Loops and Trees in Generic EFTs

March 10, 2021  Michigan Brown Bag seminars

In collaboration with
N. Craig, M. Jiang and D. Sutherland
EFT:

effective tool of parameterizing the low energy effect of new physics

allows the interpretation of the data, either agreement or disagreement with SM.

connect to UV via matching at threshold scale and running to experimental scale
Top-down: covariant derivative expansion

$S_{\text{eff}} \approx S[\Phi_c] + \frac{i}{2} \text{Tr} \log \left( -\frac{\delta^2 S}{\delta \Phi^2} \big|_{\Phi_c} \right)$

Bottom-up: enumerating operators

On-shell amplitudes

‘non-interference’ theorem

‘non-renormalization’

Interpreting data
LHC and future colliders provides potential excess to higher order expansion terms

extend the studies to dimension 8 operators and develop a more refined picture of the structures of EFT
In this talk

- Enumerate operators/helicity amplitude, up to dimension 8 with only four criteria
- Tree / loop classification
  - tree level operators,
  - loop level operator with large log enhancement, rational terms
Enumerating EFT operators/Helicity amplitude

<table>
<thead>
<tr>
<th>Field</th>
<th>Rep</th>
<th>Helicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>(0,0)</td>
<td>0</td>
</tr>
<tr>
<td>( \psi_\alpha )</td>
<td>( \left( \frac{1}{2},0 \right) )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \bar{\psi}_\dot{\alpha} )</td>
<td>( (0,\frac{1}{2}) )</td>
<td>( -\frac{1}{2} )</td>
</tr>
<tr>
<td>( F_{\alpha\beta} )</td>
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<td>1</td>
</tr>
<tr>
<td>( \bar{F}_{\dot{\alpha}\dot{\beta}} )</td>
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</tr>
<tr>
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<td>( \left( \frac{1}{2},\frac{1}{2} \right) )</td>
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\[
F_{\mu\nu}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}} \equiv F_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} + \bar{F}_{\dot{\alpha}\dot{\beta}}\epsilon_{\alpha\beta}
\]
## Enumerating EFT operators/Helicity amplitude

<table>
<thead>
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<th>Notes</th>
<th>Condition</th>
</tr>
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### Enumerating EFT operators/Helicity amplitude

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- ★ contains more than one field, otherwise, total derivative;
- ★ non vanishing Lorentz invariant:
  - a, even number of dotted and un-dotted indices;
  - b, no contraction within field strength tensor to form Lorentz invariant;

![Diagram of helicity amplitudes and operators](image_url)
## Enumerating EFT operators/Helicity amplitude

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- ★ contains more than one field, otherwise, total derivative;
- ★ non vanishing Lorentz invariant:
  - a, even number of dotted and un-dotted indices;
  - b, no contraction within field strength tensor to form Lorentz invariant;
- ★ EOM (IBP): on-shell amplitude.

less than four fields, no covariant derivatives.

\[
\begin{array}{cccc}
\sum h & F^3 & F\psi^2\phi & F^2\phi^2 \\
2 & F^2D^2 & F\psi\bar{\psi}D & \phi^2D^2 \\
1 & F\psi^2\phi & \psi^2\bar{\psi} & \phi^4D^2 \\
0 & F\bar{\psi}D^2 & F\bar{\psi}\phi D^2 & \phi^4D^2 \\
\end{array}
\]
Tree/Loop Classification: tree level operator

Weakly coupled UV completion

\[ \mathcal{L}_{UV} = -\frac{1}{2} \begin{pmatrix} \Phi & \Psi & \bar{\Psi} & V^\mu \end{pmatrix} \begin{pmatrix} D^2 + M^2 + \lambda \phi^2 & y\psi & y\bar{\psi} & 0 \\ y\psi & M + y\phi & -i\bar{\psi} & 0 \\ y\bar{\psi} & i\bar{\psi} & M + y\phi & 0 \\ 0 & 0 & 0 & -g_{\mu\nu}(D^2 + M^2 + \lambda \phi^2) + D_\nu D_\mu - [D_\mu, D_\nu] \end{pmatrix} \begin{pmatrix} \Phi \\ \Psi \\ \bar{\Psi} \\ V^\nu \end{pmatrix} \]

\[ \equiv -\frac{1}{2} H^T Q H - H^T J + O(H^3) \]

\[ H_c = -Q^{-1} J + O(J^2) \]

\[ \mathcal{L}_{EFT} = \frac{1}{2} J^T Q^{-1} J + O(J^3) \]

