Langrange, Euer and Kovalevskaya tops

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A relevant problem in classical mechanics is the rotation of a rigid body, such as a spinning top, under the influence of gravity. The general problem is known to be a chaotic system. However, three famous cases are integrable, meaning they are not chaotic; the cases are Lagrange, Euler, and Kovalevskaya top, the only integrable cases when the system is subject to spin around a fixed point.

The easy case is the Euler top; this case describes a free top without any particular symmetry moving without external torque and for which the fixed point is the center of gravity. The Lagrange top has symmetry, in which two moments of inertia are the same, and the center of gravity lies on the symmetry axis. The Kovalevskaya top is the more challenging case; it is a special symmetric top with a unique ratio of the moments of inertia, which satisfies the relation.

$$I_1 = I_2 = 2I_2.$$

In other words, two moments of inertia are equal, and the third is half as large. In addition, the center of gravity lies in the plane perpendicular to the symmetry axis.

These examples help to understand the concepts of nutation and precession, two fundamental concepts in astronomy. You can see an explanation from the astronomical point of view in this video

This project is an introduction to rigid body and integrable systems.