Inconsistency of SuperFluid DM with Milky Way Observables

Oren Slone, Princeton University

Great things from the 80’s
Great things from the 80’s

III. THE ROTATION CURVES

We assume that the emission arises from H\textsc{ii} regions which are moving in planar circular orbits about the center of each galaxy. The observed line-of-sight velocities along the major axis can then be projected to velocities in the plane of each galaxy, with \( V(R) = (V_{\text{obs}} - V_0)/\sin i \). For galaxies for which the major axis (\( \theta_m \)) is displaced from the position angle of the spectrum, \( \theta_s \), the circular velocity is given by

\[
R = \sqrt{\frac{I}{m} (F_{\text{obs}} - F_0) \left[ \sec^2 i - \tan^2 i \cos^2 (\theta_m - \theta_s) \right]^{1/2}}.
\]

Values for \( \theta_s \) and \( i \) are listed in columns (7) and (8) of Table 1. The adopted rotation curve is formed from both sides of the major axis. In general, velocities are reasonably symmetrical on both sides of the major axis; the principal exceptions are NGC 3672, 1421, 4321, and 7541. A simple way to determine the symmetry properties of the velocities is to trace a smooth curve through the points in Figure 4, then rotate the tracing paper 180° about the origin and compare the traced line with the plotted points. The adopted rotation curves are plotted in Figure 5, arranged by increasing linear radii, and the velocities are listed in Table 2.

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**Figure 5**—Mean velocities in the plane of the galaxy, as a function of linear distance from the nucleus for 21 Sc galaxies, arranged according to increasing linear radius. Curve drawn is rotation curve formed from mean of velocities on both sides of the major axis. Vertical bar marks the location of \( R_2 \), the isophote of 25 mag arcsec\(^{-2} \); those with upper and lower extensions mark \( R_1 \), i.e., \( R_1 \) corrected for inclination and galactic extinction. Dashed line from the nucleus indicates regions in which velocities are not available, due to small scale. Dashed lines at larger \( R \) indicate a velocity fall faster than Keplerian.
A Naive Solution

\[ \nabla^2 \Phi = 4\pi G \rho \]

Amazingly: Still not clear-cut on galactic scales
Outline

- Missing Mass and Galaxy Scale Observables
- Features of Various Classes of Solutions
- Superfluid Dark Matter
- Framework to Test Various Models using MW data
- Results and Conclusions
The Missing Mass Problem on Galactic Scales, 2019

- Flat Rotation Curves
- Issues with Small Scales:
  - Missing Satellites (maybe solved)
  - Too Big To Fail
  - Core vs Cusp
- DM Correlates with Baryons:
Galaxy Scale Observables
The Diversity Problem

- Diversity of inner rotation curves even for galaxies with similar halo and stellar mass.
- Rotation curves correlate with baryons
Galaxy Scale Observables

The Diversity Problem

DM dominated galaxies!

- Low surface brightness - halo is cored
- High surface brightness - halo is cusped
- Self similar if scaled to baryonic scale radius

$V_f \approx 79-91 \text{ km/s}$

Kamada et al., 2016
Galaxy Scale Observables

Renzo’s Rule

Sancisi, 2003
Galaxy Scale Observables
The Radial Acceleration Relation (RAR)

A tight correlation and an acceleration scale appear in rotation curve data from the SPARC catalog.

Lelli et. al, 2017

McGaugh, Lelli, 2017
Galaxy Scale Observables
The Baryonic Tully-Fisher Relation

A result of the information in the low end of the RAR

\[ g_{\text{obs}} \propto \sqrt{g_{\text{bar}}} \Rightarrow \frac{V_f^2}{R} \propto \frac{\sqrt{GM_{\text{bar}}}}{R}. \]
A Mass Discrepancy Acceleration Relation (MDAR) appears to be a feature of galaxies:

\[
a = \begin{cases} 
  a_N & a \gg a_0 \\
  \sqrt{a_0 a_N} & a \ll a_0
\end{cases}
\]

An acceleration scale appears in the data

\[a_0 \sim 1.2 \times 10^{-10} \text{ m/sec}^2 \sim \frac{1}{6} H_0\]
Galaxy Scale Observables
What models resolve these issues?

