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UV/IR Mixing and the Hierarchy Problem

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Based (mainly) on
- *IR Dynamics from UV Divergences: UV/IR Mixing, NCFT, and the Hierarchy Problem* [1909.01365, JHEP] **with N. Craig**
- *The Hierarchy Problem: From the Fundamentals to the Frontiers* [2009.11870, PhD thesis]

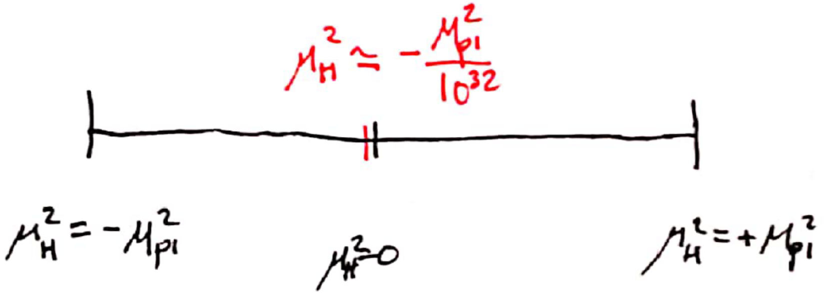
UMichigan HEP Seminar, 11/11/20

Fine-tuning problems come from asking big, important questions

Why is there macroscopic structure?

It just is

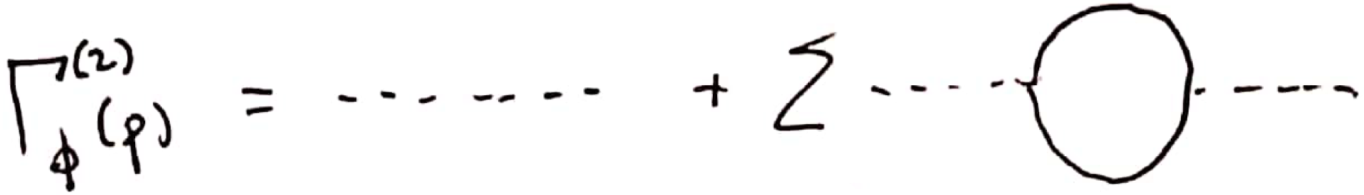
The Hierarchy Problem



There is no hierarchy problem in the Standard Model

Our toy model of the SM – a single scalar whose mass is an input parameter

$$S = \int \mathrm{d}^4x \left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_0^2\phi^2 - V(\phi) - g\phi\mathcal{O}(\psi_i\psi_i) \right)$$



$$m_{\mathrm{phys}}^2 = m_0^2 + \frac{g^2}{(4\pi)^2}\frac{1}{\epsilon}\sum_i M_i^2$$

Mass not protected by a symmetry, so gets large corrections

The Higgs mass is an *input* so just choose the bare mass to give the right answer

Hierarchy problem when Higgs mass is an *output*

Now imagine in the UV there is an SU(2) global symmetry

$$\Phi^{\mathrm{a}} = \begin{pmatrix} \psi \\ \phi \end{pmatrix}$$

Where we've measured ϕ to be very light, but ψ must be heavy

$$S = \int \mathrm{d}^4 x \left(-\frac{1}{2}\left(\partial_\mu \Phi\right)^\dagger \partial^\mu \Phi - \frac{1}{2} M_0^2 \Phi^\dagger \Phi - \lambda_0 \Phi^\dagger \Sigma \Sigma^\dagger \Phi - V(\Phi) \right)$$

Tree-level $\quad m_\psi^2 = M_0^2 \quad\quad m_\phi^2 = M_0^2 + \lambda_0 v^2$

Loop-level $\quad m_\psi^2 = M_0^2 + \frac{1}{\epsilon}\left(M_\Sigma^2 + \cdots\right) \quad\quad m_\phi^2 = \left[M_0^2 + \frac{1}{\epsilon}\left(M_\Sigma^2 + \cdots\right)\right] + \lambda_0 v^2$

Renormalized $\quad m_\psi^2 = M_{\mathrm{phys}}^2 \quad\quad m_\phi^2 = M_{\mathrm{phys}}^2 + \lambda_{\mathrm{phys}} v_{\mathrm{phys}}^2$

$$\lambda_{\mathrm{phys}} = -1.00000000000000000000000000001 \times \frac{M_{phys}^2}{v_{phys}^2}$$

Fine-tuned unless $\mathrm{m}_\phi \sim$ scale of new physics

The Hierarchy Problem:
From the Fundamentals to the Frontiers

Contents

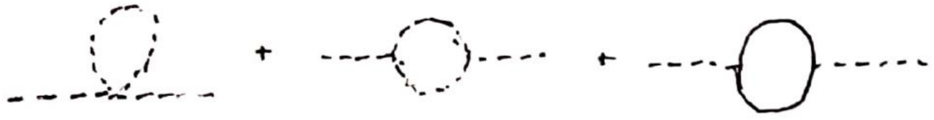
How to get a light scalar: Classic edition

Introduce UV structure to forbid large contributions, and IR dynamics to break that structure to the observed SM EFT

E.g. Supersymmetry

$Q|\text{fermion}\rangle = |\text{boson}\rangle \qquad Q|\text{boson}\rangle = |\text{fermion}\rangle$

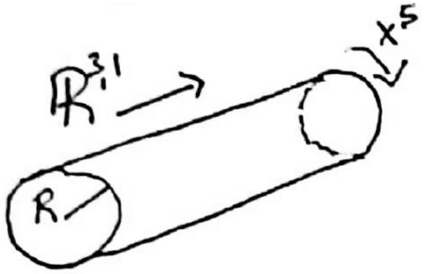
Must be broken!



$$\sim \Lambda^2 \times 0 + \left(m^2 - \tilde{m}^2\right)\log\frac{\Lambda^2}{m^2}$$

