

RICCI FLOW AND RICCI SOLITONS

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The Ricci flow was introduced by Hamilton in the '80's as an analogue of the heat equation for Riemannian metrics; later, it was famously used by Perelman to prove the Poincaré conjecture. The flow naturally deforms the metric by “shrinking” parts where the curvature is positive and “pushing out” parts where the curvature is negative. Because of this, the flow collapses many natural shapes in finite time – for instance, a round sphere will shrink down to a point. This collapsing behavior can be problematic if we flow for long periods of time. For example, a dumbbell with a really long handle bar might have the center of the bar pinch before the spherical ends collapse, leaving us with an irregular surface.

To better understand collapsing, one can carefully zoom in near a pinched point as we move along the flow. This gives you a new manifold, which tends to preserve its shape along the flow – this is called a Ricci soliton. Classifying all possible Ricci solitons should then lead to a (more-or-less) complete understanding of how the Ricci flow can collapse.

The proposed project is to investigate explicit examples of Ricci solitons on surfaces and other special manifolds where there is a lot of symmetry. In these cases, the Ricci soliton equation can be reduced to an ODE and often solved explicitly. This will allow us to visualize several interesting geometric effects of the Ricci flow. Exact questions will depend on student background and interest.

Prerequisites:

- A class in ODE's
- Some knowledge of Riemannian geometry (e.g. MATH 433) and preferably complex numbers
- Some programming experience is a plus (preferably Mathematica)