Tree level operator: products of currents $J$
Weakly coupled UV completion

\[ \mathcal{L}_{\text{UV}} = -\frac{1}{2} \left( \Phi \bar{\Psi} \vec{\Psi} \mathbb{V}^\mu \right) \begin{pmatrix} D^2 + M^2 + \lambda \phi^2 & y \psi & y \bar{\psi} & 0 \\ y \bar{\psi} & M + y \phi - i \lambda \phi & 0 & 0 \\ y \psi & 0 & M + y \phi & 0 \\ 0 & 0 & 0 & -g_{\mu\nu}(D^2 + M^2 + g \phi^2) + D_\nu D_\mu - [D_\mu, D_\nu] \end{pmatrix} \begin{pmatrix} \Phi \\ \Psi \\ \bar{\Psi} \\ \mathbb{V}^\nu \end{pmatrix} \]

\[ = -\left( \Phi \bar{\Psi} \vec{\Psi} \mathbb{V}^\mu \right) \begin{pmatrix} y \psi \psi + y \bar{\psi} \bar{\psi} + \lambda \phi^3 \\ y \phi \psi \\ y \phi \bar{\psi} \\ g \bar{\psi} \sigma_\mu \psi + g \phi \vec{D}_\mu \phi \end{pmatrix} + O(\{\Phi, \Psi, \bar{\Psi}, \mathbb{V}\}^3) \]

\[ \equiv -\frac{1}{2} \mathbb{H}^T Q \mathbb{H} - \mathbb{H}^T \mathbb{J} + O(\mathbb{H}^3) \]

\[ \mathbb{H}_c = -Q^{-1} \mathbb{J} + O(\mathbb{J}^2) \]

\[ \mathcal{L}_{\text{EFT}} = \frac{1}{2} \mathbb{J}^T Q^{-1} \mathbb{J} + O(\mathbb{J}^3) \]

Tree level operator: products of currents J

Tree/Loop Classification: tree level operator

\( \sum h \)

\( F^3 \)

\( F^2 D^2 \)

\( F \psi^2 \phi \)

\( \psi^4 \)

\( \psi^2 \phi^2 \)

\( \psi \phi^3 \)

\( \phi^6 \)
Weakly coupled UV completion

\[
\mathcal{L}_{UV} = -\frac{1}{2} \left( \Phi \Psi \bar{\Psi} \mathcal{V}^\mu \right) \begin{pmatrix}
  D^2 + M^2 + \lambda \phi^2 & y\psi & y\bar{\psi} & 0 \\
  y\psi & M + y\phi & -i\mathcal{D} & 0 \\
  y\bar{\psi} & i\mathcal{D} & M + y\phi & 0 \\
  0 & 0 & 0 & -g_{\mu\nu}(D^2 + M^2 + g\phi^2) + D_\nu D_\mu - [D_\mu, D_\nu]
\end{pmatrix} \begin{pmatrix}
  \Phi \\
  \Psi \\
  \bar{\Psi} \\
  \mathcal{V}^\nu
\end{pmatrix}
\]

\[
\equiv -\frac{1}{2} \mathbf{H}^T Q \mathbf{H} - \mathbf{H}^T \mathcal{J} + O(\mathbf{H}^3)
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\mathbf{H}_c = -Q^{-1} \mathcal{J} + O(\mathcal{J}^2)
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\[
\mathcal{L}_{EFT} = \frac{1}{2} \mathcal{J}^T Q^{-1} \mathcal{J} + O(\mathcal{J}^3)
\]

Tree level operator: products of currents \( J \)

\[
\sum h
\]

\[
D=6
\]

log enhancement??

\[
F^3
\]

\[
F^2 D^2
\]

\[
\psi^2 \phi
\]

\[
\psi^4
\]

\[
\phi^6
\]
Tree/Loop Classification: loop level operator

\[ \frac{dc_i(\mu)}{d \log \mu} = \sum_j \frac{1}{16\pi^2} \gamma_{ij} c_j \]

‘non-renormalization’ theorem

Only marginal interactions

\[ (\frac{n_A}{\sum h_A}, \frac{n_B}{\sum h_B}, \frac{n_C}{\sum h_C}) = (n_C + 4 - n_A, \sum h_C - \sum h_A) \]