E.g. Extra dimensions

UV masses forbidden by gauge invariance

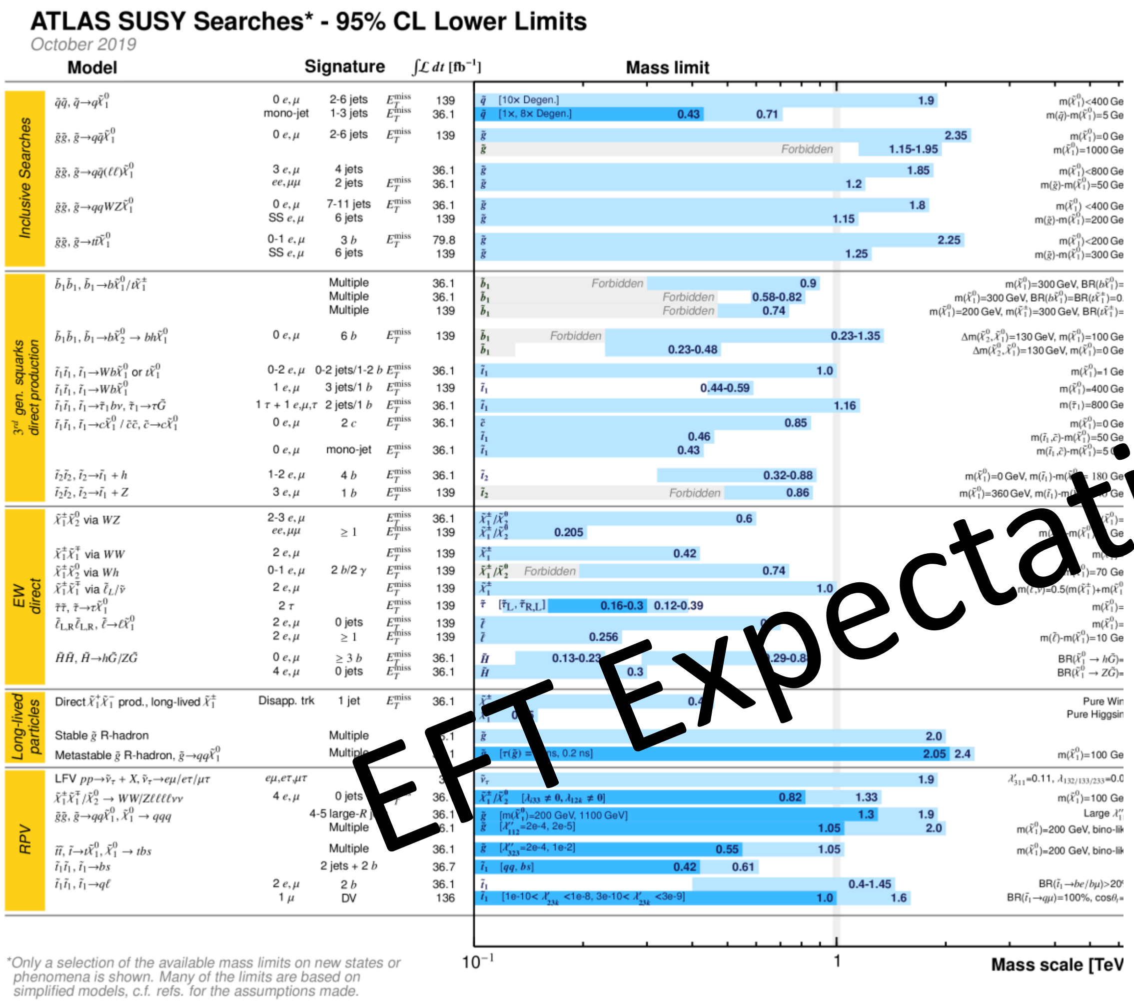


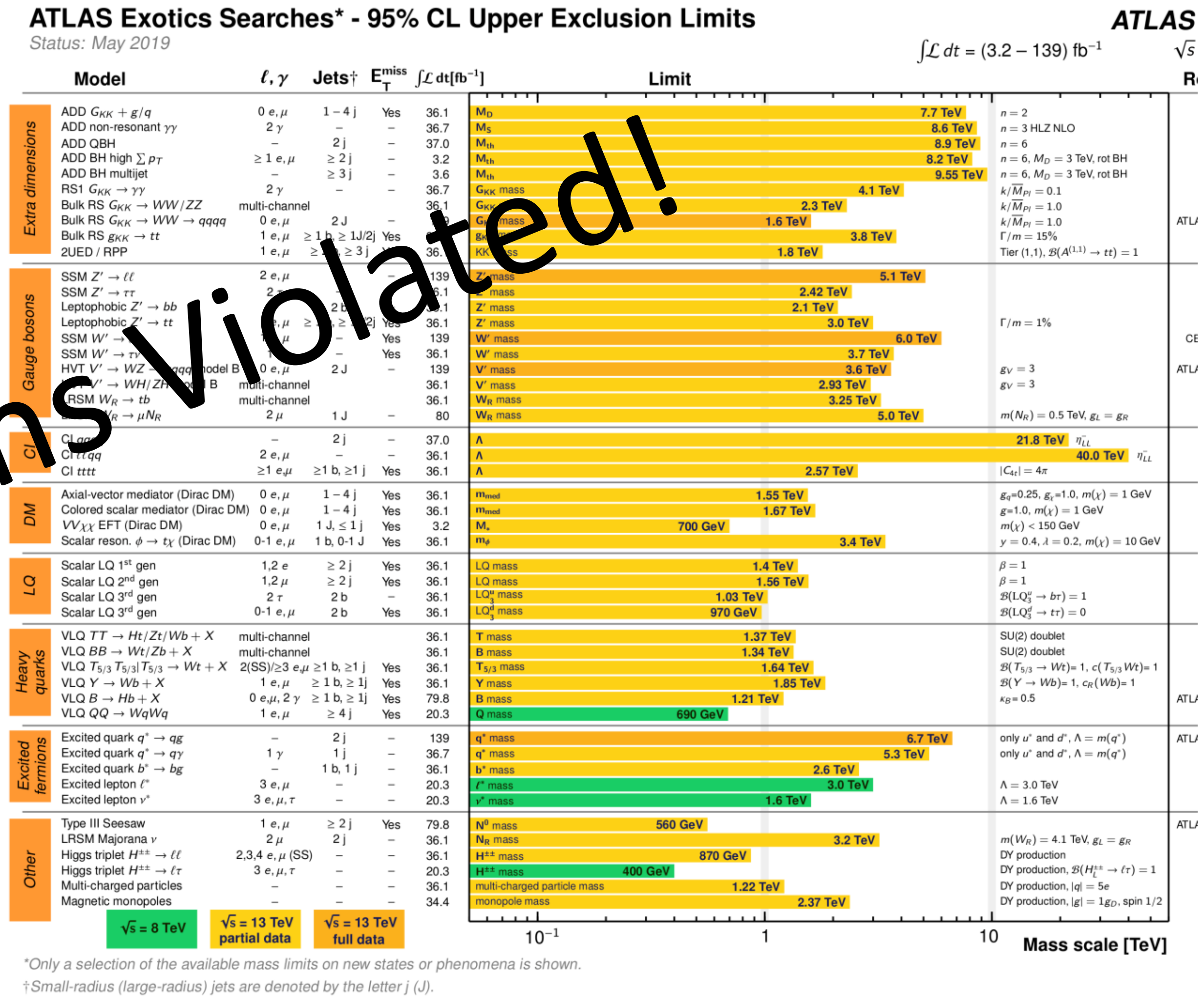
$$g_{AB} = \begin{pmatrix} g_{\mu\nu} & g_{\mu 5} = A_\mu \\ g_{5\nu} = A_\nu & g_{55} = \phi \end{pmatrix}$$

$$\mathcal{L} \supset \frac{1}{2}\frac{1}{R^2}\phi^2 + \sum_{\substack{\text{SM fields} \\ n}} \frac{n}{R}\psi\psi$$

→ Either way, expect new strongly-interacting particles near the weak scale

Data!





EFT Expectations Violated!

(With apologies to CMS)

Data!

A *maximalist* interpretation of the results is that maybe there really is no new weak-scale physics. EFT expectations really *are* violated.

Ite…
phy…

[Cr…
Alipo…
…'18;
Co… …oby-Smith '20;
Cr…

Not radical enough

Innovate!

Perhaps the violation of EFT expectations results from a *physical* violation of EFT.

[Craig, Garcia-Garcia, **SK** '18;
Craig, Garcia-Garcia, **SK** '19;
Craig, **SK** '19;
Alipour-fard, **SK** hopefully, …]

(NB: I also do things/have interests unrelated to the hierarchy problem!)

The EFT of Quantum Gravity

We have a *great* perturbative theory of quantum gravity!

$$M_{\rm pl} \approx 10^{18}\ \text{GeV}$$

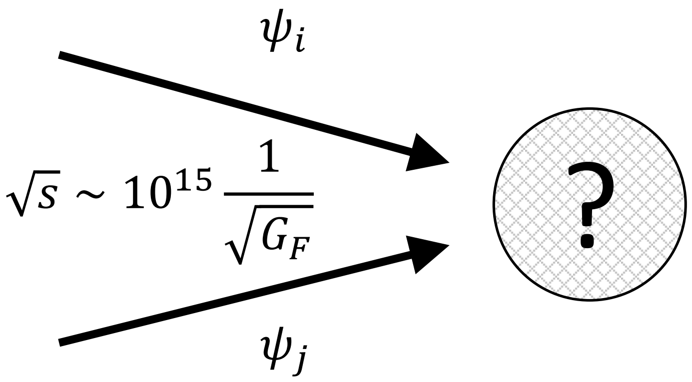
$$\mathcal{L}_{EH} = \frac{1}{2M_{\rm pl}^2}\sqrt{-g}R \approx \partial h \partial h + \frac{1}{M_{\rm pl}}\partial^2 \text{h}^3 + \frac{1}{M_{\rm pl}^2}\partial^2 \text{h}^4 + \cdots$$

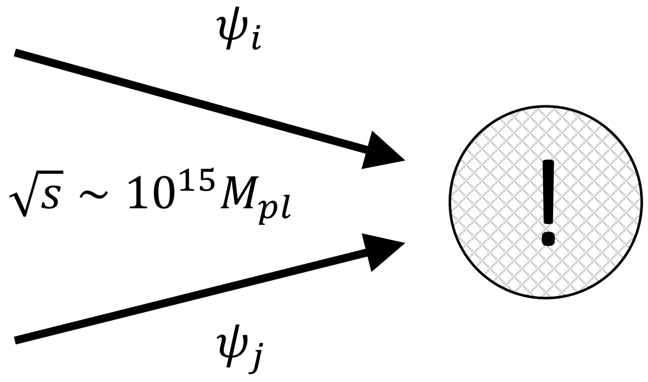
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm pl}$$

Compare with Fermi theory of the weak interactions

$$\mathcal{L} \approx \bar{\psi}_i \gamma_\mu \partial^\mu \psi^i + G_F \psi_i \psi_j \psi_k \psi_l + G_F^2\, \psi_i \psi_j \psi_k \psi_l \psi_m \psi_n + \cdots$$

$$\frac{1}{\sqrt{G_F}} \approx 300\ \text{GeV}$$





Noncommutativity

'Quantize!'