- Galaxies provide clues that DM correlates with baryons.
- Examples of solutions are:
  - CDM with baryonic feedback
  - Self Interactions SIDM
  - Modified Gravity MOND / TeVeS
  - Models with a MOND-like force e.g. Superfluid

SUMMARY OF THIS TALK
These are Preferred (even in galaxies)

Or maybe DM mimics MOND on galactic scales?
Solutions?

We should think broadly about the possible solutions + baryonic feedback

Bertone and Tait, 2018
Fitting the MDAR with a Fundamental Force

- Produce flat rotation curves: \( \Phi \propto \log r \rightarrow a \sim \frac{1}{r} \rightarrow v_c \propto \text{const} \)

- Different models do this in various ways

- They typically reduce to: \( a = \nu \left( \frac{a_N}{a_0} \right) a_N \)

- With an interpolation function with asymptotes: \( \nu(x_N) = \begin{cases} x_N^{-1/2} & x_N \ll 1 \\ 1 & x_N \gg 1 \end{cases} \)

- This reproduces the MDAR: \( a = \begin{cases} a_N & a \gg a_0 \\ \frac{a}{\sqrt{a_0 a_N}} & a \ll a_0 \end{cases} \)
Fitting the MDAR with a Fundamental Force

For example: Solar acceleration happens to live here.

Local measurements are sensitive only to small deviation in acceleration.

\[ \mathbf{a} = \nu \left( \frac{a_N}{a_0} \right) \mathbf{a}_N \]

\[ \mathbf{a} = (\nu_0 + \nu_1 a_N) \mathbf{a}_N \]

Spread of SPARC data

Lisanti, Moschella, Outmezguine, OS

16
What can we do?

1. Ask a **model independent** question:
   - Can local MW measurements fit a generic model that predicts the MDAR with a fundamental force?

2. Test a **specific realization**:
   - e.g. A specific interpolation function
   - e.g. Superfluid dark matter

   (Test these models where they’re supposed to shine!)
Consider a light scalar DM particle with mass $m$.

Require condensation to a state where the relevant DOF are phonons:

An overlapping de Broglie wavelength:

$$\frac{1}{mv} \geq \left( \frac{m}{\rho_{\text{vir}}} \right)^{1/3} \Rightarrow m \lesssim 2\text{eV}$$

With a critical temperature:

$$T_c \approx \frac{1}{3} mv^2 \approx \text{few} \left( \frac{\text{eV}}{m} \right)^{5/3} \text{mK}$$

($\sim$ known values in cold atom systems)
Superfluid DM

\[ T \approx m v_{\text{vir}} \]

**Galaxies**

- \( T_{\text{gal}} \approx 0.1 \text{mK} \)
- Super Fluid Phase
- MOND-Like Emergent Force

**Galaxy Clusters**

- \( T_{\text{cluster}} \approx 10 \text{mK} \)
- Cold DM
- Standard DM Dynamics
Superfluid DM

\[ \mathcal{L}_{\text{DM}, T=0} = \frac{2\Lambda(2m)^{3/2}}{3}X \sqrt{|X|} - \alpha \frac{\Lambda}{M_{\text{Pl}}} \phi \rho_b \]

\[ X = -m\Phi - (\vec{\nabla}\phi)^2/2m. \]

\[ \rho_{\text{SF}} = \frac{\partial \mathcal{L}}{\partial \Phi} \]

\[ \vec{a} = \vec{a}_b + \vec{a}_{\text{DM}} + \vec{a}_{\text{phonon}} \]
Superfluid DM

The SF Interpolation Function:

\[
\bar{a}_\phi = \alpha \frac{\Lambda}{M_{\text{Pl}}} \nabla \phi .
\]

\[
a_\phi = \alpha^3 \alpha^2 \frac{M_{\text{Pl}}}{a_b} .
\]

\[
a_0 = \frac{\alpha^3 \Lambda^2}{M_{\text{Pl}}}
\]

E.O.M. for \( \phi \)

Solar acceleration

\( v(a_b) \)

\( a_b \) [10^{-10} m/sec^2]
Constraining These Models
Local MW Observations Provide Differentiating Power

Compare accelerations in the R and z directions:

- Data requires amplification in $a_R$ but essentially none in $a_z$.
- A spherical DM halo does precisely this:
  $$a_{DM} \approx -G\frac{M(R_0)}{R_0^2} \left(1, \frac{z}{R_0}\right)$$
- A slightly prolate halo is slightly better.
- A MOND-like force amplifies $a_R$ too little or $a_z$ too much:
  $$\frac{a_z}{a_R} = \frac{a_{z,N}}{a_{R,N}} \bigg|_{\text{disk}}$$
Local MW Observations Provide Differentiating Power

