Name: _____

University of Michigan Physics Department Graduate Qualifying Examination

Part I - Classical Physics

Saturday, January 8, 2005 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$
\begin{aligned}
\text{speed of light} \quad c &= 2.998 \times 10^8 \text{ m/s} \\
\text{electron charge} \quad e &= 1.602 \times 10^{-19} \text{ C} \\
\text{Planck's constant} \quad h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{c} \\
\hbar = h/2\pi &= 1.055 \times 10^{-34} \quad \text{J} \cdot \text{s} = 0.658 \times 10^{-15} \text{ eV} \cdot \text{s} \\
\text{Rydberg constant} \quad R_\infty &= 1.097 \times 10^6 \text{ m}^{-1} \\
\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} &= 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \\
\text{Universal gas constant} \quad R &= 8.3\text{J/K} \cdot \text{mol} \\
\text{Avogadro's number} \quad N_A &= 6 \times 10^{23} \text{ mol}^{-1} \\
\text{Boltzmann's constant} \quad k_B = R/N_A &= 1.38 \times 10^{-23} J/K = 8.617 \times 10^{-5} \text{eV/K} \\
\text{Stefan} - \text{Boltzmann constant} \quad \sigma &= 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4 \\
\text{radius of the sun} \quad R_{sun} &= 6.96 \times 10^8 \text{ m} \\
\text{radius of the moon} \quad R_{moon} &= 1.74 \times 10^6 \text{ m} \\
\text{radius of the earth} \quad R_{earth} &= 6.37 \times 10^6 \text{ m} \\
G_N &= 6.67 \times 10^{-11} m^3/kg/s^2 = 6.71 \times 10^{-39} GeV^{-2}
\end{aligned}
$$

SOME USEFUL CONVERSIONS AND COMBINATIONS

$$
\begin{aligned}
\text{fine structure constant} \quad \alpha &= ke^2/\hbar c = 1/137 \\
\text{Bohr magneton} \quad e\hbar/2m_e &= 9.27 \times 10^{-24} \text{J/T} = 5.79 \times 10^{-5} \text{eV/T} \\
hc &= 19.865 \times 10^{-26} \text{ J} \cdot \text{m} = 12.41 \times 10^3 \text{ eV} \cdot \text{Å} = 1241 \text{ MeV} \cdot \text{fm} \\
\hbar c &= 3.165 \times 10^{-26} \text{ J} \cdot \text{m} = 1973 \text{ eV} \cdot \text{Å} = 197.3 \text{ MeV} \cdot \text{fm} \\
ke^2 &= 1.44 \text{ MeV} \cdot \text{fm} \\
& 1\text{Å} = 10^{-10} \text{ m} = 10^5 \text{ fm} \qquad 1\text{eV} = 1.602 \times 10^{-19} \text{ J}
\end{aligned}
$$

SOME USEFUL RELATIONS

$$\frac{C_V}{Nk} = 9\,(T/\theta_D)^3 \int_0^{\theta_D/T} \frac{x^4 e^4 dx}{(e^x - 1)^2} \quad \text{(Debye formula)}$$

$$= 3\left[1 - \frac{1}{20}\left(\frac{\theta_D}{T}\right)^2 + \ldots\right] = \frac{12\pi^4}{5}\left(\frac{T}{\theta_D}\right)^3 (1 + \ldots)$$

$$\frac{U}{N} = \frac{3}{5}E_F\left[1 + \frac{5\pi^2}{12}\left(\frac{kT}{E_F}\right)^2 - \frac{\pi^4}{16}\left(\frac{kT}{E_F}\right)^4 + \ldots\right] \quad \text{(degenerate electron gas)}$$

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \quad \text{(nonrelativistic Fermi energy)}$$

$$r = r_\circ A^{1/3}, \qquad r_\circ = 1.2 \times 10^{-15} m \quad \text{(approximate average nuclear radius)}$$

$$E_b = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A} \quad \text{(semiempirical binding energy of a nucleus)}$$

$$\frac{n}{s} + \frac{n\prime}{s\prime} = \frac{n\prime - n}{R} \text{(refraction of paraxial rays)}$$

MASSES OF SOME ELEMENTARY PARTICLES

	Rest Mass, m_0 (kg)	m_0c^2 (MeV)
Electron	9.109×10^{-31}	0.511
Proton	1.673×10^{-27}	938.3
bNeutron	1.675×10^{-27}	939.6
Atomic mass unit (1 amu)	1.661×10^{-27}	931.5

VISIBLE LIGHT SPECTRUM

300	400	500		600		700(Nanometer)	
← Ultraviolet	Violet	Blue	Green	Yellow	Orange	Red	Infrared →

Part A: Obligatory Problems

1. Work out the normal mode frequencies for a diatomic molecule with two different masses that move along a line.

2. A dumbbell-shaped satellite consists of two "point" masses, m, separated by a massless rod of length $2L$. It is in a circular orbit around the Earth with the center of the dumbbell at a radius R_c from the center of the Earth. [Use M for the mass of the Earth and G for the graviational constant and assume $m \ll M$.]

   (a) What is the angular velocity ω of the satellite (in terms of m, R_c, G, ...)?

   (b) A motor in the satellite is used to pull the masses together. Calculate the work that it must do to pull them together. Assume the dumbbell maintains a radial orientation as shown.

