

UV/IR Mixing and the Hierarchy Problem

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Based (mainly) on

- *IR Dynamics from UV Divergences: UV/IR Mixing, NCFT, and the Hierarchy Problem* [1909.01365, JHEP] **with N. Craig**

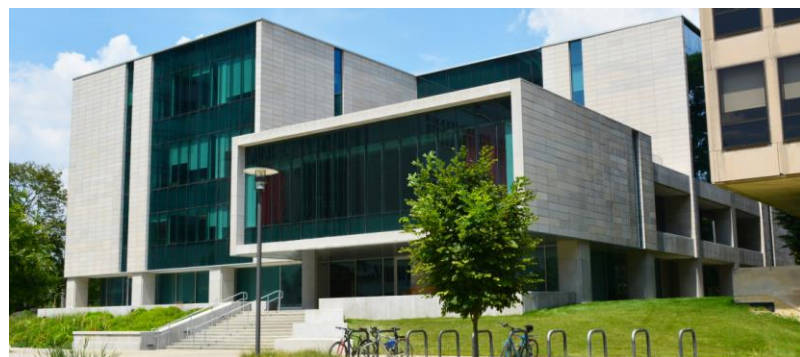
- *The Hierarchy Problem: From the Fundamentals to the Frontiers* [2009.11870, PhD thesis]



David Rittenhouse Lab, UPenn

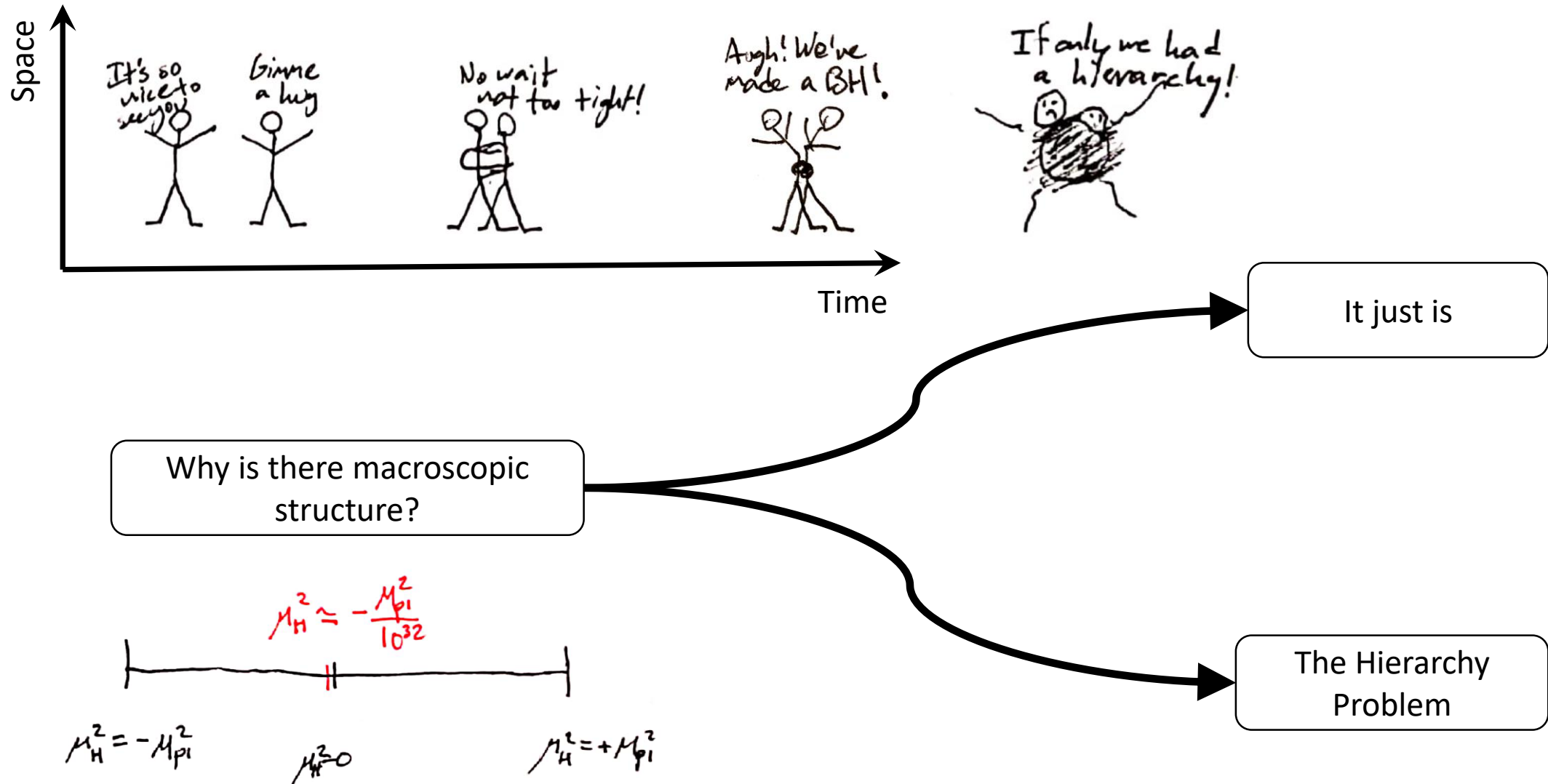


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Fine-tuning problems come from asking big, important questions



There is no hierarchy problem in the Standard Model

Our toy model of the SM – a single scalar whose mass is an input parameter

$$S = \int d^4x \left(-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_0^2 \phi^2 - V(\phi) - g \phi \mathcal{O}(\psi_i \psi_i) \right)$$

$$\Gamma_\phi^{(2)} = \text{---} + \sum \text{---} \bigcirc \text{---}$$

$$m_{\text{phys}}^2 = m_0^2 + \frac{g^2}{(4\pi)^2} \frac{1}{\epsilon} \sum_i M_i^2$$