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

[Snyder '47]!

$$\Rightarrow \ \Delta\hat{x}_\mu \Delta\hat{x}_\nu \geq \frac{|\theta_{\mu\nu}|}{2}$$

UV/IR mixing is front and center!
Separation of scales is violated!

Noncommutative Field Theory

But how to do physics on such spaces?

Transfer the noncommutativity to the fields!

Introduce 'star-product' $$f(x) \star g(x) = f(x) \exp\left(\frac{i}{2} \overleftarrow{\partial}^{\mu} \theta_{\mu\nu} \overrightarrow{\partial}^{\nu}\right) g(x)$$

Noncommutative Field Theory

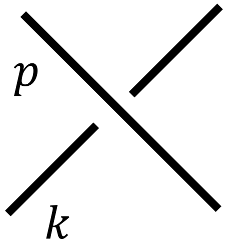
$$\mathcal{L}_{\text{int}}^{(NC)} = \frac{\lambda}{n!} \overbrace{\phi(x) \star \phi(x) \star \cdots \star \phi(x)}^{\text{n copies}}$$

$$\Rightarrow \quad \tilde{V}\left(k_1, \ldots, k_n\right) = \delta\left(k_1 + \cdots + k_n\right) \exp\left(\frac{i}{2} \sum_{i<j}^{n} k_i^{\mu} k_j^{\nu} \theta_{\mu\nu}\right)$$

Vertices no longer permutation-invariant!

Phase factors of graphs reduce to a graph-topological statement [Filk '94]

Planar graphs: Solely an overall phase involving external momenta
Nonplanar graphs: Additional phases for lines which cross



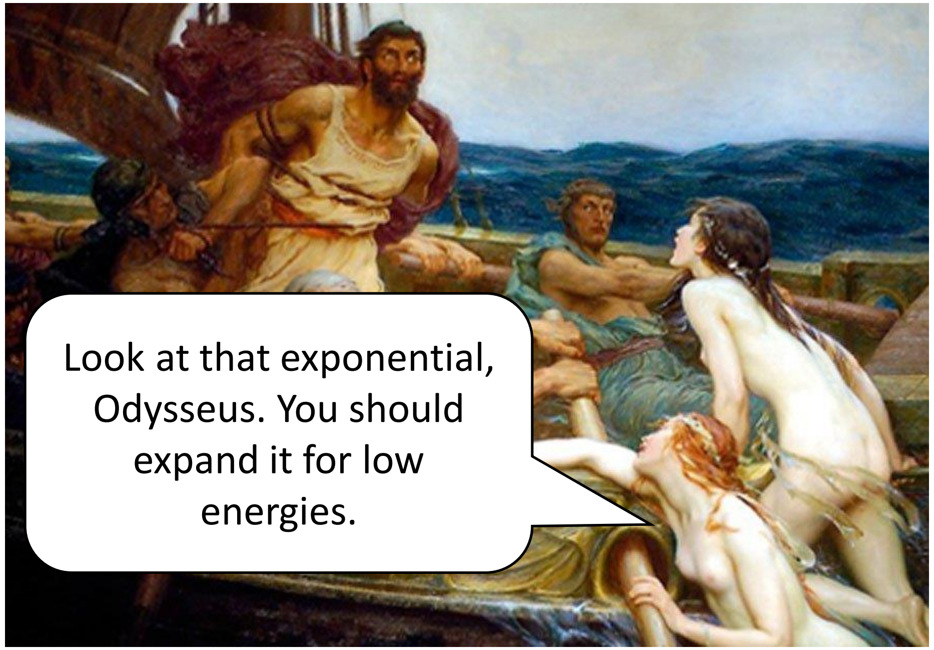
$$\sim e^{i\, p^{\mu} \theta_{\mu\nu} k^{\nu}}$$

Thou Shalt Not Expand

$$\mathcal{S}_{\text{int}}^{(NC)} \sim \frac{\lambda}{n!} \int \mathrm{d}k \; \phi(k_1)\phi(k_2)\ldots\phi(k_n)\exp\left(\frac{i}{2}\sum_{i<j}^{n} k_i^{\mu} k_j^{\nu} \theta_{\mu\nu}\right)$$

The 'theta-expanded' NCFT removes all of the UV/IR mixing!

Much past work can be ignored from our perspective



[*Ulysses and the Sirens*, Draper, 1909]

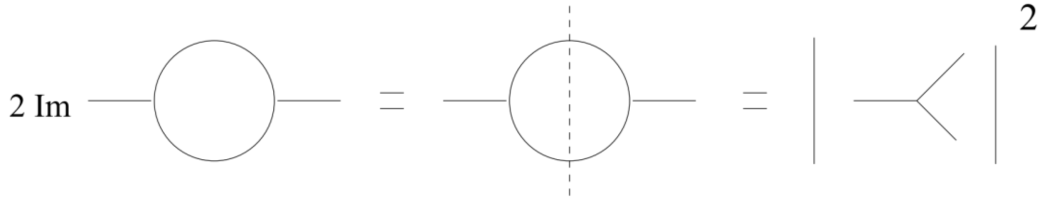
Skeletons in the Closet

- Lorentz violation!
  - Not out the window; just like turning on a magnetic field in a lab
  - Folk theorems [Collins et al. ‘04] about empirical bounds not fully applicable

See also [Calmet ‘04]

- Unitarity of Lorentzian theory with timelike noncommutativity?
  - Failure well-understood from stringy perspective [Seiberg, Susskind, Toumbas ’00]
  - Field-theoretically, issue with formulation of nonlocal-in-time theories

[Gomis et al. ‘00; Bahns et al. ’02; Bozkaya et al. ‘02; Liao & Sibold ’02; Rim & Yee ‘02; Denk & Schweda ‘03, Fischer & Putz ’03; Liao ‘04, …]

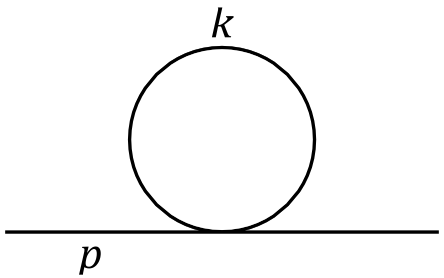


[Gomis, Mehen ’00]

(But at the end I’ll mention some preliminary progress on these.)

Euclidean ϕ^4

[Minwalla, Seiberg, Van Raamsdonk '99]



$$\sim \int \frac{d^4 k}{k^2 + m^2}$$

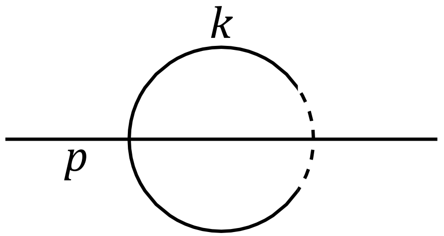
$$\sim \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2}$$

$$\text{Planar} \sim g^2 \left(\Lambda^2 - m^2 \log \frac{\Lambda^2}{m^2} + \cdots \right)$$

Use Schwinger parameters

$$\frac{1}{k^2 + m^2} = \int_0^\infty d\alpha \, e^{-\alpha (k^2 + m^2)}$$

Regulate with $e^{-\frac{1}{\Lambda^2 \alpha}}$



$$\sim \int \frac{d^4 k}{k^2 + m^2} e^{i k^\mu \theta_{\mu\nu} p^\nu}$$

$$\sim \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2 - \frac{p \circ p}{4\alpha}}$$

Completed the square
$p \circ k \equiv -p_\mu \theta^{2\mu\nu} k_\nu$

$$\text{Nonplanar} \sim g^2 \left(\Lambda_{\text{eff}}^2 - m^2 \log \frac{\Lambda_{\text{eff}}^2}{m^2} + \cdots \right)$$

$$\Lambda_{\text{eff}}^2 \equiv \frac{1}{1/\Lambda^2 + p \circ p / 4}$$

Not regulator-dependent – can also see in dim reg [Craig, SK]

[Minwalla, Seiberg, Van Raamsdonk '99]

A new pole!

