501  AIM Student Seminar  Alben  F 12:00-1pm & 3pm-4pm

Prerequisites: You must be a graduate student in the AIM program to register for this course.

Course Description: The Applied and Interdisciplinary Mathematics (AIM) student seminar is an introductory and survey course in the methods and applications of modern mathematics in the natural, social, and engineering sciences. Students will attend the weekly AIM Research Seminar where topics of current interest are presented by active researchers (both from U-M and from elsewhere). The other central aspect of the course will be a seminar to prepare students with appropriate introductory background material. The seminar will also focus on effective communication methods for interdisciplinary research. MATH 501 is primarily intended for graduate students in the Applied & Interdisciplinary Mathematics M.S. and Ph.D. programs. It is also intended for mathematically curious graduate students from other areas. Qualified undergraduates are welcome to elect the course with the instructor’s permission. Student attendance and participation at all seminar sessions is required. Students will develop and make a short presentation on some aspect of applied and interdisciplinary mathematics.

Text: None

520  Life Contingencies I  Moore  TTh 8:30-10am & 10am-11:30am

Prerequisites: Math 424 and Math 425, or permission from the instructor

Course Description: Background and Goals: Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of the Math 520-521 sequence is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. The main topics are (1) developing probability distributions for the future lifetime random variable, and (2) using those distributions to price life insurance and annuities.


523  Loss Models I  Marker  TTh 11:30am-1pm & 1-2:30pm

Prerequisites: Math 425

Course Description: The goals of this course are to understand parametric distributions for the purpose of (1) modeling frequency, severity, and aggregate insurance losses, (2) analyzing the
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effects of insurance coverage modifications, and (3) estimating future insurance losses via credibility theory.


525 Probability Theory Barvinok TTh 8:30-10am
Prerequisites: MATH 451 (Required)

Course Description: This is a fairly rigorous introduction to probability theory with some emphasis given to both theory and applications, although a knowledge of measure theory is not assumed. Topics covered are: probability spaces, conditional probability, discrete and continuous random variables, generating functions, characteristic functions, random walks, branching processes, limit theorems.


526 Discrete Stochastic Processes Nadtochiy TTh 8:30-10am
Prerequisites: Required: MATH 525 or STATS 525 or EECS 501

Course Description: The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include: Markov chains (Markov property, recurrence and transience, stationarity, ergodicity, exit probabilities and expected exit times); exponential distribution and Poisson processes (memoryless property, thinning and superposition, compound Poisson processes); Markov processes in continuous time (generators and Kolmogorov equations, embedded Markov chains, stationary distributions and limit theorems, exit probabilities and expected exit times, Markov queues); martingales (conditional expectations, gambling (trading) with martingales, optional sampling, applications to the computation of exit probabilities and expected exit times, martingale convergence); Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales). Significant applications will be an important feature of the course.
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531 Transformation Groups in Geometry Ji MWF 9:00am-10:00am

Course Description: Symmetry is familiar to most people. We can see them in symmetric objects such as Platonic solids, and symmetric patterns such as wallpaper patterns. Symmetry is also an essential notion in sciences.
But what is the mathematics theory behind these symmetric objects and patterns? The mathematics behind symmetry is group actions on spaces. In this course, we will explain the notion of groups and their basic structures, and how their properties and actions on spaces can be used to describe precisely and explain the symmetry phenomenon as pointed out above. For example, we will explain why there are only exactly 5 Platonic solids, and how these regular solids are related.
The course is a solid and concrete preparation for more advanced topics such as Lie groups, spaces on constant curvature, Fuchsian groups, Kleinian groups, differential geometry, geometric group theory and topology, where Lie groups and their discrete subgroups play an essential role.


