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Title and abstract:

Title: Geometry of Discrete Painlev\'e Equations and Applications

Abstract: The goal of this talk is to present some of the main ideas behind the theory of discrete Painlev\'e equations and to show how they can be effectively applied to understand examples of discrete Painlevé equaions that appear in applications. After explaining the geometric framework, we consider a particular example of a discrete Painlev\'e equation of type $A\_{2}^{(1)\*}$ that occurs as a reduction from the simplest rank-one two-point Schlesinger transformation of a Fuchsian system. The explicit form of this equation is quite complicated. This reflects the fact that the initial coordinates in which the equation was written were good for studying the original Fuchsian system, but not for its Schlesinger transformations. A natural question then is how to find a good coordinate form of this equation. A related question is whether our equation is equivalent to a simpler form of a discrete Painlev\'e equation obtained previously by Grammaticos, Ramani, and Ohta as a deautonomization of a particular QRT mapping. To answer this qustion, we construct a biraional representation of the extended affine Weyl symmetry group $E\_{6}^{(1)}$ of the $d-PA\_{2}^{(1)\*}$ family and then represented each equation as a word in the generators of the group. We show that these words are conjugate. This means that these very differently-looking two equations are in fact equivalent through a change of variables. Using the birational representation of $\widetilde{W}\left(E\_{6}^{(1)}\right)$ we can find this change of variables explicitly. This example is a good illustration of the applicability of Sakai's geometric theory for various applications.