1 Loop 1PI quadratic effective action

$$\frac{1}{2}\left(p^2+M^2+\frac{g^2}{96\pi^2\left(\frac{p\circ p}{4}+\frac{1}{\Lambda^2}\right)}+\dots\right)\phi(p)\phi(-p)$$

with renormalized mass $M^2=m^2+\frac{g^2\Lambda^2}{48\pi^2}-\frac{g^2m^2}{48\pi^2}\log\frac{\Lambda^2}{m^2}$

Nonperturbative in θ!

In $\Lambda\to\infty$ limit there are now two poles!

$$p^2=-m^2+\mathcal{O}(g^2)$$

$$p\circ p=-\frac{g^2}{24\pi^2m^2}+\mathcal{O}(g^4)$$

simple case $\theta^{\mu\nu}\sim {}^1/_{\Lambda_\theta^2}\to p^2\propto g^2\Lambda_\theta^4/m^2$

A new light 'particle' with nothing nearby to explain its presence!

After Wick rotation, inaccessible in s-channel

[Minwalla, Seiberg, Van Raamsdonk '99]

Wilsonian Interpretation

Normally a renormalizable Wilsonian action must satisfy
1. Correlation functions are well defined in the $\Lambda \to \infty$ limit
2. At finite Λ they differ from the limiting value by $O(\Lambda^{-1})$ for all momenta

$$\frac{1}{2}\left(p^2 + M^2 + \frac{g^2}{96\pi^2\left(\frac{p\circ p}{4} + \frac{1}{\Lambda^2}\right)} + \dots\right)\phi(p)\phi(-p)$$

At small momenta (2) is badly violated here!

Can restore a Wilsonian interpretation by introducing a new field χ

$$\delta\mathcal{L}(\Lambda) = \frac{1}{2}\partial\chi \circ \partial\chi + \frac{1}{2}\frac{\Lambda^2}{4}(\partial \circ \partial\chi)^2 + i\frac{1}{\sqrt{24\pi^2}}g\chi\phi$$

$$\delta S(\Lambda)_{1\mathrm{PI}} \sim \frac{1}{2}\left(-\frac{g^2}{96\pi^2\left(\frac{p\circ p}{4} + \frac{1}{\Lambda^2}\right)} + \frac{g^2}{24\pi^2 p\circ p}\right)\phi(p)\phi(-p)$$

Yukawa theory

$$g\varphi\overline{\psi}\psi \;\to\; g_1\varphi\star\overline{\psi}\star\psi + g_2\overline{\psi}\star\varphi\star\psi$$

[c.f. Anisimov, Banks, Dine, Graesser '01]

However T is antiunitary! $(PT)^{-1}\left(f(x)\star g(x)\right)PT = g(x)\star f(x)$

So CPT 're-cycles' the two interaction terms!

$$(CPT)^{-1}\mathcal{L}_{\text{int}}^{(\text{NC})}CPT = \mathcal{L}_{\text{int}}^{(\text{NC})} \quad\Longrightarrow\quad g_1 = g_2$$

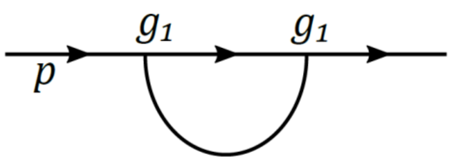
Scalar Two-Point Function

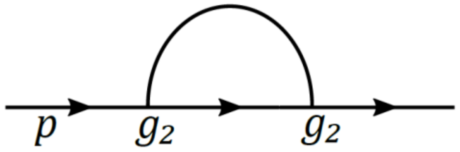
$$\text{Planar} \sim -(g_1^2 + g_2^2)\left(\Lambda^2 + \cdots\right)$$

Evaluation requires some cleverness and 'lightcone Schwinger coordinates'

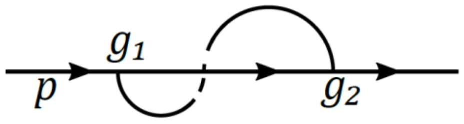
$$\text{Nonplanar} \sim -g_1 g_2\left(\Lambda_{\text{eff}}^2 + \cdots\right)$$

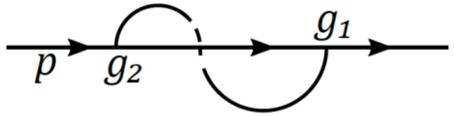
Fermion Two-Point Function





$$\text{Planar} \sim -\left(g_1^2 + g_2^2\right)\left(M\ -\frac{p\cdot\gamma}{2}\right)\log\Lambda^2 + \cdots$$





$$\text{Nonplanar} \sim -g_1 g_2\left(M\ -\frac{p\cdot\gamma}{2}\right)\log\Lambda_{\text{eff}}^2 + \cdots$$

Logarithmic UV sensitivity → only logarithmic IR feature

Softly-broken Supersymmetry - Wess Zumino

Look at the interplay between UV/IR mixing and UV finiteness

Compute one-loop two-point function again, with
Z wavefunction renormalization and δm^2 mass correction

$$\Gamma^{(2),s} \equiv Zp^2 + Z^{-1}(m^2 + \delta m^2)$$

$$Z = 1 + \frac{y^2}{32\pi^2} \log\left[\frac{\Lambda\Lambda_{\text{eff}}}{M^2}\right] + \ldots$$

$$\delta m^2 = \frac{y^2}{32\pi^2}\left(M^2 - m^2\right) \log\left[\frac{\Lambda\Lambda_{\text{eff}}}{M^2}\right] + \ldots$$

For $\Lambda, \Lambda_{\text{eff}}$ large

The transmogrification accords with the intuition we've now built

Softly-broken Wess Zumino

So a UV finite theory has no IR effects from UV/IR mixing
A UV sensitive theory has this surprising IR feature

How do these connect? With soft breaking we can transition between the two.

Taking instead $M \gg \Lambda, \Lambda_{\text{eff}}$ of the full result

$$\delta m^2 = \frac{y^2}{256\pi^2}\left(6M^2 + 16\Lambda^2 + 8\Lambda_{\text{eff}}^2\right) + \ldots$$

EFT has been broken in a controlled way!

UV/IR mixing requires lack of UV finiteness! So not a module to tack on to theory with hierarchy problem.
On the other hand, one says that in this theory there just never is a hierarchy problem.

Though perhaps we can have our cake and eat it too with noncommutative orbifold field theory?

Some Early Words on Current Directions

Reformulate Lorentz invariant version of NCFT

$$S = \int d^6\theta W(\theta) \int d^4x \mathcal{L}_\star(\phi)$$

[Snyder '47; Doplicher et al. '95; Kase et al '02; Carlson et al. '02; Heckman & Verlinde '14; Much & Vergara '17, many large literatures in various directions,…]

$$S_{int} \sim \int d^6\theta W(\theta) \int \left(\prod_{k=1}^{n} d^4k_i\right) \delta\left(\sum k_i\right) \tilde{\phi}(k_1)\ldots\tilde{\phi}(k_n) \exp\frac{i}{2}\left(\sum_{i<j} k_i^\mu \theta_{\mu\nu} k_j^\nu\right)$$

$$\tilde{V}(k_1,\ldots,k_n) = \delta(\sum k_i) \int d^6\theta W(\theta) \exp\frac{i}{2}\left(\sum_{i<j} k_i^\mu k_j^\nu\right)\theta_{\mu\nu} = \delta(\sum k_i)\widetilde{W}(K_{[\mu\ \nu]})$$

$$\Gamma^{(2)}_{1,\mathrm{nonplanar}} = \frac{g^2}{6(2\pi)^4} \int \frac{\mathrm{d}^4k}{k^2+m^2} \int d^6\theta W(\theta) e^{ik^\mu\theta_{\mu\nu}p^\nu}$$

$$\Gamma^{(2)}_{1,\mathrm{nonplanar}} \sim g^2 \int \frac{\mathrm{d}^4k}{k^2+m^2} \widetilde{W}(k^{[\mu}p^{\nu]})$$