555 Intro to Complex Variable Bothner TTH 1-2:30pm

Prerequisites: Advanced calculus, including multivariable concepts (e.g. Math 215 or Math 452) and Introductory Analysis at the level of Math 451

Course Description: Many notions from advanced calculus that seem unintuitive make perfect sense when understood in the extended sense made available with the simple idea of extending the real number system with the square root of -1 to obtain complex numbers. This course is a first introduction to the theory and application of complex-valued functions of a complex variable, with special emphasis on the class of analytic functions and their uses in solving practical problems. Applications include the elegant evaluation of definite integrals, the solution of polynomial equations, steady planar fluid flow, and electrostatics. Math 555 and
Math 596 are courses that cover similar material, but Math 555 is more focused on breadth and applications, while Math 596 goes into certain topics with greater depth.

The audience for this course are graduate students in mathematics, engineering, and the physical sciences, and advanced undergraduate students with sufficient background. Math 555 is a core course for the graduate program in Applied and Interdisciplinary Mathematics.

(Optional) Basic Complex Analysis, Barry Simon. (A comprehensive course in analysis; part 2A) ISBN 978-1-4704-1100-8

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**556  Applied Functional Analysis  Schotland  TTh 10-11:30am**

*Prerequisites:* Undergraduate analysis, linear algebra and complex variables. Some exposure to partial differential equations is desirable but not essential.

**Course Description:** Introduction to topics in functional analysis that are used in the analysis of ordinary and partial differential equations. Metric and normed linear spaces, Banach spaces and the contraction mapping theorem, Hilbert spaces and spectral theory of compact operators, distributions and Fourier transforms, Sobolev spaces and applications to elliptic PDEs.

**Text:** (Required) Applied Analysis, Hunter and Nachtergaele

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**558  Advanced Ordinary Differential Equations and Dynamical Systems  Simon  TTh 2:30-4pm**

*Prerequisites:* Basic Linear Algebra, Ordinary Differential Equations (Math 216), Multivariable Calculus (215). Some exposure to more advanced mathematics e.g. Advanced Calculus (Math 450/451) or Advanced Mathematical Methods (Math 454)

**Course Description:** This course surveys a broad range of differential equations topics, with a focus on techniques and results that are useful in applications. Topics covered include: dynamics in dimension 1 and 1.5, bifurcations, Poincaré map, existence, uniqueness, and perturbation theory, linear systems, spectral theorems, linearization at equilibria for nonlinear systems, phase plane solutions of linear and nonlinear systems, stable and unstable manifolds, Lyapunov functions, gradient flows and Hamiltonian systems, periodic solutions, stability, omega-limit set, Poincaré -Bendixson and Bendixson-duLac Theorems, bifurcation theory, and chaotic dynamics.

565  Combinatorics and Graph Theory  Lam  TTh 10:00-11:30am
Prerequisites: Linear Algebra, experience with proofs and abstract mathematics

Course Description: The goal of the course is to provide an introduction to some basic notions, techniques, and results in combinatorics. The first part of the course will be devoted to graph theory. Topics we may discuss include graph colorings, Ramsey theory, extremal graph theory, and planar graphs. The second part of the course will cover a number of topics related to partially ordered sets, matroids, projective geometries and hyperplane arrangements.


568  Mathematical & Computational Neuroscience  Booth  MW 8:30-10am
Prerequisites: Math 462 or Math 463, or graduate standing

Course Description: Computational neuroscience investigates the brain at many different levels, from single cell activity, to small local network computation, to the dynamics of large neuronal populations. As such, this course introduces students to modeling and quantitative techniques used to investigate neural activity at these different levels.

Topics to be covered include:

Text: None

571  Numerical Methods  Karni  MWF 11am-12pm
for Scientific Computing I
Prerequisites: Linear algebra on the level of Math 417, 419 or Math 513, working knowledge of (any) computer programming language.
Course Description: This course is an introduction to numerical methods for solving linear systems of equations and for computing eigenvalues of a matrix. Topics include singular value decomposition, QR factorization, Gram-Schmidt orthogonalization, least squares problems, condition number, Gaussian elimination, iterative methods (Arnoldi, GMRES, conjugate gradient), preconditioning, methods for computing eigenvalues (e.g. power method, QR algorithm, shifts). The course should be very useful for students in applied and computational mathematics, and in any area of scientific computing and engineering.

Text: (Optional) Numerical Linear Algebra, L.N. Trefethen and D. Bau.