Superfluid Dark Matter is even more predictive:

Galactic Acceleration
- CDM
- SFDM
- Data

Vertical Velocity Dispersions
- $\sigma_z$ [km/s]

Rotation Curve
- $v_c$ [km/s]
Local MW Observations Provide Differentiating Power

A new criterion for any theory which attempts to reproduce the MDAR.
Local MW Observations Provide Differentiating Power

- In principle: measure $a$ and $a_N$ and you’re done!
- However measurements are imperfect:
  - Baryonic profile is not perfectly measured.
  - Accelerations are not directly measured. Velocities and velocity dispersions are.
- Therefore: Adopt a **Bayesian Approach**
Local MW Observations Provide Differentiating Power
Bayesian Approach

- Given a model: $M = \text{CDM vs SFDM} / \text{a generic MOND-like force}$
- With parameters: $\theta_M$
- Construct a likelihood function: $\mathcal{L}(\theta_M) \propto \exp \left[ -\frac{1}{2} \sum_{j=1}^{N} \left( \frac{X_{j,\text{obs}} - X_j(\theta_M)}{\delta X_{j,\text{obs}}} \right)^2 \right]$
- $X_{\text{obs}}$: a set of measured values imposed as constraints
- $X(\theta_M)$: the corresponding model predictions
- Impose reasonable priors on $\theta_M$ and recover posterior distributions
Analysis Procedure:
Testing CDM vs SFDM / MOND-like
Analysis Procedure
Milky Way Model

Model baryonic profile:
- Double exponential stellar disk
- Double exponential gas disk
- Hernquist stellar bulge

SFDM/MOND-like
For MOND use a Taylor expansion of the interpolation func

Dark Matter
A generalized NFW profile

Perform a Markov Chain Monte Carlo analysis and fit parameters to MW measurements
\[ \rho_B = \rho_{*,\text{bulge}} + \rho_{*,\text{disk}} + \rho_{g,\text{disk}} \]
Analysis Procedure
 Milky Way Observables

- Local stellar surface density
- Local gas surface density
- Disk scale radii (stars and gas)
- Disk scale heights (stars and gas)
- Bulge mass
- Rotation curve
- Vertical acceleration
Analysis Procedure
Milky Way Observables

- Local stellar surface density
- Local gas surface density
- Disk scale radii (stars and gas)
- Disk scale heights (stars and gas)
- Bulge mass
- Rotation curve
- Vertical acceleration

\[
v_c(R) = \sqrt{R \cdot a(R)}
\]

Eilers et. al., 2018
### Analysis Procedure

**Milky Way Observables**

- Local stellar surface density
- Local gas surface density
- Disk scale radii (stars and gas)
- Disk scale heights (stars and gas)
- Bulge mass
- Rotation curve

**Vertical acceleration**

Inferred from 9000 K-dwarfs in the SEGUE sub-survey of the SDSS

\[
\sigma_{i,z}(z)^2 = \frac{n_i(0) \sigma_{i,z}(0)^2}{n_i(z)} \\
+ \frac{1}{n_i(z)} \int_0^z n_i(z') a_z(z') \, dz'
\]
Analysis Procedure
Ensure Self Consistency

- Only use measurements from locations where non-linear effects are negligible
- Only use measurements which were not inferred dynamically under the assumption of DM
RESULTS
Results for any MOND-like Model

\[ a = \nu \left( \frac{a_N}{a_0} \right) a_N \rightarrow a = (\nu_0 + \nu_1 a_N) a_N. \]
Results of MCMC Scans

Interpolation Function Parameters

Interpolation function fitted to RAR:

\[ \nu(a_N/a_0) = \frac{1}{1 - e^{-\sqrt{a_N/a_0}}} \]

with

\[ a_0 = 1.20 \pm 0.24 \times 10^{-10} \text{ m s}^{-2} \]

Excluded at 95% confidence
Results of MCMC Scans

Tension with MW Observations

Driven by stellar surface density constraint

\[
M_{*,\text{disk}} = \frac{2\pi h_{*,R}^2 \Sigma_{*,\text{obs}}^\text{max} \exp(R_\odot/h_{*,R})}{1 - \exp(-z_{\text{max}}/h_{*,z})}
\]