Easy example

$$W(\theta) = \frac{\lambda^3}{\pi^3}\exp(-\lambda\theta^2) \rightarrow \widetilde{W}(k^{[\mu}p^{\nu]}) = \exp(-\frac{\pi^2}{\lambda}(k^{[\mu}p^{\nu]})^2)$$

$$\Gamma^{(2)}_{1,\text{planar}} = \frac{g^2}{48\pi^2}\int \frac{\mathrm{d}\alpha}{\alpha^2} e^{-\alpha m^2}$$

Old $\Gamma^{(2)}_{1,\text{nonplanar}} = \frac{g^2}{96\pi^2}\int \frac{\mathrm{d}\alpha}{\alpha^2} e^{-\alpha m^2 - \frac{p \circ p}{4\alpha}}$

New $\Gamma^{(2)}_{1,\text{nonplanar}} \sim g^2\lambda^2 \int \mathrm{d}\alpha e^{-\alpha m^2}\left[\frac{1}{(\alpha\lambda)(\alpha\lambda+\pi^2p^2)} + \frac{1}{(\pi^2p^2)^{1/2}(\alpha\lambda+\pi^2p^2)^{3/2}}\text{ArcTanh}\left(\sqrt{\frac{\pi^2p^2+\alpha\lambda}{\pi^2p^2}}\right)\right.$

$$\Gamma^{(2)}_{1,\text{full}} = p^2 + M^2(g,\Lambda) + \frac{g^2\lambda}{p^2}(1+\log\Lambda)$$

$$p^2 = -M^2 + \mathcal{O}(g^2) \qquad \& \qquad p^2 = -g^2\lambda/M^2 + \mathcal{O}(g^4)$$

Conclusions

- Our EFT expectations have been violated! Perhaps by physical breakdown of EFT
- In NCFT this can generate an IR scale *ex nihilo*
- This behavior persists and with interesting properties as you go closer to the SM
- UV/IR mixing requires UV sensitivity which puts this strategy in stark contrast to others
- Lots of directions to investigate

The Hierarchy Problem: From the fundamentals to the frontiers

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Backup Slides

Wilsonian Interpretation Redux

$$\Delta S_{\mathrm{1PI}}(\Lambda) = -\frac{1}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{g_1 g_2}{2\pi^2}\left(\Lambda_{\mathrm{eff}}^2 - \frac{4}{p\circ p}\right)\varphi(p)\varphi(-p)$$

Then for any cutoff Λ, we have the behavior for small $p \circ p$

$$\Gamma_1^s(p) = -\frac{2g_1 g_2}{\pi^2 p\circ p} + \ldots$$

Opposite sign from ϕ^4 theory!

Introduce auxiliary field, or just talk about a modified propagator after integrating it out

$$m^2 + (p_i + p_j)^2 - \frac{2g_1 g_2}{\pi^2}\frac{1}{(p_i+p_j)\circ(p_i+p_j)}$$

New light pole at $$s = \frac{2g_1 g_2}{\pi^2}\frac{1-\beta^2}{1+\beta^2}\frac{\Lambda_\theta^4}{m^2}$$

[but see e.g. Bozkaya et al ‘02]

Future Directions

In Wilsonian EFT, nonlocality for $p \gtrsim \Lambda \leftrightarrow x \lesssim 1/\Lambda$

Particles in NCFT are like rods of length $L \sim p\theta$ [Sheikh-Jabbari '99, Bigatti & Susskind '00, Seiberg, Susskind, Toumbas '00, …]

So 'extra' nonlocality for $1/\Lambda^2 \lesssim p\theta^2 p$

Could this sort of nonlocality appear in other models?

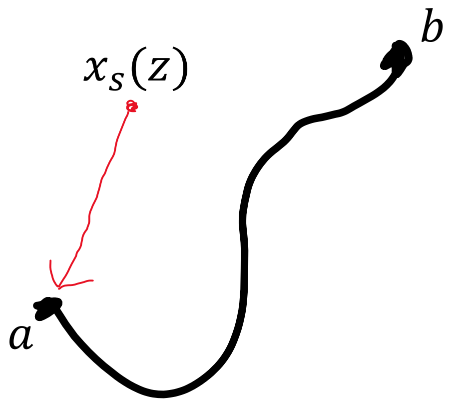
Generalized Uncertainty Principle: $\Delta x \gtrsim \frac{\hbar}{\Delta p} + \ell_p^2 \Delta p$

[originally Maggiore '93, review Tawfik & Diab, '15]

Similar claims in String Theory as well [Gross & Mende '88, Konishi, Paffuti, Provero '90, Yoneya, '00, …]

Future Directions

$$F(z) = \int_a^b \mathrm{d}x \; f(x,z)$$

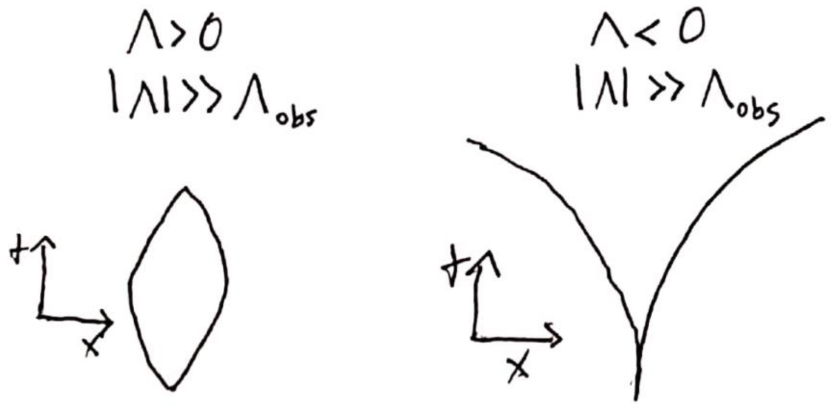


Alternatively, try to understand general nonlocal theories

[Tomboulis '15, Tomboulis & Chin '18] look carefully at a disjoint class of nonlocal theories

Presumably the magic of NCFT is associated with 'endpoint singularities'

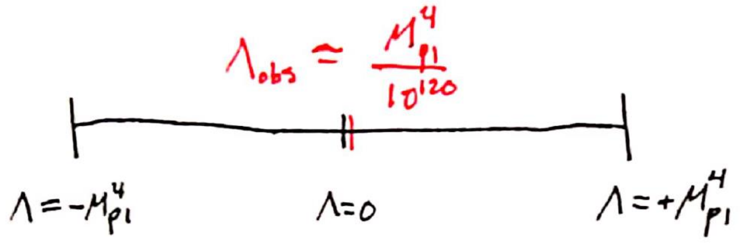
Fine-tuning problems come from asking big, important questions



Why is there a macroscopic universe?