573 Financial Math I Hermann TTh 1-2:30pm
Prerequisites: MATH 526

Course Description: This is an introductory course in Financial Mathematics. This course starts with the basic version of Mathematical Theory of Asset Pricing and Hedging (Fundamental Theorem of Asset Pricing in discrete time and discrete space). This theory is applied to problems of Pricing and Hedging of simple Financial Derivatives. Finally, the continuous time version of the proposed methods is presented, culminating with the Black-Scholes model. A part of the course is devoted to the problems of Optimal Investment in discrete time (including Markowitz Theory and CAPM) and Risk Management (VaR and its extensions). This course shows how one can formulate and solve relevant problems of financial industry via mathematical (in particular, probabilistic) methods.


591 Differential Topology Scott MWF 10-11:00am
Prerequisites: Math 451 and Math 590

Course Description: This is one of the basic courses for students beginning the PhD program in mathematics. The approach is rigorous and emphasizes abstract concepts and proofs. The first 2 weeks of the course will be devoted to general topology, and the remainder of the course will be devoted to differential topology.

Note: this course has been recently restructured and is more advanced than it was in previous years.
Topics may include: Product and quotient topology, CW-complexes, group actions, topological groups, topological manifolds, smooth manifolds, manifolds with boundary, smooth maps, partitions of unity, tangent vectors and differentials, the tangent bundle, submersions, immersions and embeddings, smooth submanifolds, Sard's Theorem, the Whitney embedding theorem, transversality, Lie groups, vector fields, Lie brackets, Lie algebra, multilinear algebra, vector bundles, differential forms, exterior derivatives, orientation, De Rham cohomology groups, homotopy invariance, degree theory.


593  Algebra 1               Derksen            MWF 2:00-3:00pm
Prerequisites: Previous exposure to point-set topology and familiarity with abstract algebra will be assumed.

Course Description: This course covers part of the required topics for the QR algebra exam for graduate students. We will study rings and their modules and some topics in linear algebras such as tensor products, exterior powers of vector spaces, the Jordan and rational canonical normal form of a matrix and real symmetric bilinear forms.


596  Complex Analysis        Barrett            MWF 9:00am-10:00am
Prerequisites: Solid background in rigorous undergraduate analysis

Course Description: This course covers the Complex Analysis portion of the syllabus for the Qualifying Review Exam in Analysis.

Topics to be covered include:

Complex elementary functions, conformal mapping, the Riemann sphere, linear fractional transformations, rational functions

Complex derivatives, Cauchy-Riemann equations

Contour integration, Cauchy's theorem, Cauchy-Green formula, Cauchy's integral formula and consequences, power series expansion and consequences
Harmonic functions, maximum principle, Dirichlet's problem

Isolated singularities, residues, application to computation of definite integrals, meromorphic functions, argument principle, Rouche's theorem

Equicontinuity, Montel's theorem, Schwartz's lemma, Riemann mapping theorem

Homework will be assigned weekly, and midterm and final exams will be given.

**Text:** (Required) Complex Analysis by Gamelin

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602  Real Analysis II  Jonsson  MWF 12:00-1:00pm

*Prerequisites:* Linear algebra, real and complex analysis, point set topology

**Course Description:** Functional analysis is the study of spaces of functions and is of great importance not only to analysis, but also to applied mathematics and fields such as physics and economics.

The goal of this course is to introduce the students to the basic concepts, methods, and results of functional analysis. We will cover the fundamental theorems involving Banach and Hilbert spaces, such as the Hahn-Banach Theorem, the Open Mapping Theorem and the Riesz Representation Theorem. Additional possible topics include spectral theory, Sobolev spaces, and Fourier analysis.

Grades will be based on written homework and class participation.


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606  Advanced Stochastic Analysis  Muhle-Karbe  TTH 11:30am-1pm

*Prerequisites:* Math 625

**Course Description:** This is a PhD level course in Stochastic Analysis and applications to Quantitative Finance. The aim of this course is to teach the advanced probabilistic techniques and concepts from the theory of stochastic processes, in order to provide a sufficient knowledge base for the students to conduct research in Financial Mathematics. It is a research-level extension of Math 506.
612  Algebra II:  
Lie Algebras and Their Representations  
Stembridge  
MWF 1-2:00pm  
Prerequisites: Math 593 and Math 594 (or equivalent).

