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**Title: Painlev\'e functions, Fredholm determinants and combinatorics**

Abstract:  I am going to explain explicit construction of general solutions to isomonodromy equations, with the main focus on the Painlev\'e VI equation. I will start by deriving Fredholm determinant representation of the Painlev\'e VI tau function. The corresponding integral operator acts in the direct sum of two copies of $L^2(S^1)$. Its kernel is expressed in terms of hypergeometric fundamental solutions of two auxiliary 3-point Fuchsian systems whose monodromy is determined by monodromy of the associated linear problem via a decomposition of $\mathbb C\mathbb P^1\backslash\{4\text{ points}\}$ into two pairs of pants. In the Fourier basis, this kernel is given by an infinite Cauchy matrix. I am going to show that the principal minor expansion of the Fredholm determinant yields a combinatorial series representation for the general solution to Painlev\'e VI in the form of a sum over pairs of Young diagrams.