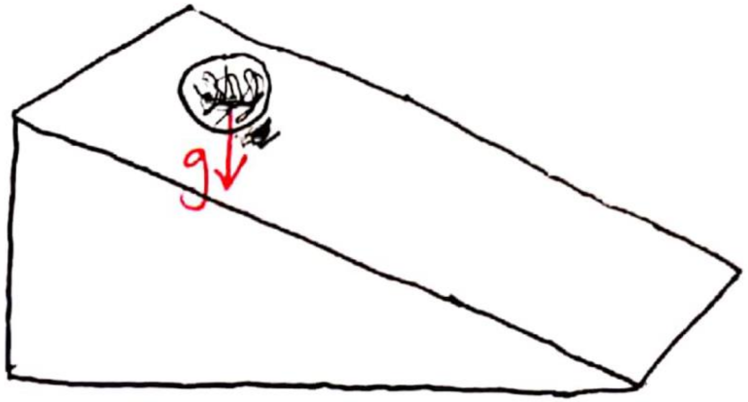
It just is

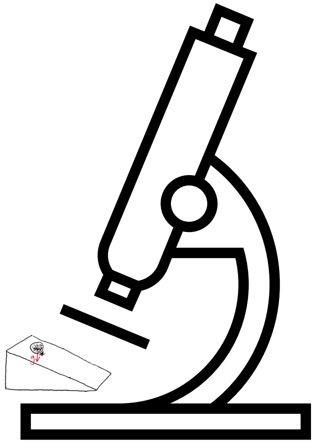
The Cosmological Constant Problem

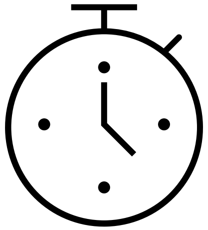


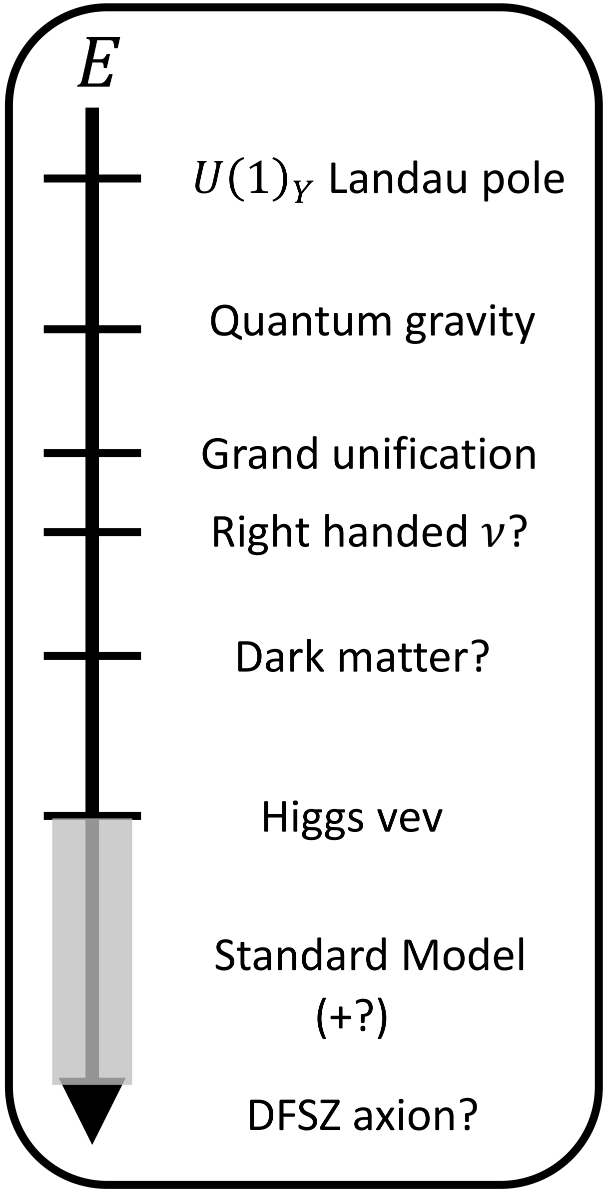
Effective Field Theory

Focus on the important degrees of freedom!









Correct your leading order description in a perturbative expansion depending on how much precision you want

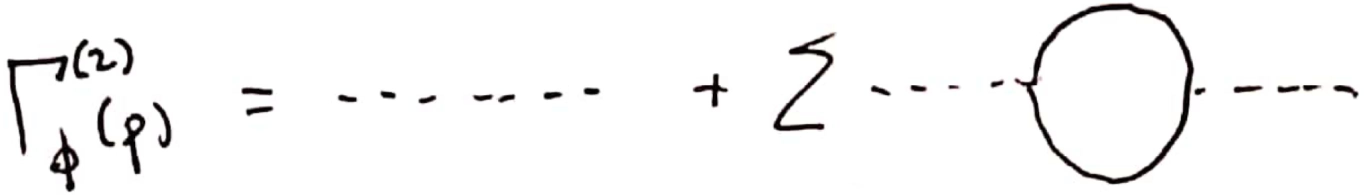
$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = \frac{m\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} \approx m\boldsymbol{v} + \frac{1}{2} m \frac{v^2}{c^2}\boldsymbol{v} + \cdots$$

$$\mathcal{L}(\phi_{SM}) = \mathcal{L}_{SM}(\phi_{SM}) + \frac{\mathcal{L}^{(5)}(\phi_{SM})}{\Lambda} + \frac{\mathcal{L}^{(6)}(\phi_{SM})}{\Lambda^2} + \cdots$$

There is no hierarchy problem in the Standard Model

Our toy model of the SM – a single scalar whose mass is an input parameter

$$S = \int \mathrm{d}^4 x \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_0^2 \phi^2 - V(\phi) - g\phi \mathcal{O}(\psi_i \psi_i) \right)$$



$$m_{\mathrm{phys}}^2 = m_0^2 + \frac{g^2}{(4\pi)^2}\frac{1}{\epsilon}\sum_i M_i^2$$

Mass not protected by a symmetry, so gets large corrections

The Higgs mass is an *input* so just choose the bare mass to give the right answer

Hierarchy problem when Higgs mass is an *output*

Now imagine in the UV there is an SU(2) global symmetry

$$\Phi^{\mathrm{a}} = \begin{pmatrix} \psi \\ \phi \end{pmatrix}$$

Where we've measured ϕ to be very light, but ψ must be heavy

$$S = \int \mathrm{d}^4x \left(-\frac{1}{2}\left(\partial_\mu \Phi\right)^\dagger \partial^\mu \Phi - \frac{1}{2} M_0^2 \Phi^\dagger \Phi - \lambda_0 \Phi^\dagger \Sigma \Sigma^\dagger \Phi - V(\Phi) \right)$$

Tree-level	$m_\psi^2 = M_0^2$	$m_\phi^2 = M_0^2 + \lambda_0 v^2$
Loop-level	$m_\psi^2 = M_0^2 + \frac{1}{\epsilon}\left(M_\Sigma^2 + \cdots\right)$	$m_\phi^2 = \left[M_0^2 + \frac{1}{\epsilon}\left(M_\Sigma^2 + \cdots\right)\right] + \lambda_0 v^2$
Renormalized	$m_\psi^2 = M_{\mathrm{phys}}^2$	$m_\phi^2 = M_{\mathrm{phys}}^2 + \lambda_{\mathrm{phys}} v_{\mathrm{phys}}^2$

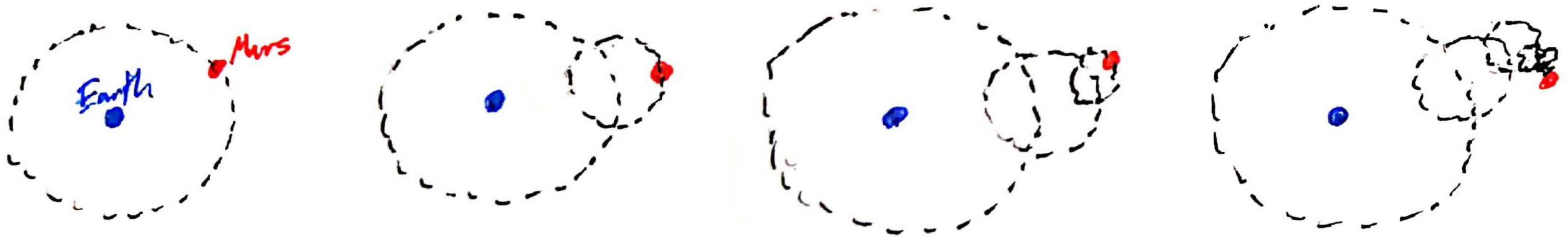
$$\lambda_{\mathrm{phys}} = -1.00000000000000000000000000001 \times \frac{M_{phys}^2}{v^2}$$

Fine-tuned unless $\mathrm{m}_\phi \sim$ scale of new physics

Waiter, there's Philosophy in my Physics

“Who cares? We can fit the data by tuning those parameters.”