Course Description:  Semisimple Lie algebras and their representations are at the crossroads of many important branches of mathematics--witness the ubiquity of Dynkin diagrams, for example. This course should be valuable for those interested in representation theory and Lie theory, as well as for those with interests in allied areas, such as algebraic combinatorics, and non-commutative algebra.

We will cover the basic theory of Lie algebras, with emphasis on the complex semisimple case. We plan to cover most of the topics in Humphreys' book, with priority given to the finite-dimensional representation theory.


614  Commutative Algebra  
Hochster  
MWF 2:00pm-3:00pm  
Prerequisites: Math 593 and Math 594 or permission of instructor

Course Description:  The course is an introduction to commutative algebra, a subject that has interactions with algebraic geometry, number theory, combinatorics and several complex variables. The emphasis will be on Noetherian rings. Topics include behavior of Spec and the Zariski topology, localization, integral extensions and integral closure, behavior of primes and chains of primes for ring extensions, particularly integral extensions, Noether normalization, Hilbert’s Nullstellensatz and applications to affine algebraic geometry, discrete valuation rings, normal Noetherian rings, Dedekind domains, rings and modules that are Noetherian or Artinian, primary decomposition, tensor products and flatness, the Artin-Rees lemma, the Krull height theorem and dimension theory, completion, and properties of complete local rings. Lecture notes will be provided – there is no text.

Text: None

623  Computational Finance  
Keller  
TTh 10-11:30am  
Prerequisites: Differential equations (e.g. Math 316); probability theory (e.g. Math 525/526, Stat 515); numerical analysis (Math 471 or Math 472); mathematical finance (Math 423 or Math 574, Math 506 or permission from instructor); programming (e.g. C, Matlab, Mathematica, Java).

Course Description:  This is a course in computational methods in finance and financial modeling. The course starts with the introduction of numerical methods for solving differential equations of evolution, including partial differential equations (PDEs) of parabolic type.
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Convergence and stability of explicit and implicit numerical schemes are analyzed. Examples include the generalized Black-Scholes PDE for pricing European, American and Asian options. Another part of the course is concerned with Monte Carlo methods. This includes pseudo random number generators (with applications to option pricing) and numerical methods for solving stochastic differential equations (with applications to stochastic volatility models). A part of the coursework requires programming in a high-level language.


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625  Probability and Random Processes I: Bayraktar  TTh 10-11:30am

**Course Description:** A Ph.D. level course on probability theory and the theory of martingales in discrete time. (Main examples will be drawn from mathematical finance, optimal stopping/control problems.) Topics covered include measure theory and integration; convex analysis on the space of positive measurable functions; characteristic functions; convergence concepts; limit theorems; conditional expectation; martingales (uniform integrability, martingale convergence theorems, optional sampling theorem); (time permitting) basic concepts in continuous time stochastic analysis, Poisson random measures.

**Text:** (Required) Probability with Martingales, David Williams. (Required) Probability and Stochastics, Erhan Cinlar.

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631  Algebraic Geometry I: Mustata  MWF 11:00am-12pm

**Prerequisites:** Familiarity with basic commutative algebra

**Course Description:** The course is the first part of a 2-semester course giving an introduction to algebraic geometry. In the first semester we will focus on varieties over an algebraically closed field. A special emphasis will be put on concrete geometric examples.

The following is a rough outline of the material to be covered during the Fall semester:

1) Affine algebraic subsets and Hilbert’s Nullstellensatz
2) Projective and quasiprojective varieties; morphisms of algebraic varieties.
3) Dimension theory.
4) Sheaves and ringed spaces; general algebraic varieties.
5) Complex algebraic varieties.
6) Separatedness and completeness.
7) Coherent sheaves on algebraic varieties.
8) Normal and nonsingular varieties.
9) Divisors and line bundles.
10) Introduction to toric varieties.