   (c) Which if the following remain constant during this maneuver? (Circle all that remain constant.)

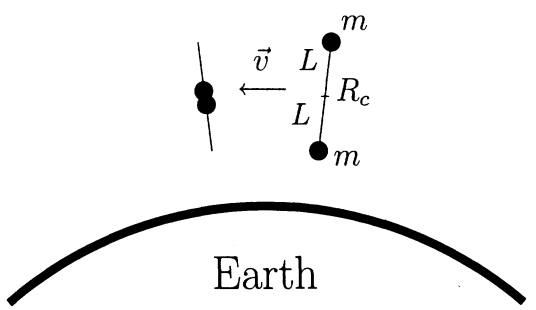
      i. Total mechanical energy of the satellite (*i.e.*, kinetic plus gravitational potential)?

      ii. Total angular momentum of the Earth-satellite system?

      iii. Total mechanical energy of th Earth-satellite system (kinetic plus gravitational potential)?

      iv. The radius of the center-of-mass of the satellite?

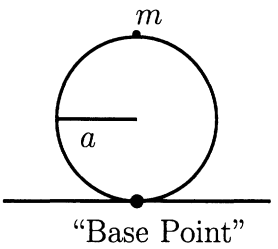
      v. The circular shape of the orbit?



[There is space on the next page to write out solution.]

Extra space for working out solution to problem 2:

3. A particle of mass m is placed on top of a smooth sphere of radius a that is at rest on a concrete floor. The sphere and the floor touch at the "Base Point." After being nudged an infinitesimal amount, the particle begins to slide down the sphere due to gravity. How far from the "Base Point" does the particle hit the floor?



4. Consider a simple circuit consisting of an inductor L in parallel with a capacitor C. The circuit is driven by a voltage supply with peak voltage V_0 and angular frequency ω. Suppose the voltage supply has an internal resistance R. Compute the average power W dissipated in the circuit.

5. Consider two particles with charges $e_{1,2}$ and masses $m_{1,2}$. Show that the magnetic moment $\vec{M}$ is proportional to the angular momentum $\vec{L}$

$$\vec{M} = g\vec{L}$$

and determine the proportionality constant g. You can assume both particles are non-relativistic.

6. A co-axial line consists of two long conducting, concentric, cylindrical shells. The inner cylinder has radius a and the other one has radius b. The line is terminated by a resistive load at one end. The inner conductor is held at fixed potential V relative to the other conductor. A constant current I flows along the inner conductor, through the resistive load and back along the other conductor.

   (a) Find the (magnitude and direction of the) Poynting vector everywhere on a plane that is perpendicular to both conductors and cuts them far from either end.

   (b) Integrate the Poynting vector over the plane to find the power transmitted along the line by the electromagnetic field. Compare this with the power dissipated in the resistive load.

7. **Adiabatic compression of an ideal gas:** The isothermal compressibility of a gas is

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T,$$

where p and V are the pressure and volume of the gas. The adiabatic compressibility is given by the same expression but at constant entropy, rather than constant temperature.

(a) Find an expression for κ_T in terms of the pressure for an ideal gas obeying $pV = nRT$, where n is the number of moles of gas and R is the ideal gas constant.

(b) Show that for *any* gas (ideal or not) the ratio κ_T/κ_S is equal to the ratio of the heat capacities at constant pressure and volume $C_p/C_V = \gamma$.

(c) The value of γ for an ideal gas turns out to be a constant. Hence show that when compressed adiabatically, the ideal gas follows a line in the $p - V$ plane given by $pV^\gamma =$ constant. (You will probably want to use your results from the first two parts of this question.)

8. A variable optical attenuator is constructed by inserting a half-wave (retardation) plate between crossed linear polarizers. Describe the polarization state immediately after the wave-plate when the fast axis makes an angle θ with respect to the second polarizer. What is the transmission through the second polarizer as a function of θ?

Part B: Optional Problems

9. **Work done using a heat engine:** Two identical objects with constant heat capacities C are initially at temperatures T_h and T_c, with $T_h > T_c$. Work is extracted from them by running a perfectly efficient heat engine between them until they reach the same temperature and no more work can be extracted.

   (a) What is their final temperature T_f?

   (b) How much work is extracted from them in total?

10. A thin film of optical thickness $\lambda/4$ is coated on a glass substrate. In terms of the indices of refraction of the glass (n_2) and the surrounding media (n_0), determine the index of refraction n_1 of the film, such that there is no reflection of light of wavelength λ for illumination at normal incidence. *Hint: You may find the following sum useful:*

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad \text{for } |x| < 1.$$

11. (a) Consider a circular orbit for the Kepler problem. From elementary physics show that $E = -T$, where T is the kinetic energy.

(b) Give a proof that $E = -\langle T \rangle$ for an elliptical orbit.

(c) Suppose a mass in a nearly circular orbit is subjected to a tangential impulse so that $v \to v + \delta v$, where $\delta v / v \ll 1$. Now the orbit is no longer circular, of course. Show that the semi-major axis a obeys $\delta a / a = 2\delta v / v$. *Hint: If the force is* k/r^2, *then* $a = -k/2E$.

12. Estimate the maximum height of the bar that a pole vaulter can get over. Would the main corrections to your estimate increase or decrease the height? Explain.