Driven by local value of rotation curve constraint

\[
v_c(R) = \sqrt{R \cdot a(R)} \bigg|_{z=0}
\]
Results of MCMC Scans
Stellar Scale Radius vs Stellar Bulge Mass

Driven by local surface density and rotation curve

Interpolation Functions
- \( \nu = \left(1 - e^{-\sqrt{a_0/a_N}}\right)^{-1} \)
- \( \nu = \nu_0 + \nu_1 \cdot a_N \)
- prior

Central panel of Fig. 2 illustrates the correlation between stellar disk mass, bulge mass, and scale radius, approximately following the same procedure as in the previous sections: the MOND scan picks out an acceleration enhancement that is close to unity. The right panel of Fig. 2 shows the correlation of \( M_{\ast,\text{bulge}} \) vs \( h_{\ast,R} \). The shaded regions indicate the range of parameters consistent with the local observational points. The results of MCMC scans suggest that the tension for a MOND-like force is mostly driven by local surface density and rotation curve.
Results of MCMC Scans
Bulge Mass is Poorly Constrained

<table>
<thead>
<tr>
<th>Reference</th>
<th>$M_B^\text{B} \pm 1\sigma$ $(10^{10} M_\odot)$</th>
<th>$R_0$ assumed (kpc)</th>
<th>Constraint type</th>
<th>$\beta^a$</th>
<th>$M_*^B \pm 1\sigma(R_0 = 8.33\text{kpc})$ $(10^{10} M_\odot)$</th>
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<tr>
<td>Kent (1992)</td>
<td>$1.69 \pm 0.85$</td>
<td>8.0</td>
<td>Dynamical</td>
<td>1</td>
<td>$1.76 \pm 0.88$</td>
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<tr>
<td>Dwek et al. (1995)</td>
<td>$2.11 \pm 0.81$</td>
<td>8.5</td>
<td>Photometric</td>
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<td>$2.02 \pm 0.78$</td>
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<tr>
<td>Han &amp; Gould (1995)</td>
<td>$1.69 \pm 0.85$</td>
<td>8.0</td>
<td>Dynamical</td>
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<td>$1.76 \pm 0.88$</td>
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<tr>
<td>Blum (1995)</td>
<td>$2.63 \pm 1.32$</td>
<td>8.0</td>
<td>Dynamical</td>
<td>1</td>
<td>$2.74 \pm 1.37$</td>
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<tr>
<td>Zhao (1996)</td>
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<td>8.0</td>
<td>Dynamical</td>
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<td>$2.15 \pm 1.08$</td>
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<td>Bissantz et al. (1997)</td>
<td>$0.81 \pm 0.22$</td>
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<td>0</td>
<td>$0.81 \pm 0.22$</td>
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<td>Freudenreich (1998)$^b$</td>
<td>$0.48 \pm 0.65$</td>
<td>...</td>
<td>Photometric</td>
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<td>$0.48 \pm 0.65$</td>
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<td>Dehnen &amp; Binney (1998)</td>
<td>$0.61 \pm 0.38$</td>
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<td>$1/2$</td>
<td>$0.62 \pm 0.38$</td>
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<td>Sevenster et al. (1999)</td>
<td>$1.60 \pm 0.80$</td>
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<td>Dynamical</td>
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<td>$1.66 \pm 0.83$</td>
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<td>Klipin et al. (2002)</td>
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<td>1</td>
<td>$0.98 \pm 0.31$</td>
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<tr>
<td>Bissantz &amp; Gerhard (2002)$^c$</td>
<td>$0.84 \pm 0.09$</td>
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<td>Han &amp; Gould (2003)</td>
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<td>Picaud &amp; Robin (2004)</td>
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<td>Wyse (2006)</td>
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<td>Historical review</td>
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<td>López-Corredoira et al. (2007)</td>
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<td>$0.65 \pm 0.33$</td>
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<td>Calchi Novati et al. (2008)</td>
<td>$1.50 \pm 0.38$</td>
<td>8.0</td>
<td>Microlensing</td>
<td>0</td>
<td>$1.50 \pm 0.38$</td>
</tr>
<tr>
<td>Widrow et al. (2008)</td>
<td>$0.90 \pm 0.11$</td>
<td>7.94</td>
<td>Dynamical</td>
<td>1</td>
<td>$0.95 \pm 0.12$</td>
</tr>
</tbody>
</table>


Conservative Range: $0 < M_{*,\text{bulge}} < 2 \times 10^{10} M_\odot$

Reference Value: $M_{*,\text{bulge}} = 1.50 \pm 0.38 \times 10^{10} M_\odot$
Empiracally, the acceleration scale is at, then the Fourier Transform of

\[ \nu(a_N) = \nu_0 + \nu_1 a_N \]

and during the period when the mass is moving through the horizon is

\[ T \sim g \]

with dimensions \( S^2 \).