Text: None

636  Topics in Differential Geometry: Burns  TTh 11:30-1pm
Symplectic Geometry and Mechanics: Old and New

Prerequisites: Basic Analysis, geometry

Course Description: Symplectic geometry has its origins in Hamilton’s formulation of classical mechanics. From this point of view, conserved quantities generate the flows of the relevant dynamical systems. Periodic solutions give rise to the corresponding periods and actions along the solutions and the totality of this data has been called the spectrum of the system. In recent decades, symplectic geometry has made enormous strides, including Gromov’s introduction of the theory of pseudoholomorphic curves, Floer homology theories of many types, resultant theorems on the existence of fixed points and periodic orbits (Arnold’s conjecture) and various “capacity” invariants which give far-reaching extensions and applications of the notion of the spectrum of a Hamiltonian system. Recent work of Leonid Polterovich and coworkers have begun to look at the applications of some of these new techniques in symplectic geometry to the study of functions on a symplectic manifold viewed as the Lie algebra of the group of Hamiltonian flows. The Lie bracket is given by Poisson brackets of functions, the classical analogue of the uncertainty principle in quantum mechanics. That these brackets have continuity properties under continuous convergence allows one to construct “quasi-states” which will have applications in geometric quantization (which will be reviewed) and quantum noise.

The course will start off with a review of some of the classical concepts and basic symplectic geometry, with application to some standard mechanical examples. Then we will review some of the modern “wonders of symplectic geometry” (Polterovich) as well as give a rapid survey of Floer homologies. Finally, we will present the work on the space of Hamiltonians (functions) and its surprising internal geometry.

The course will be based on course notes, but our primary reference will be “Function Theory on Symplectic Manifolds”, by Polterovich and Daniel Rosen, which is the recommended text for the course. Some problem sets will be distributed, and student presentations of reading will be organized for later in the term.

656 Introduction to Partial Differential Equations Bieri MWF 10-11:00am

Prerequisites: Math 558, 596, and 597 or permission of instructor

Course Description: Partial differential equations are at the core of models in science, technology, economics and related fields. These equations and their solutions have interesting structures that are studied by methods of analysis, geometry, probability and other mathematical fields. This course will introduce students in pure and applied mathematics to concepts and methods that mathematicians have developed to understand and analyze the properties of solutions to partial differential equations. Topics to be covered will include nonlinear first order equations, linear elliptic, hyperbolic and parabolic equations. The method of characteristics, energy methods, maximum principle, Fourier transform and Sobolev spaces. Course material will be taken mainly from the first chapters of Evans and John.

Grading: The course grade will be based on homework and a final project.

Subsequent Courses: Math 657 Nonlinear Partial Differential Equations

Text: (Required) Partial Differential Equations by L.C. Evans, 2nd Edition AMS.
(Optional) Partial Differential Equations by Fritz John, Springer.

665 Combinatorial Theory II: Speyer MWF 1-2:00pm

Course Description: This will be an introduction to Coxeter groups. We'll start by describing the finite reflection groups and describing their geometry and combinatorics in detail, and then discuss the properties of the various orders placed on Coxeter groups. This is essential background material to the study of algebraic groups, cluster algebras and much else in algebraic combinatorics.

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675  Analytic Theory of Numbers:             Ho          MW 11:30am-1pm

**Prerequisites:** Math 575, Math 596

**Course Description:** This is a first course in analytic number theory, which focuses on studying problems in number theory using analytic tools. One of the main topics in the course will be understanding the prime number theorem. We will also study topics such as arithmetic functions, Dirichlet's theorem on arithmetic progressions, and sieve methods.

**Text:** None

678  Intro to Modular Forms                 Lagarias      TTH 2:30-4pm

**Prerequisites:** Familiarity with one complex variable.

**Course Description:** Modular forms involve a wonderful overlap of arithmetic, algebra, analysis and geometry.

This is a basic course on modular forms, expected to take an analytic viewpoint but covering algebraic aspects. It will cover the modular group, classical modular forms (holomorphic and non-holomorphic) Eisenstein series, and related spectral theory for SL(2, R). This will include Hecke operators, and the connection of Dirichlet series with Euler products. The course will include elliptic functions, theta functions and theta identities.