Mass not protected by a symmetry, so gets large corrections

The Higgs mass is an *input* so just choose the bare mass to give the right answer

Hierarchy problem when Higgs mass is an *output*

Now imagine in the UV there is an SU(2) global symmetry

$$\Phi^a = \begin{pmatrix} \psi \\ \phi \end{pmatrix} \quad \text{Where we've measured } \phi \text{ to be very light, but } \psi \text{ must be heavy}$$

$$\mathcal{S} = \int d^4x \left(-\frac{1}{2} (\partial_\mu \Phi)^\dagger \partial^\mu \Phi - \frac{1}{2} M_0^2 \Phi^\dagger \Phi - \lambda_0 \Phi^\dagger \Sigma \Sigma^\dagger \Phi - V(\Phi) \right)$$

Tree-level	$m_\psi^2 = M_0^2$	$m_\phi^2 = M_0^2 + \lambda_0 v^2$
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Loop-level	$m_\psi^2 = M_0^2 + \frac{1}{\epsilon} (M_\Sigma^2 + \dots)$	$m_\phi^2 = \left[M_0^2 + \frac{1}{\epsilon} (M_\Sigma^2 + \dots) \right] + \lambda_0 v^2$
------------	--	---

Renormalized	$m_\psi^2 = M_{\text{phys}}^2$	$m_\phi^2 = M_{\text{phys}}^2 + \lambda_{\text{phys}} v_{\text{phys}}^2$
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$\lambda_{\text{phys}} = -1.00000000000000000000000000000001 \times \frac{M_{\text{phys}}^2}{v_{\text{phys}}^2}$ Fine-tuned unless $m_\phi \sim \text{scale of new physics}$

The Hierarchy Problem: From the Fundamentals to the Frontiers

Contents

Acknowledgements	v
Curriculum Vitae	viii
Abstract	x
Permissions and Attributions	xi
Preface	xii
1 Effective Field Theory	1
1.1 EFT Basics	2
1.1.1 Scale-dependence	3
1.1.2 Bottom-up or Top-down	5
1.2 Renormalization	14
Loops Are Necessary	14
1.2.1 To Remove Divergences	17
Physical Input is Required	17
Renormalizability	21
Wilsonian Renormalization of ϕ^4	23
Renormalization and Locality	29
1.2.2 To Repair Perturbation Theory	31
Renormalization Group Equations	31
Decoupling	34
Renormalized Perturbation Theory	36
Continuum Renormalization	38
Renormalization Group Improvement	42
1.2.3 To Relate Theories	44
Mass-Independent Schemes and Matching	44
Flowing in Theory Space	51
Trivialities	57
1.2.4 To Reiterate	58

1.3 Naturalness	59
1.3.1 Technical Naturalness and Fine-Tuning	61
Technical Naturalness and Masses	66
1.3.2 Spurion Analysis	67
1.3.3 Dimensional Transmutation	70
2 The Hierarchy Problem	72
2.1 The Higgs in the Standard Model	72
2.2 Nonsolutions to the Hierarchy Problem	74
2.2.1 An End to Reductionism	74
2.2.2 Waiter, there's Philosophy in my Physics	78
2.2.3 The Lonely Higgs	86
2.2.4 Mass-Independent Regulators	88
2.3 The Hierarchy Problem	90
3 The Classic Strategies	92
3.1 Supersymmetry	95
3.2 Extra Dimensions	106
3.2.1 Technology: Kaluza-Klein Reduction	107
3.2.2 Quantum Gravity at the TeV Scale	110
3.2.3 Technology: Orbifold Reduction	112
3.2.4 Nonlocal Symmetry-Breaking	118
3.3 Compositeness	121
3.3.1 Technicolor	121
3.3.2 A Composite Goldstone Higgs	123
4 The Loerarchy Problem	131
4.1 The 'Little Hierarchy Problem'	131
4.1.1 The Twin Higgs	132
4.1.2 Neutral Naturalness or The Return Of The Orbifold	136
Example 1: Folded Supersymmetry	138
Example 2: The F-plet Higgs	142
4.2 The Loerarchy Problem	144
4.3 Violations of Effective Field Theory	146
4.3.1 Gravity and EFT	147
5 Neutral Naturalness in the Sky	154
5.1 Particle Cosmology	154
5.2 Asymmetric Reheating	158
5.2.1 Introduction	158
5.2.2 Thermal History of the Mirror Twin	160
Twin Degrees of Freedom	160
Decoupling	161

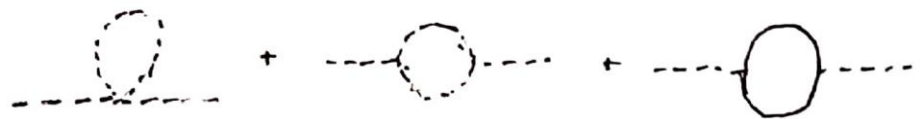
How to get a light scalar: Classic edition

Introduce UV structure to forbid large contributions, and IR dynamics to break that structure to the observed SM EFT

E.g. Supersymmetry

$$Q|\text{fermion}\rangle = |\text{boson}\rangle \quad Q|\text{boson}\rangle = |\text{fermion}\rangle$$

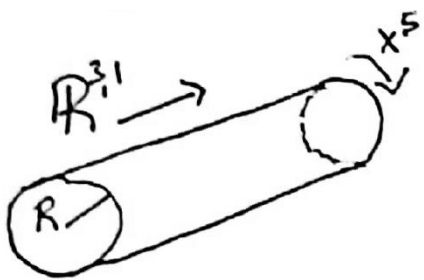
Must be broken!



$$\sim \Lambda^2 \times 0 + (m^2 - \tilde{m}^2) \log \frac{\Lambda^2}{m^2}$$

E.g. Extra dimensions

UV masses forbidden by gauge invariance



$$g_{AB} = \begin{pmatrix} g_{\mu\nu} & g_{\mu 5} = A_\mu \\ \hline g_{5\nu} = A_\nu & g_{55} = \phi \end{pmatrix}$$

$$\mathcal{L} \supset \frac{1}{2} \frac{1}{R^2} \phi^2 + \sum_{\text{SM Fields } \psi} \frac{1}{R} \bar{\psi} \psi$$

→ Either way, expect new strongly-interacting particles near the weak scale

Data!

