    - Arc length and curvature of curves.
    - Applications to mechanics: force, velocity and acceleration.
4. Partial differentiation
    - Clairaut's Theorem on mixed partial derivatives.
    - Tangent planes and linear approximations.
    - The chain rule.
    - Directional differentiation and gradient vectors.
    - Optimization: finding extreme values of functions. Lagrange multipliers.
5. Multiple integrals
    - Double and triple integrals and iterated integrals.
    - Change of variables in multiple integrals. Jacobians.
    - Surface integrals and change of parametrization.
6. Vector calculus
    - Vector fields.
    - Line integrals and the fundamental theorem of calculus.
    - Green's theorem and its use in applications.
    - Curl and divergence of vector fields.
    - Stokes' theorem and its use in applications.
    - The divergence theorem and its use in applications.

Complex Variables

(Brown and Churchill)

1. Complex numbers and complex arithmetic
    - Algebra of complex numbers.
    - Modulus, argument, complex conjugation.
    - Exponential (polar) form.
    - Roots of complex numbers.
2. Analytic functions of a complex variable
    - Continuity and differentiability of complex functions.
    - The Cauchy-Riemann equations.
    - Harmonic functions and harmonic conjugation. Maximum principle.
    - Schwarz reflection principle.
3. Elementary functions
    - The exponential function.
    - The logarithm and its multivaluedness.
    - Power functions with complex exponents.
    - Trigonometric and hyperbolic functions and their inverses.
4. Integration of complex functions
    - Contour integration.
    - Estimation of moduli of contour integrals.
    - Cauchy-Goursat theorem and applications.
    - Simply and multiply-connected domains.
    - Cauchy's integral formula and differentiation.
    - Liouville's theorem. The fundamental theorem of algebra.
    - The maximum modulus principle.
5. Infinite series of complex functions
    - Power series and their convergence properties.
    - Taylor and Laurent series.
    - Absolute and uniform convergence of power series.
    - Continuity of sums of power series.
    - Integration and differentiation of power series.
    - Uniqueness of series representation. Analytic continuation.
    - Multiplication and division of power series.
6. Singularities of complex functions and applications to integration
    - The three types of isolated singular points.
    - Poles and residues. Cauchy's residue theorem. Computation of residues of simple and higher-order poles. Integration of functions with an infinite number of singularities.

- Essential singularities. Behavior of functions near essential singularities.
- Branch points.
- Evaluation of improper integrals by contour integration.
- Estimation of contour integrals. Jordan's lemma.
- Paths along and around branch cuts.
- Applications to Fourier and Laplace transforms.
- Applications to analytic function theory: the argument principle and Rouché's theorem.

7. Conformal mapping

   - Linear and fractional-linear (Möbius) transformations.
   - Mappings of the upper half-plane.
   - Mappings by elementary functions: exponential, logarithmic, and power functions.
   - Square roots of polynomials.
   - Riemann surfaces.
   - General properties of conformal mappings, invertibility, preservation of angles, transformations of boundary conditions.
   - Solution of physical boundary-value problems for Laplace's equation via conformal mapping. Potentials and stream functions. Flow around obstacles. The Joukowski map.
   - Polygonal regions and the Schwarz-Christoffel transformation.

8. Boundary-value problems and the Poisson integral formula

   - The Poisson integral formula.
   - Dirichlet problems for the disk and half-plane.
   - The Schwarz integral formula.
   - Neumann problems.