Importantly, this means that if there is some entropy related to a system (eg a BH), then this translates to energy which can alter gravity. This is the general point of emergent gravity.

Now, note something known about BHs. That if the entropy and temperature are given by,

\[ S = \frac{2 \pi a}{G} \]

\[ T = \frac{\mu}{2 \pi a} \]

\[ dA/G \]

\[ dE = \mu gmR \]

\[ dM = g \]

\[ m \] with dimensions \( g \) and \( g \) and \( g \) with dimensions \( g \).

\[ B.I.C. = k \log n - 2 \log \hat{\mathcal{L}} \]

\( k \): number of model parameters

\( n \): number of data points

\( \hat{\mathcal{L}} \): maximum likelihood

Results of MCMC Scans
Comparison between the Theories

<table>
<thead>
<tr>
<th>Naming Convention</th>
<th>Functional Form</th>
<th>Prior for Scan</th>
<th>( \Delta \text{BIC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor Expansion</td>
<td>( \nu(a_N) = \nu_0 + \nu_1 a_N )</td>
<td>( \nu(a_N) &gt; 1 ) or ( \nu_1 &gt; 1.3 )</td>
<td>4.1 or 7.5</td>
</tr>
<tr>
<td>RAR [7]</td>
<td>( \nu(a_N) = \left(1 - e^{-\sqrt{a_N/a_0}}\right)^{-1} )</td>
<td>( a_0 = \text{LOGNORMAL} (1.20, 0.24^2) )</td>
<td>10.4</td>
</tr>
<tr>
<td>Simple [27, 52]</td>
<td>( \nu(a_N) = \frac{1}{2} \left(1 + \sqrt{1 + \frac{4}{a_N/a_0}}\right) )</td>
<td>( a_0 = \text{LOGNORMAL} (1.2, 0.4^2) )</td>
<td>9.6</td>
</tr>
<tr>
<td>Standard [27, 52]</td>
<td>( \nu(a_N) = \sqrt{\frac{1}{2} \left(1 + \sqrt{1 + \left(\frac{2}{a_N/a_0}\right)^2}\right)} )</td>
<td>( a_0 = \text{LOGNORMAL} (1.2, 0.4^2) )</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Bayesian Information Criterion: (a proxy for the Bayes Evidence)
Results for Superfluid DM
Results for SuperFluid DM

Full Rotation Curve and Vertical Accelerations

FIG. B2. Breakdown of contributions to radial and vertical accelerations for the CDM (left column) and SFDM (right column) models. In each case, the total acceleration is shown in gray, and the contributions from the DM halo and baryons are shown in purple and green, respectively. For the SFDM model, the orange band corresponds to the phonon-induced acceleration. The colored bands represent 68% containment around the median value of the posterior distribution, given by the dashed black lines.

Top row: Radial accelerations as a function of Galactocentric radius, $R$. The original rotational velocity data and the associated uncertainties used in the likelihood (from Ref. [21]) are given by black data points and corresponding error bars. In the radial direction, the phonon-mediated force contributes a dominant $O(1)$ fraction in the SFDM model.

Because of the large constant-density superfluid core in the SFDM model, the gravitational acceleration from DM increases linearly with radius and begins to dominate even over the contribution from phonons at large radii.