ATLAS SUSY Searches* - 95% CL Lower Limits

October 2019

Model	Signature	$\int \mathcal{L} dt$ [fb ⁻¹]	Mass limit
Inclusive Searches	$\tilde{q}\tilde{q}, \tilde{q} \rightarrow q\tilde{\chi}_1^0$	0 e, μ mono-jet	2-6 jets 1-3 jets
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow q\tilde{q}\tilde{\chi}_1^0$	0 e, μ	2-6 jets
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow q\tilde{q}(\ell\ell)\tilde{\chi}_1^0$	3 e, μ $e\ell, \mu\mu$	4 jets 2 jets
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow qqWZ\tilde{\chi}_1^0$	0 e, μ SS e, μ	7-11 jets 6 jets
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow \tilde{t}\tilde{t}\tilde{\chi}_1^0$	0-1 e, μ SS e, μ	3 h 6 jets
	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1 \rightarrow b\tilde{\chi}_1^0/\tilde{\chi}_1^0$	Multiple Multiple Multiple	36.1 36.1 139
3 rd gen. squarks direct production	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1 \rightarrow b\tilde{\chi}_1^0 \rightarrow b\tilde{h}\tilde{\chi}_1^0$	0 e, μ	6 h
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow Wb\tilde{\chi}_1^0$ or $\tilde{t}_1\tilde{\chi}_1^0$	0-2 e, μ	0-2 jets/1-2 b
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow Wb\tilde{h}\tilde{\chi}_1^0$	1 e, μ	3 jets/1 b
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow \tilde{t}_1 b\tilde{\chi}_1^0, \tilde{t}_1 \rightarrow \tau\tilde{G}$	1 $\tau + 1 e, \mu, \tau$	2 jets/1 b
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow \tilde{t}_1 \tilde{\chi}_1^0/\tilde{\ell}\tilde{\ell}, \tilde{t}_1 \rightarrow \tilde{c}\tilde{\chi}_1^0$	0 e, μ	2 c
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow \tilde{t}_1 \tilde{\chi}_1^0$	0 e, μ	mono-jet
EW direct	$\tilde{\chi}_1^0\tilde{\chi}_2^0$ via WZ	2-3 e, μ $e\ell, \mu\mu$	≥ 1
	$\tilde{\chi}_1^0\tilde{\chi}_2^0$ via WW	2 e, μ	≥ 1
	$\tilde{\chi}_1^0\tilde{\chi}_2^0$ via Wh	0-1 e, μ	2 $h/2 \gamma$
	$\tilde{\chi}_1^0\tilde{\chi}_2^0$ via $\tilde{\ell}_i\tilde{\nu}$	2 e, μ	0 jets
	$\tilde{\chi}_1^0\tilde{\chi}_2^0$ via $\tilde{\ell}_i\tilde{\nu}$	2 e, μ	0 jets
	$\tilde{\chi}_1^0\tilde{\chi}_2^0$ via $\tilde{\ell}_i\tilde{\nu}$	2 e, μ	≥ 1
Long-lived particles	Direct $\tilde{\chi}_1^0\tilde{\chi}_1^0$ prod., long-lived $\tilde{\chi}_1^0$	Disapp. trk	1 jet
	Stable \tilde{g} R-hadron	Multiple	0 jets
	Metastable \tilde{g} R-hadron, $\tilde{g} \rightarrow q\tilde{q}\tilde{\chi}_1^0$	Multiple	0 jets
	LFV $\tilde{pp} \rightarrow \tilde{\nu}_e + X, \tilde{\nu}_e \rightarrow e\mu/\tau/\mu$	$e\mu, e\tau, \mu\tau$	0 jets
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow q\tilde{q}\tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow qq$	4 e, μ	0 jets
	$\tilde{g}\tilde{g}, \tilde{g} \rightarrow q\tilde{q}\tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow qq$	4-5 large-R	0 jets
RPV	$\tilde{u}\tilde{t}, \tilde{t} \rightarrow \tilde{u}\tilde{\chi}_1^0, \tilde{\chi}_1^0 \rightarrow tbs$	Multiple	2 jets + 2 b
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow bs$	Multiple	2 jets + 2 b
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow q\ell$	2 e, μ	2 h
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow q\ell$	1 μ	DV
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow q\ell$	1 μ	DV
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1 \rightarrow q\ell$	1 μ	DV