A model being not literally impossible is a **very low bar** for a scientific theory



$$r(\theta) = \sum_{i=1}^{N} r_n \sin\frac{\theta}{n}$$

A perfect fit, a perfect theory

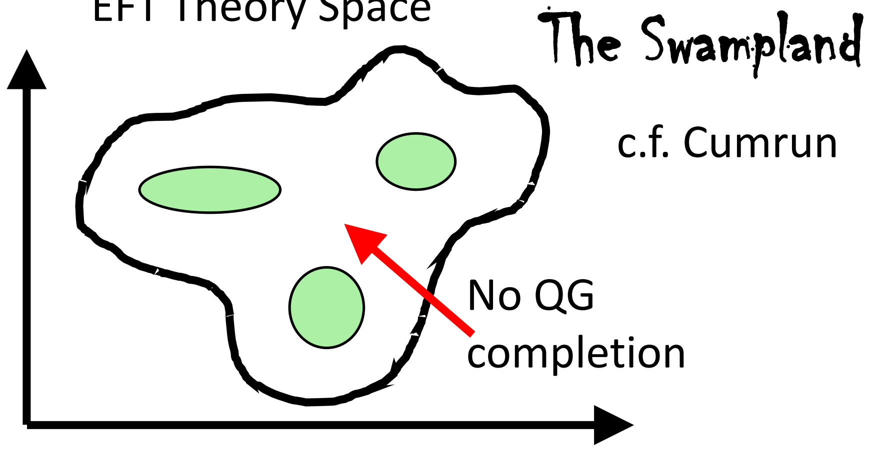
To EFT or not to EFT?

Gravity very generally motivates looking at `UV/IR mixing' effects!

BH thermodynamics
c.f. Stephen

$T_{BH} \propto 1/M_{BH}$

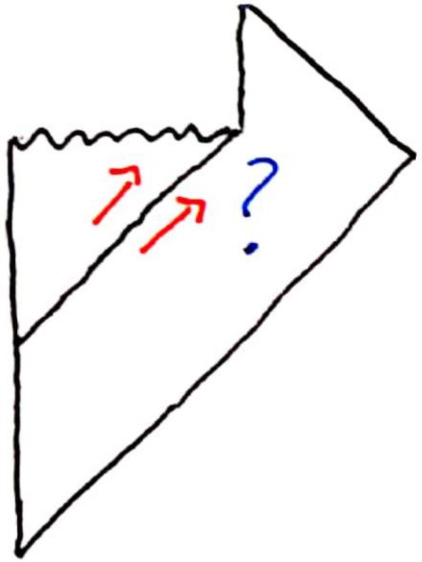
EFT Theory Space



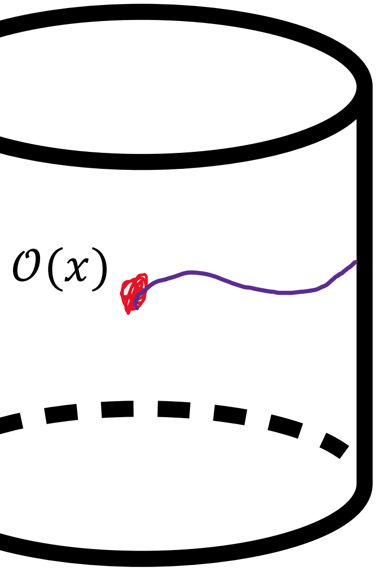
c.f. Cumrun

BH info and firewalls?!

c.f. Don

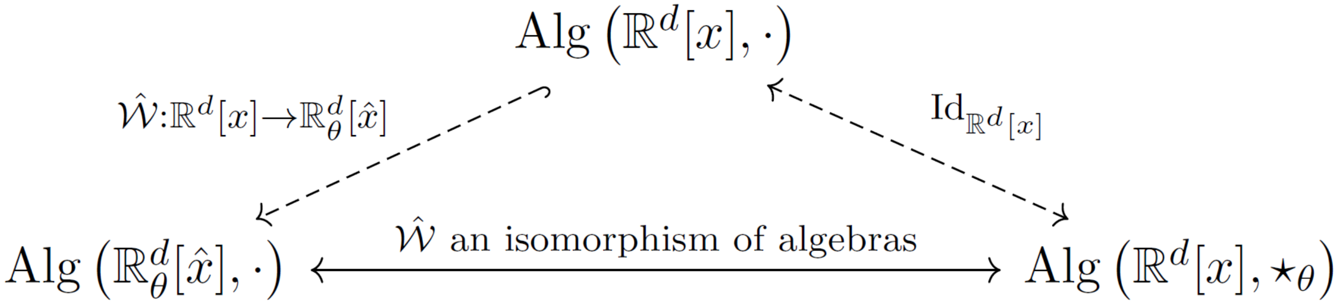


Necessity of nonlocality in gravitational observables



c.f. Steve

NC Quantization



$$\hat{\mathcal{W}}\left[f \star_\theta g\right] = \hat{\mathcal{W}}\left[f\right] \cdot \hat{\mathcal{W}}\left[g\right]$$

$$\begin{aligned} e^{ikx} \star_\theta e^{ik'x} &= \hat{\mathcal{W}}^{-1}\left[\hat{\mathcal{W}}\left[e^{ikx}\right] \cdot \hat{\mathcal{W}}\left[e^{ik'x}\right]\right] \\ &= \hat{\mathcal{W}}^{-1}\left[e^{-\frac{i}{2}\theta^{ij}k_i k'_j} e^{i(k+k')\cdot\hat{x}}\right] \end{aligned}$$

$$e^{ikx} \star_\theta e^{ik'x} \equiv e^{-\frac{i}{2}\theta^{\mu\nu}k_\mu k'_\nu} e^{i(k+k')\cdot x}$$

Correspondence Principle

[5] We note here that the failure of a 'correspondence principle' between commutative and noncommutative theories as $\theta^{\mu\nu} \to 0$ is clearly intrinsically linked to the appearance of UV/IR mixing. This failure doesn't violate Kontsevich's proof of the existence of deformation quantization for any symplectic manifold [81], as that is confined solely to 'formal' deformation quantization — that is, the production of a formal power series expansion of the algebra of observables in terms of the deformation parameter. As was noted in Section 2 and is now on prime display, the physics of the theory with nonperturbative θ-dependence is starkly different from that of any truncation.

Renormalizability

[7] There has been much work on understanding renormalizability of NCFTs, especially with an eye toward finding a mathematically well-defined four-dimensional quantum field theory with a non-trivial continuum limit. Renormalizability has been proven for modifications of NCFTs where the free action is supplemented by an additional term which adjusts its long-distance behavior. Such an action is manufactured either by requiring it manifest 'Langman-Szabo' duality [82] $p_\mu \leftrightarrow 2(\theta^{-1})_{\mu\nu}x^\nu$ [83, 84] or by adding a $1/p \circ p$ term to the free Lagrangian [85], the latter of which directly has the interpretation of adding 'somewhere to put the $1/p \circ p$ counterterm'. For recent reviews of these and related efforts we refer the reader to [86, 87]. It would be interesting to understand fully the extent to which the physics of these schemes agrees with the interpretation of the IR effects as coming from auxiliary fields [37, 61].

Pole Accessibility

[11]Note that this peculiar connection regarding (in)accessibility is due to the Lorentz violation. While the normal pole which is inaccessible in Euclidean signature becomes accessible for timelike momenta in Lorentzian signature, the Wick rotation affects the noncommutative momentum contraction differently. When taking $x_4 \to -ix_0$, one also rotates $\theta_{4\nu} \to -i\theta_{0\nu}$ such that Equation 2.1 continues to hold for the same numerical $\theta_{\mu\nu}$. For the simplest configuration of full-rank noncommutativity with $\theta_{\mu\nu}$ block-off-diagonal and only one eigenvalue $1/\Lambda_\theta^2$, the Euclidean $p{\circ}p = p^2/\Lambda_\theta^4$ becomes a Lorentzian $p{\circ}p = (p_0^2 - p_1^2 + p_2^2 + p_3^2)/\Lambda_\theta^4$. So a noncommutative pole which is inaccessible in the Euclidean theory becomes accessible in the Lorentzian theory for spacelike momenta, while a noncommutative pole which can be accessed in the Euclidean theory becomes accessible in the s-channel in Lorentzian signature.

