Name: _____

University of Michigan Physics Department
Graduate Qualifying Examination

Part I - Classical Physics

Saturday, January 10, 2009 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$
\begin{aligned}
\text{speed of light} \quad c &= 2.998 \times 10^{8}\ \mathrm{m/s} \\
\text{electron charge} \quad e &= 1.602 \times 10^{-19}\ \mathrm{C} \\
\text{Planck's constant} \quad h &= 6.626 \times 10^{-34}\ \mathrm{J \cdot s} = 4.136 \times 10^{-15}\ \mathrm{eV \cdot s} \\
\hbar = h/2\pi &= 1.055 \times 10^{-34}\ \mathrm{J \cdot s} = 0.658 \times 10^{-15}\ \mathrm{eV \cdot s} \\
\text{Rydberg constant} \quad R_{\infty} &= 1.097 \times 10^{7}\ \mathrm{m}^{-1} \\
\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} &= 8.988 \times 10^{9}\ \mathrm{N \cdot m^2/C^2} \\
\text{Universal gas constant} \quad R &= 8.3\ \mathrm{J/K \cdot mol} \\
\text{Avogadro's number} \quad N_A &= 6 \times 10^{23}\ \mathrm{mol}^{-1} \\
\text{Boltzmann's constant} \quad k_B = R/N_A &= 1.38 \times 10^{-23} J/K = 8.617 \times 10^{-5} \mathrm{eV/K} \\
\text{Stefan} - \text{Boltzmann constant} \quad \sigma &= 5.6703 \times 10^{-8}\ \mathrm{W/m^2K^4} \\
\text{radius of the sun} \quad R_{sun} &= 6.96 \times 10^{8}\ \mathrm{m} \\
\text{radius of the moon} \quad R_{moon} &= 1.74 \times 10^{6}\ \mathrm{m} \\
\text{radius of the earth} \quad R_{earth} &= 6.37 \times 10^{6}\ \mathrm{m} \\
\text{Gravitational constant} \quad G_N &= 6.67 \times 10^{-11} m^3/kg/s^2 = 6.71 \times 10^{-39} GeV^{-2}
\end{aligned}
$$

PART A: Obligatory Problems

1. (Mechanics) The elliptical orbit of a planet of mass m around a star of mass M is given by r=R/(1+ϵcosθ) where r is the distance between the star and planet, and θ is the angle between the vector pointing from the star to the planet and a fixed direction in the plane of the orbit. Let's assume that astronomers have measured the following three quantities:

   r_{min}, the minimum value of r

   r_{max}, the maximum value of r

   and ω, the value of dθ/dt when r=r_{min}

   (a) Express the eccentricity ϵ in terms of r_{min} and r_{max}.

   (b) Express R in terms of r_{min} and r_{max}.

   (c) Express dθ/dt at r=r_{max} in terms of r_{min}, r_{max}, and ω.

   (d) Express the energy of the system in terms of G, M, m, r_{min}, and ω.

2. (Mechanics) Consider two pendulums hanging from the same ceiling. Both pendulums have mass m and length ℓ. They are connected by a weak spring with spring constant k. The spring is not stretched when the pendulums are hanging straight down (their equilbrium positions).

   (a) Determine the Lagrangian for the system.

   (b) Find the Lagrangian in the limit of small oscillations of the pendulums.

   (c) Find the frequencies of the normal modes (i.e., the eigenfrequencies) of the system.

3. (Mechanics) Assume that the earth can be approximated as a uniform sphere with a density of 5.5×10^3 kg/m^3 and radius $R_{earth} = 6.37 \times 10^6$ m.

   (a) How much energy would have to be supplied to break up the earth completely (i.e., remove all mass to infinity)?

   (b) Estimate the number of WWII nuclear bombs (20 kT TNT / bomb) this would require. (1 gram TNT is equivalent to 4.2 kJoule)

   (c) If the earth were then reassembled rapidly (i.e, so fast that no energy is radiated away), would it melt? Justify your answer.

4. (E&M) A metal bar slides frictionless on two parallel conducting wires of zero resistance as shown in Fig. 1. The two wires are a distance ℓ apart and connected with a resistor R on one side. A constant and uniform magnetic field B points into the page everywhere.

Figure 1

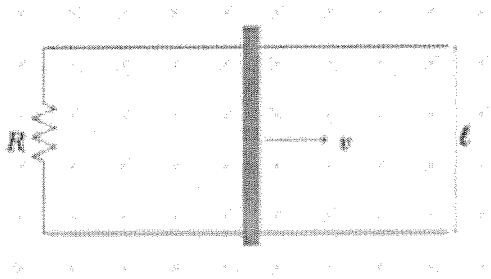
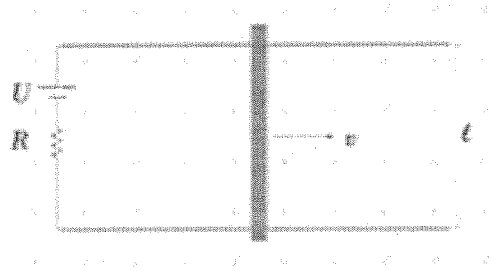


Figure 2



(a) If the bar slides to the right at speed v, what current flows through the resistor? What is the direction of the current?

(b) What is the magnitude and direction of the magnetic force on the bar?

(c) How much power is dissipated in the resistor?

(d) Discuss conservation of energy in this system.

(e) A battery with voltage U is now added to the circuit as shown in Fig. 2. If the mass of the bar is M, obtain an expression for the velocity of the bar as a function of time t, assuming it starts from rest at $t = 0$.