*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

ATLAS Exotics Searches* - 95% CL Upper Exclusion Limits

Status: May 2019

$\int \mathcal{L} dt = (3.2 - 139) \text{ fb}^{-1}$ \sqrt{s}

Model	ℓ, γ	Jets [†]	$E_{\text{miss}}^{\text{min}}$	$\int \mathcal{L} dt$ [fb ⁻¹]	Limit
Extra dimensions	ADD $G_{KK} + g/q$	0 e, μ	1-4 j	Yes	36.1
	ADD non-resonant $\gamma\gamma$	2 γ	-	-	36.7
	ADD QBH	-	2 j	-	37.0
	ADD BH high Σp_T	$\geq 1 e, \mu$	$\geq 2 j$	-	3.2
	ADD BH multijet	-	$\geq 3 j$	-	3.6
	RS1 $G_{KK} \rightarrow \gamma\gamma$	2 γ	-	-	36.7
	Bulk RS $G_{KK} \rightarrow WW/ZZ$	multi-channel	-	-	36.1
	Bulk RS $G_{KK} \rightarrow WW \rightarrow qq\bar{q}\bar{q}$	0 e, μ	$\geq 2 J$	-	36.1
	Bulk RS $G_{KK} \rightarrow tt$	1 e, μ	$\geq 1 b, \geq 1 J/2 j$	Yes	36.1
	2UED / RPP	1 e, μ	$\geq 3 j$	-	36.1
Gauge bosons	SSM $Z' \rightarrow \ell\ell$	2 e, μ	-	-	139
	SSM $Z' \rightarrow \tau\tau$	2 e, μ	-	-	139
	Leptophobic $Z' \rightarrow b\bar{b}$	0 e, μ	$\geq 2 b$	-	36.1
	Leptophobic $Z' \rightarrow t\bar{t}$	0 e, μ	$\geq 2 t$	Yes	36.1
	SSM $W' \rightarrow \ell\ell$	2 e, μ	-	-	139
	SSM $W' \rightarrow \tau\tau$	2 e, μ	-	-	139
	HVT $V' \rightarrow WZ$	0 e, μ	2 J	-	139
	HVT $V' \rightarrow WH/Z\tilde{h}$	0 e, μ	2 J	-	139
	RSM $W_R \rightarrow t\bar{b}$	multi-channel	-	-	36.1
	$W_R \rightarrow \mu N_R$	2 μ	1 J	-	80
DM	Cl $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$	-	2 j	-	37.0
	Cl $pp \rightarrow q\bar{q}$	2 e, μ	-	-	36.1
	Cl $pp \rightarrow t\bar{t}$	$\geq 1 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1
	Cl $pp \rightarrow t\bar{t}t\bar{t}$	$\geq 1 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1
LQ	Axial-vector mediator (Dirac DM)	0 e, μ	1-4 j	Yes	36.1
	Colored scalar mediator (Dirac DM)	0 e, μ	1-4 j	Yes	36.1
	VV $\chi\chi$ EFT (Dirac DM)	0 e, μ	1 J, $\leq 1 j$	Yes	3.2
	Scalar reson. $\phi \rightarrow t\chi$ (Dirac DM)	0-1 e, μ	1 b, 0-1 J	Yes	36.1
Heavy quarks	VLQ $TT \rightarrow Ht/Zt/Wb + X$	multi-channel	-	-	36.1
	VLQ $BB \rightarrow Wt/Zb + X$	multi-channel	-	-	36.1
	VLQ $T_{5/3} T_{5/3} \rightarrow Wt + X$	2(SS)/ $\geq 3 e, \mu \geq 1 b, \geq 1 j$	Yes	36.1	
	VLQ $Y \rightarrow Wb + X$	1 e, μ	$\geq 1 b, \geq 1 j$	Yes	36.1
Excited fermions	Excited quark $q^* \rightarrow qg$	-	$\geq 2 j$	-	139
	Excited quark $q^* \rightarrow q\gamma$	1 γ	1 j	-	36.7
	Excited quark $b^* \rightarrow bg$	-	1 b, 1 j	-	36.1
	Excited lepton ℓ^*	3 e, μ, τ	-	-	20.3
Other	Excited lepton ν^*	3 e, μ, τ	-	-	20.3
	Type III Seesaw	1 e, μ	$\geq 2 j$	Yes	79.8
	LRS Majorana ν	2 μ	2 j	-	36.1
	Higgs triplet $H^{\pm\pm} \rightarrow \ell\ell$	2,3,4 e, μ (SS)	-	-	36.1
Higgs triplet $H^{\pm\pm} \rightarrow \ell\tau$	3 e, μ, τ	-	-	20.3	
Multi-charged particles	-	-	-	-	36.1
Magnetic monopoles	-	-	-	-	34.4

*Only a selection of the available mass limits on new states or phenomena is shown.

[†]Small-radius (large-radius) jets are denoted by the letter j (J).

(With apologies to CMS)

Data!



A *maximalist* interpretation of the results is that maybe there really is no new weak-scale physics. EFT expectations really *are* violated.

Innovate!

Perhaps the violation of EFT expectations results from a *physical* violation of EFT

lte
phy.

Not radical enough

[C

Alipo

Al

'18;

Con

Cr

boby-Smith '20;

(NB: I also do things/have interests unrelated to the hierarchy problem!)

The EFT of Quantum Gravity

We have a *great* perturbative theory of quantum gravity!

$$\mathcal{L}_{EH} = \frac{1}{2M_{\text{pl}}^2} \sqrt{-g} R \approx \partial h \partial h + \frac{1}{M_{\text{pl}}} \partial^2 h^3 + \frac{1}{M_{\text{pl}}^2} \partial^2 h^4 + \dots$$

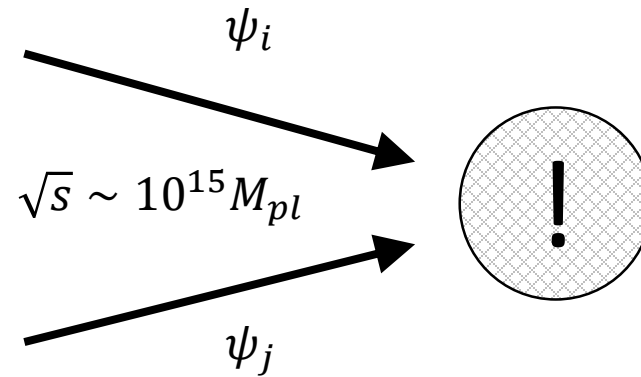
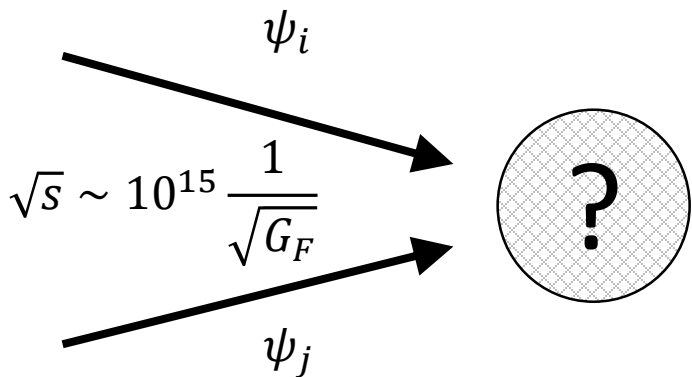
$$M_{\text{pl}} \approx 10^{18} \text{ GeV}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\text{pl}}$$