No kinematic singularity for two particle cut, non-renormalization
Tree/Loop Classification: loop level operator

\[ \frac{dc_i(\mu)}{d \log \mu} = \sum_j \frac{1}{16\pi^2} \gamma_{ij} c_j \]

\[ (\psi^+)^4 \]

\[ \Sigma h \]

\[ n = |\Sigma h| \]

\[ d = 4 \]

non-renormalization' theorem


only marginal interactions

c_j operator insertion

\[ \pm \]

\[ \mp \]

\[ \pm \]

\[ \mp \]

\[ A \]

\[ B \]

\[ C \]

\[ \sum h_A \]

\[ \sum h_B \]

\[ \sum h_C \]

\[ \sum h \]

\[ n = |\sum h| \]

\[ (\psi^+)^4 \]

\[ \sum h \]

\[ n = |\sum h| \]

\[ d = 4 \]

\[ (\psi^+)^4 \]

\[ \sum h \]

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\[ d = 4 \]

\[ (\psi^+)^4 \]

\[ \sum h \]

\[ n = |\sum h| \]

\[ d = 4 \]
Tree/Loop Classification: loop level operator

\[ \frac{dc_i(\mu)}{d \log \mu} = \sum_j \frac{1}{16\pi^2} \gamma_{ij} c_j \]

'non-renormalization' theorem

\[
\left( \frac{n_A}{\sum h_A} \right) + \left( \frac{-4}{0} \right) + \left( \frac{n_B}{\sum h_B} \right) = \left( \frac{n_C}{\sum h_C} \right)
\]

only marginal interactions

\[ (n_B, \sum h_B) = (n_C + 4 - n_A, \sum h_C - \sum h_A) \]

no kinematic singularity for two particle cut, non-renormalization

\[ D = 6 \text{ for tree level } c_j \text{ only} \]
Tree/Loop Classification: loop level operator

\[ \frac{d c_i(\mu)}{d \log \mu} = \sum_j \frac{1}{16\pi^2} \gamma_{ij} c_j \]

\[ \psi^4 \]

For tree level \( c_j \) only

**non-renormalization** theorem


only marginal interactions

\( c_j \) operator insertion

\( c_i \) operator

\[
\left( \frac{n_A}{\sum h_A} \right) + \left( \frac{n_B}{\sum h_B} \right) = \left( \frac{n_C}{\sum h_C} \right)
\]

no kinematic singularity for two particle cut, non-renormalization

\((n_B, \sum h_B) = (n_C + 4 - n_A, \sum h_C - \sum h_A)\)

Tree level only renormalize tree level, except for \( \psi^4 \)

\[ D = 6 \] for tree level \( c_j \) only
Tree/Loop Classification: loop level operator

★ contains more than one field, otherwise, total derivative;  
★ non vanishing Lorentz invariant:  
  a, even number of dotted and un-dotted indices;  
  b, no contraction within field strength tensor to form Lorentz invariant;  
★ EOM (IBP): on-shell amplitude.  
  less than four fields, no covariant derivatives.

Tree level operator: products of currents $J$

‘non-renormalization’ theorem  

D=8

Tree level: # of field strength $\leq 1$

Sparsity of the anomalous dimension matrix persists
Contributions to observable: size of the Wilson coefficient

\[ \phi^2 F^2 : h \rightarrow \gamma\gamma \]
Loop level operator
and no large log enhancement
from the running of a tree-level operator

\[ F^2 \phi^2 D^2 : ZZ\gamma, Z\gamma\gamma \]
Loop level operator
and no large log enhancement
from the running of a tree-level operator
Contributions to observable: interference term first

$$BSM : \frac{1}{\Lambda^2} \quad \sigma \propto |SM|^2 + 2\Re(SM \ast BSM) + |BSM|^2$$

$$\frac{1}{\Lambda^2} \quad \frac{1}{\Lambda^4}$$

Some interference term vanishes due to helicity selection rule, at tree level and $$d = 6$$

finite mass converts the loop level generate amplitude (4,2) to (4,0):

$$\frac{m^2}{\Lambda^2}$$

d=8 interference with SM: $$\frac{1}{\Lambda^4}$$

Data: $$\Lambda \gg m$$  interference suppressed by only loop factors will be dominating
One loop helicity structure:

<table>
<thead>
<tr>
<th>(4, 2) process</th>
<th>Tree level</th>
<th>one loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V^+V^+V^+V^-$</td>
<td>N</td>
<td>Y(n.A.)</td>
</tr>
<tr>
<td>$V^+V^+\psi^+\psi^-$</td>
<td>N</td>
<td>Y(n.A.)</td>
</tr>
<tr>
<td>$V^+V^+\phi\phi$</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$V^+\psi^+\psi^+\phi$</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$\psi^+\psi^+\psi^+\psi^+$</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