Bottom row: Vertical accelerations as a function of height above the Galactic midplane, $z$. In the vertical direction, the phonon-mediated force contributes a dominant $O(1)$ fraction in the SFDM model. Since the phonon-mediated force enhances the vertical acceleration by the same factor as the radial acceleration, the SFDM model is unable to simultaneously reproduce the correct radial and vertical accelerations. In particular, because the SFDM model correctly fits the rotation curve, it over-predicts the vertical acceleration.
Results for SuperFluid DM

Model Parameters

CDM Model Parameters

\[ \rho_{DM}(R_0) = 0.36^{+0.01}_{-0.01} \]

\[ \gamma = 1.79^{+0.09}_{-0.11} \]

\[ r_s = 72.66^{+18.69}_{-24.06} \]

SFDM Model Parameters

\[ a_0 = 1.15^{+0.25}_{-0.20} \]

\[ \bar{a} = 911.93^{+52.29}_{-47.63} \]

\[ \rho_0 = 2.74^{+0.20}_{-0.20} \]
Results for SuperFluid DM

Baryonic Parameters

Bayes Factor: 
\( \ln BF = 32 \)
Additional Tests

Redo analysis with:

- Only one mono-abundance population for velocity dispersions
- Various choices of priors for all parameters
- Artificially enhanced errors by factor of 2

⟹ Qualitatively same results for all cross checks
## Summary of the Results

<table>
<thead>
<tr>
<th>Methodology</th>
<th>ΔBIC</th>
<th>Evidence Level</th>
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<tbody>
<tr>
<td>Local accelerations only</td>
<td>≈ 4</td>
<td>POSITIVE EVIDENCE (with $\nu \approx 1$)</td>
</tr>
<tr>
<td>Taylor interpolation func</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local accelerations only</td>
<td>≈ 10</td>
<td>STRONG EVIDENCE</td>
</tr>
<tr>
<td>Specific interpolation func</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All rotation curve and velocity dispersion data</td>
<td>≈ 30</td>
<td>DECISIVE EVIDENCE</td>
</tr>
<tr>
<td>Superfluid DM</td>
<td></td>
<td></td>
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</tbody>
</table>
Conclusions

- Standard lore is that “MOND-like forces work on Galactic scales”. This is not precisely true.

- Our results establish a new criterion for any DM model which attempts to reproduce the MDAR.

- SFDM is a representative example of a broad class of such theories.

- MW measurements seem to prefer CDM over these models.
A strictly MOND-like force has trouble simultaneously explaining rotation curves and velocity dispersions... so, probably something else
THANK YOU
Results of MCMC Scans

Tension between models for any Scalar Enhancement

Each axis is the local enhancement of acceleration in the R/z directions or an independent measurement of the local value of the interpolation function
Some general comments
(and more on MOND-like forces)
Some Comments

- Could be done for any model where dynamics are predicted locally by baryons
- The starting point could have been something of the form:
  \[ \nabla \left( \mu \left( \frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right) = 4\pi G \rho \quad \Rightarrow \quad \Phi \propto \log r \]
- This equation is non-linear and difficult to calculate
- Is VERY model dependent
- Starting from an acceleration relation can map onto other theories
Non-linear effects must be accounted for!

Potential problems include:

- A possible non-trivial correction to the acceleration relation.
- Small perturbations to a smooth potential can cause large effects.
MOND / Superfluid DM
A Divergenceless Field

Poisson Equation:

\[ \nabla (\nabla \Phi_N) = 4\pi G \rho \]

MONDian Poisson Equation:

\[ \nabla \left( \mu \left( \frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right) = 4\pi G \rho \]

Acceleration Relation known up to a divergenceless field:

\[ a = \nu \left( \frac{a_N}{a_0} \right) a_N + S \]
MOND
A Divergenceless Field

Can be shown that $S=0$ for 1D symmetrical potentials, or:

$$\nabla |\nabla \Phi_N| \times \nabla \Phi_N = 0$$

$$|\nabla \Phi_N| = f(\Phi_N)$$
MOND / Superfluid DM
Small Perturbations

The External Field Effect (EFE) is small as long as:

\[
D \gg 0.1 \text{ kpc} \times \left[ \nu \left( \frac{a_{N,BG}}{a_0} \right) \cdot \frac{m_{\text{pert}}}{10^7 M_\odot} \cdot \frac{2 \cdot 10^{-10} \text{ m/s}^2}{a_{\text{loc}}} \right]^{1/2}
\]

\[a_{\text{loc}} = \frac{v_e^2}{R_0} \approx 2 \cdot 10^{-10} \text{ m/s}^2.\]
MOND
So for a local MW study:

Using

\[ \mathbf{a} = \nu \left( \frac{a_N}{a_0} \right) \mathbf{a}_N \]

with

\[ \nu(x_N) \rightarrow \nu_0 + \nu_1 \cdot x_N \]

- A good local approximation.
- Holds for many MOND-like theories.
- Independent of specific interpolation function.