1. Given a set $S$ with $n$ elements ($n$ is a positive integer), what is the number of subsets of subsets of $S$? (More precisely, we want to count the number of pairs of subsets $(X, Y)$ with $X \subseteq Y \subseteq S$.)
2. What is the smallest positive integer $k$ such that the sum of the decimal digits of $k(10^{2002} - 1)$ is not equal to 18018?
3. Suppose that $f \in C^\infty(-1, 1)$, and suppose that there exist nonzero points $x_1, x_2, x_3, \ldots$ in $(-1, 1)$ with $\lim_{n \to \infty} x_n = 0$ and $f(x_n) = 0$ for all $n$. Prove that every derivative of $f$ vanishes at $x = 0$. (By $f \in C^\infty(-1, 1)$ we mean that $f$ is a real valued function on the interval $(-1, 1)$ whose $k$-th derivative exists for any $k$.)
4. Start with the sequence
   \[1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 4, \ldots\]
   (each number $n$ appears $n$ times) and form its partial sums
   \[1, 3, 5, 8, 11, 14, 18, 22, 26, 30, \ldots\]
   Identify all the prime numbers in the latter sequence.
5. A permutation of the set $X = \{1, 2, \ldots, 2n\}$ is called complementing if there exists an $n$-element subset $Y \subseteq X$ such that $\pi(Y)$ is the complement of $Y$. Show that the number of complementing permutations is a square.
6. Show that if $s$ is a real number, $s > 1$, then
   \[\log \frac{s}{s - 1} = \int_1^\infty \frac{1 - 1/x}{\log x} x^{-s} \, dx.\]
   (In the formula, log stands for the natural logarithm.)
7. Is there a binary operation $\ast$ on a set $S$ that consists of three distinct elements that is commutative, i.e., $x \ast y = y \ast x$ for all $x, y \in S$, and also satisfies $x \ast (x \ast y) = y$ for all $x, y \in S$?
8. An interior point $P$ in an equilateral triangle $ABC$ is connected to the vertices, and perpendiculars are dropped to the sides, hitting $AB$, $BC$, and $CA$ at $X$, $Y$, and $Z$ respectively. Is it necessarily true that the sum of the lengths of $AX$, $BY$, and $CZ$ is equal to half the perimeter of the triangle?

---

\textit{Date:} April 7, 2002.
9. Let \( n \geq 2 \) be a positive integer. Show that every complex number \( c \) with \( |c| \leq n \) can be written as \( c = a_1 + a_2 + \cdots + a_n \) where \( |a_j| = 1 \) for every \( j \).

10. Consider the sequence of first digits in the successive powers of 2:

\[2, 4, 8, 1, 3, 6, 1, \ldots\]

Does one of the digits 7 and 8 appear more often in the sequence than the other one? (We say for example that 5 appears more often than 6 in the sequence if there exists a positive integer \( N \) such that for all \( n \geq N \), 5 appears more often than 6 among the first \( n \) terms of the sequence.)