- d. 4
- d. 6

---

$V^+V^+V^+V^-$

- V^+V^+\psi^+\psi^-
- V^+V^+\phi\phi
- V^+\psi^+\psi^+\phi
- $\psi^+\psi^+\psi^+\psi^+$

- d. 4
- d. 6

- Y(n.A.)
- Y
- Y
- Y

---

d. 6 part is one loop, two loop level suppression!
One loop helicity structure:

Interference: vanishes at one loop!

no interference theorem are particularly robust against some radiative corrections.
## Tree/Loop Classification—Rational term

<table>
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<th>$(4, 2)$</th>
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<tr>
<td></td>
<td>$\psi^2 \bar{\psi}^2$</td>
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</tr>
<tr>
<td>$(4, 0)$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
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<td>$R$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(4, -2)$</td>
<td>$\bar{\psi}^4$</td>
<td>$\times$</td>
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<tr>
<td></td>
<td>$\bar{\psi}^4$</td>
<td>$\times$</td>
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## Tree/Loop Classification—Rational term

Lorentz symmetry, some selection rules

---

angular momentum conservation?

### Lorentz frame

$$J \geq |\Delta h_f|$$

***COM-frame***

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<th>(4, 2)</th>
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<tr>
<td>$\psi^2 \bar{\psi}^2$</td>
<td>×</td>
<td>0</td>
</tr>
<tr>
<td>$\phi^4 D^2$</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$\phi^2 \psi \bar{\psi} D$</td>
<td>×</td>
<td>0</td>
</tr>
<tr>
<td>$F \bar{\psi}^2 \phi$</td>
<td>×</td>
<td>R</td>
</tr>
<tr>
<td>$F^2 \phi^2$</td>
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Tree/Loop Classification—Rational term

\[(n/2, m/2) : 2J = n + m\]

\[n, m : \text{\# of un-contracted un-dotted, dotted indices}\]

highest angular momentum

\[J \geq |\Delta h_f| \quad \text{COM-frame}\]

\[F^2 \phi^2 : F_{\alpha\beta} F^{\alpha\beta} \phi^2\]

<table>
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<th>(J)</th>
<th>(F_{\alpha\beta} F^{\alpha\beta})</th>
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<tr>
<td>((4, 0))</td>
<td>0</td>
<td>(V^-V^-)</td>
</tr>
<tr>
<td>((4, 2))</td>
<td>0</td>
<td>(\rightarrow)</td>
</tr>
<tr>
<td>((4, 2))</td>
<td>0</td>
<td>(V^-\phi)</td>
</tr>
<tr>
<td>((4, -2))</td>
<td>1</td>
<td>(\rightarrow \psi^- \psi^-)</td>
</tr>
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Tree/Loop Classification— Rational term

Lorentz symmetry, some selection rules

angular momentum conservation

\[
\left( \frac{n}{2}, \frac{m}{2} \right): 2J = n + m
\]

\( n, m \) : \# of un-contracted un-dotted, dotted indices

highest angular momentum

\[ J \geq |\Delta h_f| \quad \text{COM-frame} \]

more dedicate explanation with partial wave expansion, see:

Lorentz symmetry, some selection rules

angular momentum conservation

\[
\left( \frac{n}{2}, \frac{m}{2} \right) : 2J = n + m
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\( n, m \): # of un-contracted
un-dotted, dotted indices

highest angular momentum

\[
J \geq |\Delta h_f| \quad \text{COM-frame}
\]

more dedicate explanation with
partial wave expansion, see:


Observation:
(4,0) amplitude cannot be generated at d=6 until two loop
Conclusions

With the magic coordinate, for generic EFT of scalars, fermions and vectors:

we enumerated classes of Lorentz structures of operators with **FOUR** criteria up to d=8;

determined the tree/loop classification and interference pattern
  Field strength operators are hard to be tree level
  Tree tends to renormalize tree only at d = 6
  Amplitude from tree operator interferes with d=4 tree amplitude
  Non-interference up to two loop

determined the helicity amplitudes up to the full one loop level
  some amplitudes at d=6 vanish completely at one loop in a weakly coupled UV theory, calls for explanations in the UV theory frame work
Thank you