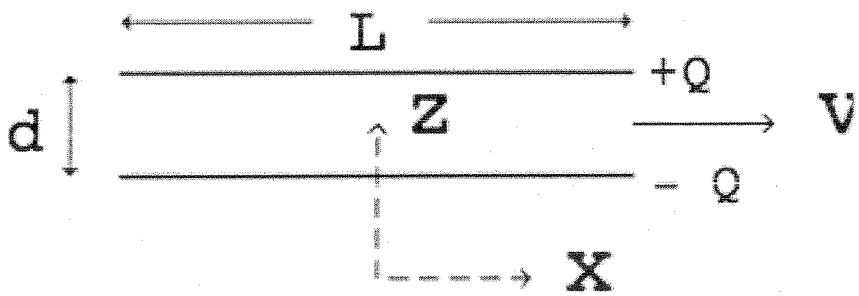
5. (E&M) The LHC ATLAS experiment uses a particle detector (drift tube) that consists of a thin, conducting wire coaxial with a surrounding aluminum tube. The radius of the wire is 0.025 mm, and the inner radius of the tube is 14.6 mm. The aluminum tube is grounded (V=0), and the wire is at +3080 V. What is the magnitude of the electric field (in V/m) at the surface of the wire?

6. (E&M) Consider a parallel plate capacitor as shown in the figure below (in profile). Each plate is a square of size $L \times L$, with charge $+Q$ on the upper plate and charge $-Q$ on the lower plate. The separation of the plates is d. Ignore edge effects.

(a) Determine the electric field produced by the charges. Give magnitude and direction of the magnetic field above, between, and below the plates. Now consider the capacitor moving with speed v to the right ($v << c$).

(b) Determine the magnetic field generated by the moving plates. Give magnitude and direction of the magnetic field above, between, and below the plates.

(c) Determine the electromagnetic energy density. At what velocity does the energy density due to the magnetic field become equivalent to the electric energy density? Justify your answer.



7. (Optics)

(a) What type of aberration is present in lenses and absent from mirrors?

(b) For maximal polarization by reflection, what is the angle between the reflected and refracted rays?

(c) What types of phenomena provide direct experimental proof of the transverse nature of light?

(d) What is the range of frequencies for visible light?

(e) For a given thick lens, what condition must a ray not parallel to the axis satisfy to go through the lens undeviated?

(f) A double slit is illuminated first by red light and then by violet light. Which color gives the wider interference pattern beyond the double slit? Justify your answer.

8. (Thermodynamics) N_2 and O_2 are the most common molecules in Earth's atmosphere. At sea level the ratio of these molecules is about 3 to 1 ($n_{N_2}/n_{O_2} = 3$).

(a) What is the ratio at an elevation of 5,000 m? You can assume the temperature of the atmosphere is 300 K independent of height. The mass of N_2 and O_2 molecules are 4.69×10^{-26} kg and 5.36×10^{-26} kg, respectively.

(b) Potatoes can be cooked by putting them in boiling water. Discuss, if at an elevation of 5,000 m the cooking time (the time needed to get a tender potato) will be significantly different than that at sea level.

PART B: Optional Problems

9. (Mechanics) Consider a central force field with a force law of the form

$$F(r) = -\frac{K}{r^2} - \frac{P}{r^4}.$$

This force law is like that of gravity with an extra attractive force. Show that this force law allows for some stable orbits, and find the condition necessary for circular stable orbits to exist.

10. (E&M) Consider a small superconducting coil of area 0.1 cm^2, 1000 wire turns and current 10 A that is located at position $(0, 0, 1\ \text{cm})$. The coil windings are parallel to the xy-plane, with counter-clockwise currents. The coil is magnetically levitated by a planar type-1 superconductor filling all volume with negative z (i.e., the surface of the superconductor is extending in the xy-plane).

(a) What is the magnetic-moment vector of the small coil?

(b) The boundary conditions at the surface of the superconductor can be satisfied by a single image coil. What is the magnetic-moment vector of the image coil, and where is the image dipole located?

(c) Determine the surface current density vector on the surface of the superconductor.

(d) What is the mass of the coil?

Notes:

1) Consider all dipoles as ideal dipoles, *i.e.* assume that their physical size is negligibly small.

2) A type-1 superconductor expels all magnetic fields from its interior. Hence, the boundary conditions at the surface require that all magnetic fields immediately outside the surface must be parallel to the surface.

3) To solve d), you do not need c).

11. (Optics) A "single-mode" optical fiber actually supports two field modes that have near-identical, near-Gaussian intensity distributions but orthogonal linear polarizations. The refractive indices for the two modes may differ slightly due to some deviation of the fiber profile from cylindrical symmetry. Assume that a piece of such a fiber extends along the z-axis from $z = 0$ to $z = L$, where L is the fiber length, and that the field modes correspond to polarizations along the x and y-directions, with respective refractive indices of $n_x = 1.4475$ and $n_y = 1.4476$. The fiber piece is used to turn linearly polarized light (wavelength 852 nm) into circularly polarized light. The light enters the fiber at $z = 0$ and exits at $z = L$.

(a) Specify all polarization directions (between 0 and π) of the incident light with respect to the x-axis and all possible fiber lengths that produce circularly polarized light.

(b) Identify the solution that will produce left-circularly polarized light for the shortest possible fiber length.

12. (Thermodynamics) The (reversible) Otto cycle shown below approximates a gasoline engine. It consists of heating an ideal gas at constant volume V_1, adiabatic expansion, cooling at volume V_2 and adiabatic compression. (There is also an exhaust cycle to replenish the fuel.) Find the efficiency in terms of the compression ratio, V_2/V_1.