Some other possible analytic topics: mock theta functions, mock modular forms, weakly holomorphic Maass forms, quasimodular forms. I am also interested in connections with integrable systems. This means modular forms with the extra elliptic curve variable present, i.e. Jacobi forms.

Applications may include the theory of partitions, representations of quadratic forms, connections to elliptic curves, with and without complex multiplication.

There will be no official textbook. Some references are given below.

The exact material to be covered is less than the above but it is expected to favor analysis.

2. F. Diamond and J.Shurman, A First course in Modular Forms, Springer-Verlag. (optional)
3. Igor Dolgachev. Lectures on Modular Forms.
682  Set Theory                  Fernández-Bretón     TTh 2:30-4pm

**Prerequisites:** None

**Course Description:** The main theme of this course is that it will be an introduction to independence proofs, that is, proofs that certain statements are independent from the axioms of set theory. Historically the first example of this phenomenon is the Continuum Hypothesis, which was listed at the very top of Hilbert's famous list of problems. In 1930 Gödel proved that it is not possible to prove the negation of the Continuum Hypothesis from the axioms of Set Theory, and in 1960 Cohen provided closure to this problem by proving that the Continuum Hypothesis is not provable from these axioms either. In this course we will introduce the necessary technical tools to understand Gödel's and Cohen's results.


695  Algebraic Topology I      Kriz      MWF 11:00am-12:00pm

**Prerequisites:** Course in algebraic topology is recommended

**Course Description:** This is a course in algebraic topology for anyone who has seen the fundamental group and homology. Traditionally (20 years ago), the syllabus included cohomology, the universal coefficient theorem, products, Tor and Ext and duality. We will cover all those topics, while also building a modern framework of derived categories and derived functors using the Cartan-Eilenberg method. We will apply these methods to modules, topological spaces, simplicial sets, sheaves and, if time permits, spectra (=generalized cohomology theories). The class is graded based on homework.

**Text:** None

697  Topic in Topology:       Canary     MWF 12:00-1pm

**Course Description:** If $S$ is a closed orientable surface of genus at least 2, the Teichmuller $T(S)$ of $S$ is the space of isometry classes of marked hyperbolic surfaces homeomorphic to $S$. Equivalently, one may think of $T(S)$ as the space of conformal classes of marked Riemann surfaces which are homeomorphic to $S$, or as the space of conjugacy classes of representations
of the fundamental group of $S$ into $\text{PSL}(2,\mathbb{R})$.

We will begin with a brief introduction to hyperbolic geometry, develop basic properties of hyperbolic surfaces, and discuss the Fenchel-Nielsen parameterization. Depending on student interest, we may also discuss the complex analytic viewpoint on Teichmüller space.

We will then discuss the space of convex projective structures on $S$, which may be identified with the Hitchin component of (conjugacy classes of) representations of the fundamental group of $S$ into $\text{PSL}(3,\mathbb{R})$. We will survey the Goldman parameterization of the Hitchin component, which generalizes the Fenchel-Nielsen coordinates, and work of Benoist on the dynamics of strictly convex projective manifolds.

These two deformation spaces are motivating examples of Higher Teichmüller spaces of geometrically meaningful representations of groups into semi-simple Lie groups. As time permits, we will cover more advanced topics, e.g. Hitchin representations into $\text{PSL}(n,\mathbb{R})$ or, more generally, Anosov representations.

Text: None

711  Advanced Algebra: Snowden  MWF 10:00am-11:00am
Representation theory of symmetric groups

Course Description: Representation theory of the symmetric groups, in characteristic zero and positive characteristic.

Text: None

731  Topics in Algebraic Geometry I: Bhatt  TTh 11:30am-1pm
Abelian Varieties

Prerequisites: Math 631 and 632.

Course Description: The first half of this course shall go over the basic theory of abelian varieties, while the second half will be dedicated to more advanced topics.

Text: (Optional) Abelian Varieties by Mumford.