Start in Euclidean space

$X_a = (x_4, x_1, x_2, x_3)$ $[x_a, x_b] = i\theta_{ab}$, $\theta_{ab} = \theta\begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}$, $g_{ab} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = g^{ab}$

$p_a = (p_4, p_1, p_2, p_3) \to p^a = g^{ab} p_b = \begin{pmatrix} p_4 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$

$\theta_a{}^b = \theta_{ac} g^{bc} = \theta\begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix} = \theta^a{}_b$, $\theta^{ab} = \theta_c{}^b g^{ac} = \theta\begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}$

If you tried $\theta^{2a}{}_b = \theta^{ac}\theta_{bc}$ you'd get the wrong sign for ↳

$\theta^{2a}{}_b = \theta^{ac}\theta_{cb} = \theta^2\begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$p\theta p_{Euclidean} > 0$ in agreement w/ everything

$p\theta p_{Euclidean} = -\theta^2 (p_4, p_1, p_2, p_3) \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}\begin{pmatrix} p_4 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \theta^2(p_4^2 + p_1^2 + p_2^2 + p_3^2) = p^2_{Euclidean}\theta^2$

Now Wick rotate in coords & metric & NC-tensor

$x_4 \to -i x_0$, $\theta_{4a} \to -i\theta_{0\nu}$, $g_{0a} \to -g_{0a}$

So that now

$[x_\mu, x_\nu] = i\theta_{\mu\nu}$, $\theta_{\mu\nu} = \theta\begin{pmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}$ so still real, $g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = g^{\mu\nu}$

Now $p_\mu = (p_0, p_1, p_2, p_3) \to p^\mu = g^{\mu\nu} p_\nu = \begin{pmatrix} -p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$

$p^2_{Mink} = p_\mu p^\mu = -p_0^2 + p_1^2 + p_2^2 + p_3^2$ in agreement w/ Srednicki conventions

$\theta_\mu{}^\nu = \theta_{\mu\rho} g^{\nu\rho} = \theta\begin{pmatrix} 0 & 1 & & \\ 1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}$, $\theta^\mu{}_\nu = \theta\begin{pmatrix} 0 & -1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}$, $\theta^{\mu\nu} = \theta_\rho{}^\nu g^{\mu\rho} = \begin{pmatrix} 0 & -1 & & \\ 1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}$

$\theta^{2\mu}{}_\nu = \theta^{\mu\rho}\theta_{\rho\nu} = \theta^2\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$p\theta p_{Mink} = -\theta^2 (p_0, p_1, p_2, p_3) \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}\begin{pmatrix} -p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \theta^2(p_0^2 - p_1^2 + p_2^2 + p_3^2)$

This agrees w/ Gomis, Mehen Eq 2.11
and with what I have in Footnote 9.

For timelike momenta $p^2_{Mink} = -m^2$, $p_0 > |p|$ so $p\theta p_{Mink} > 0$

Prop $m^2 + p^2 + \xi \frac{1}{p\theta p}$. If $\xi > 0$, inaccessible in Euclidean & for timelike mom
and the converse

Dim Reg

$$\Gamma^{(2)}_{1,\text{planar}} = \frac{\tilde{g}^2 \tilde{\mu}^{\epsilon}}{3\,(4\pi)^{d/2}} (m^2)^{\frac{d}{2}-1} \Gamma(1-\frac{d}{2})$$

$$\Gamma^{(2)}_{1,\text{nonplanar}} = \frac{\tilde{g}^2 \tilde{\mu}^{\epsilon}}{6\,(4\pi)^{d/2}} 2^{\frac{d}{2}} (m^2)^{\frac{1}{2}(\frac{d}{2}-1)} \left(\sqrt{p \circ p}\right)^{1-\frac{d}{2}} K_{\frac{d}{2}-1}\left(m\sqrt{p \circ p}\right)$$

$$\Gamma^{(2)}_{1,\text{planar}} = -\frac{\tilde{g}^2 m^2}{3(4\pi)^2} \left[\frac{2}{\epsilon} + \ln \frac{\mu^2}{m^2}\right]$$

UV Limit First ->

$$\Gamma^{(2)}_{1,\text{nonplanar}} = \frac{g^2 m^2}{6(4\pi)^2} \left[\frac{4}{m^2 p \circ p} - \ln \frac{4}{m^2 p \circ p} - 1 + 2\gamma\right]$$

IR Limit First ->

$$\Gamma^{(2)}_{1,\text{nonplanar}} = -\frac{\tilde{g}^2 m^2}{6(4\pi)^2} \left[\frac{2}{\epsilon} + \ln \frac{\mu^2}{m^2}\right]$$

Strong UV/IR Duality

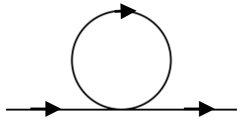
$$\text{Commutative } \Lambda^2 \rightarrow \text{Noncommutative } \frac{1}{p \circ p}$$

Is this always a necessary relationship? Not quite.

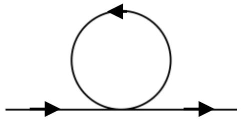
E.g. Self-interaction of complex scalar with global U(1)

[Ruiz Ruiz, '02]

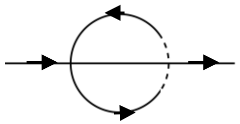
$$V = m^2|\phi|^2 + \frac{\lambda_1}{4}\phi^* \star \phi \star \phi^* \star \phi + \frac{\lambda_2}{4}\phi^* \star \phi^* \star \phi \star \phi$$



λ_1



λ_2



λ_2

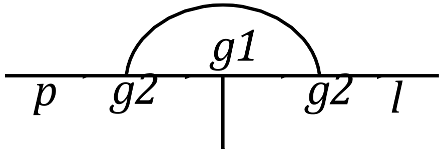
So are there other examples where it does occur?

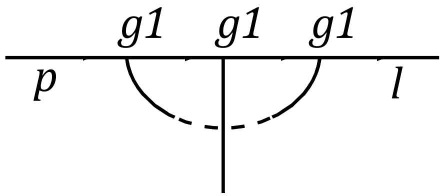
CPT

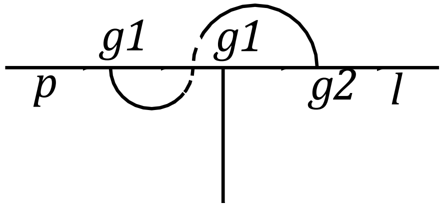
[17] We note that while the CPT theorem has only been proven in NCFT without space-time noncommutativity [109–112], the difficulty in the general case is related to the issues with unitarity discussed in Section 2, and we expect it should hold in a sensible formulation of the space-time case as well.

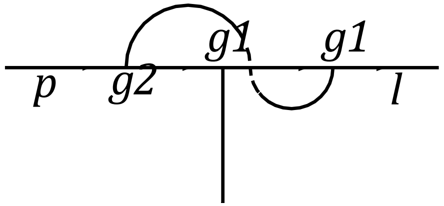
[18] We should note that in the construction of noncommutative QED it has been argued that it is sensible to assign θ the anomalous charge conjugation transformation $C : \theta^{\mu\nu} \to -\theta^{\mu\nu}$ ([113] and many others since). The argument is that charged particles in noncommutative space act in some senses like dipoles whose dipole moment is proportional to θ, and so charge conjugation should naturally reverse these dipole moments. Here, however, our particles are uncharged, and thus we have no basis for arguing in this manner. Furthermore, such an anomalous transformation makes charge conjugation relate theories on *different* noncommutative spaces $\mathcal{M}_\theta \to \mathcal{M}_{-\theta}$. The heuristic picture of the CPT theorem (that is, the reason we care about CPT being a symmetry of our physical theories) is that after Wick rotating to Euclidean space, such a transformation belongs to the connected component of the Euclidean rotation group [114], and so is effectively a symmetry of spacetime. So it is at the least not clear that defining a CPT transformation that takes one to a different space accords with the reason CPT should be satisfied in the first place.

Three Point Function









$$\lim_{p,\ell\to 0} \Gamma^{\varphi\overline{\psi}\psi}_{3,np}(p,\ell) = \frac{g_1^2(g_1+2g_2)}{16\pi^2}\log\left(\Lambda^2\right) + \text{ finite}$$

$$\lim_{\Lambda\to\infty} \Gamma^{\varphi\overline{\psi}\psi}_{3,np}(p,\ell) = \frac{g_1^2}{16\pi^2}\left[g_1\log\left(\frac{4}{(p+\ell)\circ(p+\ell)}\right) + g_2\log\left(\frac{4}{p\circ p}\right) + g_2\log\left(\frac{4}{\ell\circ\ell}\right)\right] + \text{ finite}$$