EXPONENTIAL SUMS

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Roughly speaking, an exponential sum is a sum of the form

$$\sum_{x \in \mathbb{F}_p} f(x)\, e^{\frac{2\pi x i}{p}},$$

where $f \colon \mathbb{F}_p \to \mathbb{C}$ is a complex valued function. Sums of this sort appear in number theory and representation theory. They often enjoy very interesting properties. For example, Kloosterman sums have equidistributation properties, and Gaussian periods generate amazing pictures!

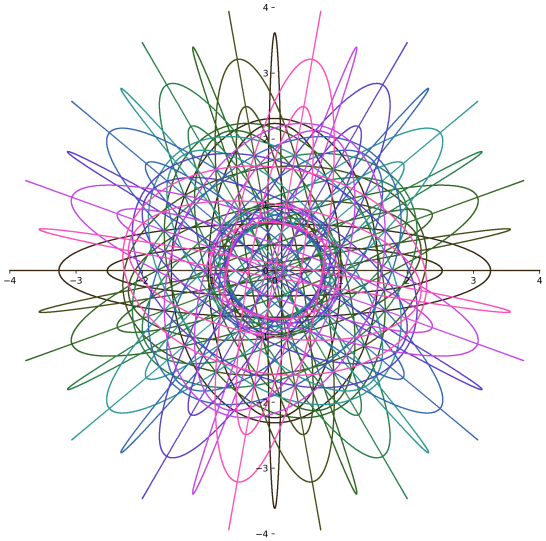

FIGURE 0.1. A Gaussian period, from Samantha Platt's website.

In this project, we will write code to explore these properties and show numerical evidence for them.

Prerequisites.

- Familiarity with finite fields.
- Experience with linear algebra, familiarity with matrices and their characteristic polynomial. Familiarity with the Jordan decomposition.
- Coding experience. We will mostly be using SageMath and JavaScript.