Compare with Fermi theory of the weak interactions

$$\mathcal{L} \approx \bar{\psi}_i \gamma_\mu \partial^\mu \psi^i + G_F \psi_i \psi_j \psi_k \psi_l + G_F^2 \psi_i \psi_j \psi_k \psi_l \psi_m \psi_n + \dots$$

$$\frac{1}{\sqrt{G_F}} \approx 300 \text{ GeV}$$



Noncommutativity

'Quantize!'

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

[Snyder '47]!

$$\Rightarrow \Delta\hat{x}_\mu \Delta\hat{x}_\nu \geq \frac{|\theta_{\mu\nu}|}{2}$$

*UV/IR mixing is front and center!
Separation of scales is violated!*

Noncommutative Field Theory

But how to do physics on such spaces?

Transfer the noncommutativity to the fields!

Introduce 'star-product' $f(x) \star g(x) = f(x) \exp\left(\frac{i}{2} \overleftarrow{\partial}^\mu \theta_{\mu\nu} \overrightarrow{\partial}^\nu\right) g(x)$

Noncommutative Field Theory

$$\mathcal{L}_{\text{int}}^{(NC)} = \frac{\lambda}{n!} \overbrace{\phi(x) \star \phi(x) \star \dots \star \phi(x)}^{n \text{ copies}}$$

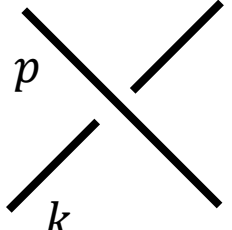
$$\Rightarrow \tilde{V}(k_1, \dots, k_n) = \delta(k_1 + \dots + k_n) \exp\left(\frac{i}{2} \sum_{i < j}^n k_i^\mu k_j^\nu \theta_{\mu\nu}\right)$$

Vertices no longer permutation-invariant!

Phase factors of graphs reduce to a graph-topological statement [Filk '94]

Planar graphs: Solely an overall phase involving external momenta

Nonplanar graphs: Additional phases for lines which cross



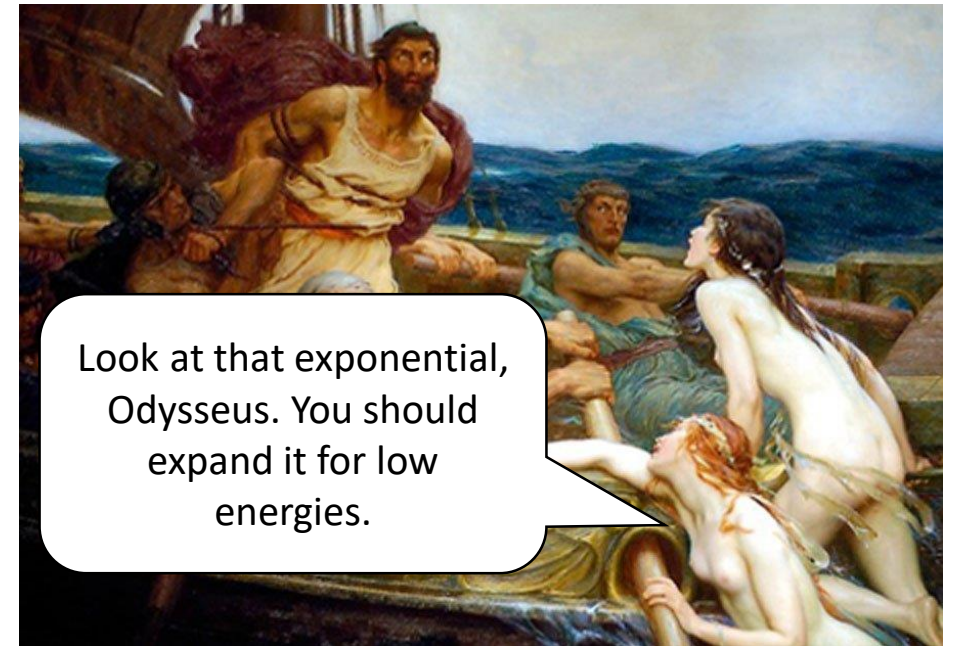
$$\sim e^{i p^\mu \theta_{\mu\nu} k^\nu}$$

Thou Shalt Not Expand

$$\mathcal{S}_{\text{int}}^{(NC)} \sim \frac{\lambda}{n!} \int dk \phi(k_1) \phi(k_2) \dots \phi(k_n) \exp \left(\frac{i}{2} \sum_{i < j}^n k_i^\mu k_j^\nu \theta_{\mu\nu} \right)$$

The ‘theta-expanded’ NCFT
removes all of the UV/IR mixing!

