Mysteries of Spin
- magical ‘rotation’ in the quantum world
- observable
- discrete
- random with certain probability distribution

Basic Concepts

Definition 1 (The Vector Space $V_J$): The space of all polynomials $\psi(\theta, \phi)$ of the form
$$\psi(\theta, \phi) = a_0 \theta^j + a_1 \theta^{j-2} \phi^2 + \cdots + a_N \phi^J,$$
where $a_0, a_1, \ldots, a_N$ are complex numbers. This space forms the foundation for exploring quantum spin in mathematical terms.

Definition 2 (The Spin Operators $S_j$): Operators defined for acting on $V_J$, given by the expressions:
$$S_0 = 1, S_1 = \frac{\partial}{\partial \phi} \text{ and } S_2 = -i (\frac{\partial}{\partial \theta} - i \frac{\partial}{\partial \phi}).$$
These operators are crucial for understanding the dynamics of quantum spin systems.

Methods and Results

We used Matlab to generate functioning codes to explore potential patterns and match each scenario with corresponding eigenvalues.

function for plotting
- plot the probability distributions on a sphere

Theorem 1
The Husimi function of $|\psi|^2$ composed with Hopf fibration, is given by:
$$h: \mathbb{S}^2 \to \mathbb{R}, \ h(\zeta, \phi) = \left(1 + \frac{1}{\sqrt{2}} \right) \left| \left( \frac{\zeta}{1 + \frac{1}{\sqrt{2}}} \right) \right|^2,$$
where $\zeta$ is a complex variable and $\phi$ is a real variable

Cases for $S_2$ system

Theorem 2
If $E_j$ is an eigenvalue of $S_2$, then the maximum of the Husimi function concentrates at level curves $|\zeta| = \frac{1}{2}$. 

References