Much past work can be ignored from our perspective



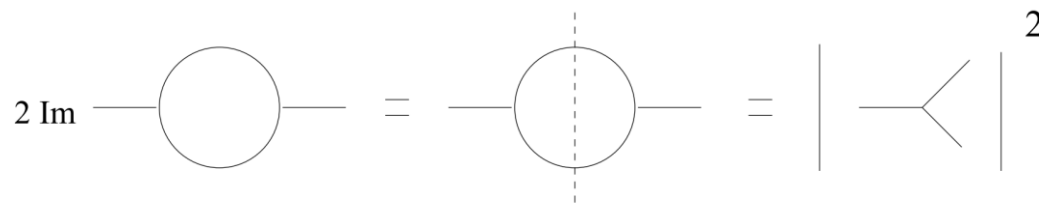
[Ulysses and the Sirens, Draper, 1909]

Skeletons in the Closet

- Lorentz violation!
 - Not out the window; just like turning on a magnetic field in a lab
 - Folk theorems [Collins et al. '04] about empirical bounds not fully applicable

See also [Calmet '04]

- Unitarity of Lorentzian theory with timelike noncommutativity?
 - Failure well-understood from stringy perspective [Seiberg, Susskind, Toumbas '00]
 - Field-theoretically, issue with formulation of nonlocal-in-time theories



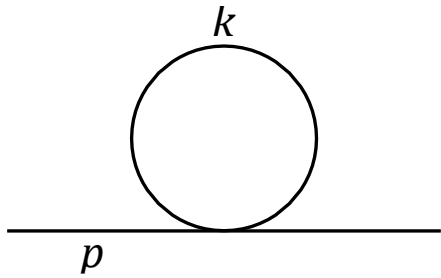
[Gomis, Mehen '00]

[Gomis et al. '00; Bahns et al. '02;
Bozkaya et al. '02; Liao & Sibold '02;
Rim & Yee '02; Denk & Schweda '03,
Fischer & Putz '03; Liao '04, ...]

(But at the end I'll mention some preliminary progress on these.)

Euclidean ϕ^4

[Minwalla, Seiberg, Van Raamsdonk '99]



$$\sim \int \frac{d^4 k}{k^2 + m^2}$$

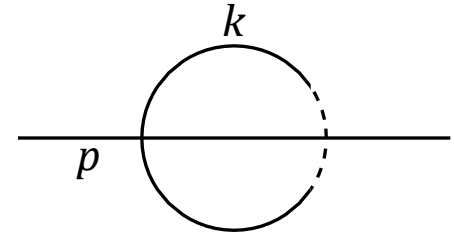
$$\sim \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2}$$

$$\text{Planar} \sim g^2 \left(\Lambda^2 - m^2 \log \frac{\Lambda^2}{m^2} + \dots \right)$$

Use Schwinger parameters

$$\frac{1}{k^2 + m^2} = \int_0^\infty d\alpha e^{-\alpha(k^2 + m^2)}$$

Regulate with $e^{-\frac{1}{\Lambda^2 \alpha}}$



$$\sim \int \frac{d^4 k}{k^2 + m^2} e^{ik^\mu \theta_{\mu\nu} p^\nu}$$

$$\sim \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2 - \frac{p \circ p}{4\alpha}}$$

Completed the square
 $p \circ k \equiv -p_\mu \theta^{2\mu\nu} k_\nu$

$$\text{Nonplanar} \sim g^2 \left(\Lambda_{\text{eff}}^2 - m^2 \log \frac{\Lambda_{\text{eff}}^2}{m^2} + \dots \right)$$

$$\Lambda_{\text{eff}}^2 \equiv \frac{1}{1/\Lambda^2 + p \circ p/4}$$

Not regulator-dependent – can also see in dim reg [Craig, SK]

A new pole!

1 Loop 1PI quadratic effective action

$$\frac{1}{2} \left(p^2 + M^2 + \frac{g^2}{96\pi^2 \left(\frac{p \circ p}{4} + \frac{1}{\Lambda^2} \right)} + \dots \right) \phi(p) \phi(-p)$$

with renormalized mass

$$M^2 = m^2 + \frac{g^2 \Lambda^2}{48\pi^2} - \frac{g^2 m^2}{48\pi^2} \log \frac{\Lambda^2}{m^2}$$

Nonperturbative in θ !

In $\Lambda \rightarrow \infty$ limit
there are now
two poles!

$$p^2 = -m^2 + \mathcal{O}(g^2)$$

$$p \circ p = -\frac{g^2}{24\pi^2 m^2} + \mathcal{O}(g^4)$$

simple case $\theta^{\mu\nu} \sim 1/\Lambda_\theta^2 \rightarrow p^2 \propto g^2 \Lambda_\theta^4 / m^2$

*A new light 'particle' with
nothing nearby to explain
its presence!*

*After Wick rotation,
inaccessible in s-channel*

Wilsonian Interpretation

Normally a renormalizable Wilsonian action must satisfy

1. Correlation functions are well defined in the $\Lambda \rightarrow \infty$ limit
2. At finite Λ they differ from the limiting value by $O(\Lambda^{-1})$ for all momenta

$$\frac{1}{2} \left(p^2 + M^2 + \frac{g^2}{96\pi^2 \left(\frac{p \circ p}{4} + \frac{1}{\Lambda^2} \right)} + \dots \right) \phi(p) \phi(-p)$$

At small momenta (2) is badly violated here!

Can restore a Wilsonian interpretation by introducing a new field χ

$$\delta\mathcal{L}(\Lambda) = \frac{1}{2} \partial\chi \circ \partial\chi + \frac{1}{2} \frac{\Lambda^2}{4} (\partial \circ \partial\chi)^2 + i \frac{1}{\sqrt{24\pi^2}} g\chi\phi$$

$$\delta S(\Lambda)_{1\text{PI}} \sim \frac{1}{2} \left(-\frac{g^2}{96\pi^2 \left(\frac{p \circ p}{4} + \frac{1}{\Lambda^2} \right)} + \frac{g^2}{24\pi^2 p \circ p} \right) \phi(p) \phi(-p)$$

Yukawa theory

$$g\varphi\bar{\psi}\psi \rightarrow g_1\varphi \star \bar{\psi} \star \psi + g_2\bar{\psi} \star \varphi \star \psi$$

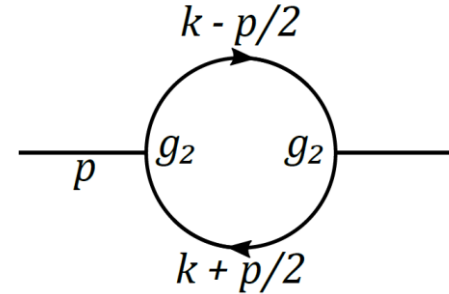
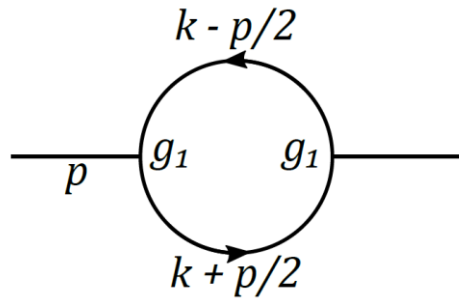
[c.f. Anisimov, Banks, Dine, Graesser '01]

However T is antiunitary! $(PT)^{-1} (f(x) \star g(x)) PT = g(x) \star f(x)$

So CPT 're-cycles' the two interaction terms!

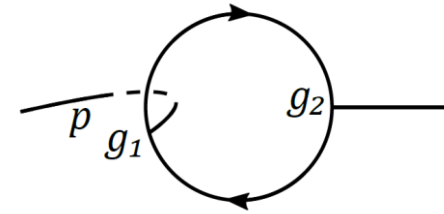
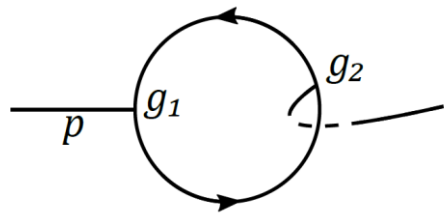
$$(CPT)^{-1} \mathcal{L}_{\text{int}}^{(\text{NC})} CPT = \mathcal{L}_{\text{int}}^{(\text{NC})} \implies g_1 = g_2$$

Scalar Two-Point Function



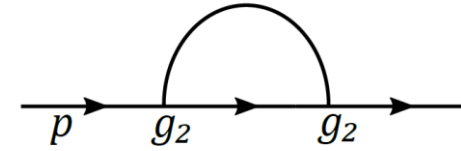
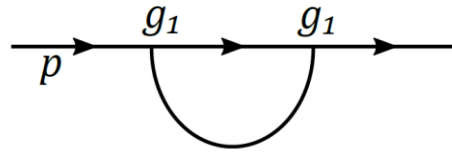
$$\text{Planar} \sim -(g_1^2 + g_2^2)(\Lambda^2 + \dots)$$

*Evaluation requires
some cleverness
and 'lightcone
Schwinger
coordinates'*

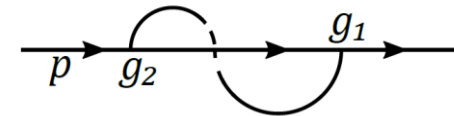
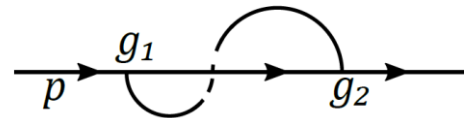


$$\text{Nonplanar} \sim -g_1 g_2 (\Lambda_{\text{eff}}^2 + \dots)$$

Fermion Two-Point Function



$$\text{Planar} \sim -(g_1^2 + g_2^2) \left(M - \frac{p \cdot \gamma}{2} \right) \log \Lambda^2 + \dots$$



$$\text{Nonplanar} \sim -g_1 g_2 \left(M - \frac{p \cdot \gamma}{2} \right) \log \Lambda_{\text{eff}}^2 + \dots$$

Logarithmic UV sensitivity \rightarrow only logarithmic IR feature

Softly-broken Supersymmetry - Wess Zumino

Look at the interplay between UV/IR mixing and UV finiteness

Compute one-loop two-point function again, with
 Z wavefunction renormalization and δm^2 mass correction

$$\Gamma^{(2),s} \equiv Z p^2 + Z^{-1}(m^2 + \delta m^2)$$

$$Z = 1 + \frac{y^2}{32\pi^2} \log \left[\frac{\Lambda \Lambda_{\text{eff}}}{M^2} \right] + \dots$$

For $\Lambda, \Lambda_{\text{eff}}$ large

$$\delta m^2 = \frac{y^2}{32\pi^2} (M^2 - m^2) \log \left[\frac{\Lambda \Lambda_{\text{eff}}}{M^2} \right] + \dots$$

The transmutation accords with the intuition we've now built

Softly-broken Wess Zumino

So a UV finite theory has no IR effects from UV/IR mixing

A UV sensitive theory has this surprising IR feature

How do these connect? With soft breaking we can transition between the two.

Taking instead $M \gg \Lambda, \Lambda_{\text{eff}}$ of the full result

$$\delta m^2 = \frac{y^2}{256\pi^2} (6M^2 + 16\Lambda^2 + 8\Lambda_{\text{eff}}^2) + \dots$$

EFT has been broken in a controlled way!

UV/IR mixing requires lack of UV finiteness! So not a module to tack on to theory with hierarchy problem.

On the other hand, one says that in this theory there just never is a hierarchy problem.

Though perhaps we can have our cake and eat it too with noncommutative orbifold field theory?

Some Early Words on Current Directions

Reformulate Lorentz invariant version of NCFT

$$S = \int d^6\theta W(\theta) \int d^4x \mathcal{L}_*(\phi)$$

[Snyder '47; Doplicher et al. '95; Kase et al '02; Carlson et al. '02; Heckman & Verlinde '14; Much & Vergara '17, many large literatures in various directions,...]

$$S_{int} \sim \int d^6\theta W(\theta) \int \left(\prod_{k=1}^n d^4k_i \right) \delta \left(\sum k_i \right) \tilde{\phi}(k_1) \dots \tilde{\phi}(k_n) \exp \frac{i}{2} \left(\sum_{i<j} k_i^\mu \theta_{\mu\nu} k_j^\nu \right)$$

$$\tilde{V}(k_1, \dots, k_n) = \delta \left(\sum k_i \right) \int d^6\theta W(\theta) \exp \frac{i}{2} \left(\sum_{i<j} k_i^\mu k_j^\nu \right) \theta_{\mu\nu} = \delta \left(\sum k_i \right) \widetilde{W}(K_{[\mu \nu]})$$

$$\Gamma_{1,\text{nonplanar}}^{(2)} = \frac{g^2}{6(2\pi)^4} \int \frac{d^4k}{k^2 + m^2} \int d^6\theta W(\theta) e^{ik^\mu \theta_{\mu\nu} p^\nu}$$

$$\Gamma_{1,\text{nonplanar}}^{(2)} \sim g^2 \int \frac{d^4k}{k^2 + m^2} \widetilde{W}(k^{[\mu} p^{\nu]})$$

Easy example

$$W(\theta) = \frac{\lambda^3}{\pi^3} \exp(-\lambda\theta^2) \rightarrow \widetilde{W}(k^{[\mu} p^{\nu]}) = \exp\left(-\frac{\pi^2}{\lambda} (k^{[\mu} p^{\nu]})^2\right)$$

$$\Gamma_{1,\text{planar}}^{(2)} = \frac{g^2}{48\pi^2} \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2}$$

$$\text{Old } \Gamma_{1,\text{nonplanar}}^{(2)} = \frac{g^2}{96\pi^2} \int \frac{d\alpha}{\alpha^2} e^{-\alpha m^2 - \frac{p \circ p}{4\alpha}}$$

$$\text{New } \Gamma_{1,\text{nonplanar}}^{(2)} \sim g^2 \lambda^2 \int d\alpha e^{-\alpha m^2} \left[\frac{1}{(\alpha\lambda)(\alpha\lambda + \pi^2 p^2)} + \frac{1}{(\pi^2 p^2)^{1/2} (\alpha\lambda + \pi^2 p^2)^{3/2}} \text{ArcTanh} \left(\sqrt{\frac{\pi^2 p^2 + \alpha\lambda}{\pi^2 p^2}} \right) \right]$$

$$\Gamma_{1,\text{full}}^{(2)} = p^2 + M^2(g, \Lambda) + \frac{g^2 \lambda}{p^2} (1 + \log \Lambda)$$

$$p^2 = -M^2 + \mathcal{O}(g^2) \quad \& \quad p^2 = -g^2 \lambda / M^2 + \mathcal{O}(g^4)$$

Conclusions

- Our EFT expectations have been violated! Perhaps by physical breakdown of EFT
- In NCFT this can generate an IR scale *ex nihilo*
- This behavior persists and with interesting properties as you go closer to the SM
- UV/IR mixing requires UV sensitivity which puts this strategy in stark contrast to others
- Lots of directions to investigate

The Hierarchy Problem: From the fundamentals to the frontiers

Contents

Acknowledgements	v
Curriculum Vitae	viii
Abstract	x
Permissions and Attributions	xi
Preface	xii
1 Effective Field Theory	1
1.1 EFT Basics	2
1.1.1 Scale-dependence	3
1.1.2 Bottom-up or Top-down	5
1.2 Renormalization	14
1.2.1 To Remove Divergences	14
Physical Input is Required	16
Renormalizability	21
Wilsonian Renormalization of ϕ^4	22
Renormalization and Locality	28
1.2.2 To Repair Perturbation Theory	31
Renormalization Group Equations	31
Decoupling	34
Renormalized Perturbation Theory	35
Continuum Renormalization	37
Renormalization Group Improvement	37
1.2.3 To Relate Theories	38
Mass-Independent Schemes and Matching	43
Flowing in Theory Space	49
Trivialities	55
1.2.4 To Reiterate	56

SMSMEFT
 ϕ^4 theory
Running couplings
Resummation Yukawa theory
Footnote UV & IR
of Grand Unification
Proton decay
first hint of hierarchy problem
transfer loop level to trees resurgance
mass indep regularization & mass indep renorm. scheme

ϕ^3 in $d=6$

Example sharing tech. not fully sensitivity for IGS
Flaw up in analogy to connection to Dirac naturalness
more explicit parts of hierarchy problem
we're always philosophizing
No MS doesn't help

1.3 Naturalness	57
1.3.1 Technical Naturalness and Fine-Tuning	59
Technical Naturalness and Masses	64
1.3.2 Spurion Analysis	65
1.3.3 Dimensional Transmutation	67
The Hierarchy Problem	69
2.1 The Higgs in the Standard Model	69
2.2 Nonsolutions to the Hierarchy Problem	71
2.2.1 An End to Reductionism	71
2.2.2 Waiter, there's Philosophy in my Physics	75
2.2.3 The Lonely Higgs	83
2.2.4 Mass-Independent Regulators	85
2.3 The Hierarchy Problem	87
3 The Classic Strategies	89
3.1 Supersymmetry	92
3.2 Extra Dimensions	102
3.2.1 Technology: Kaluza-Klein Reduction	103
3.2.2 Quantum Gravity at the TeV Scale	106
3.2.3 Technology: Orbifold Reduction	108
3.2.4 Nonlocal Symmetry-Breaking	114
3.3 Compositeness	117
3.3.1 Technicolor	117
3.3.2 A Composite Goldstone Higgs	119
4 The Loerarchy Problem	126
4.1 The 'Little Hierarchy Problem'	126
4.1.1 The Twin Higgs	127
4.1.2 Neutral Naturalness or The Return Of The Orbifold	131
Example 1: Folded Supersymmetry	133
Example 2: The Γ -plet Higgs	136
4.2 The Loerarchy Problem	138
4.3 Violations of Effective Field Theory	140
4.3.1 Gravity and EFT	141
5 Neutral Naturalness in the Sky	148
5.1 Particle Cosmology	148
5.2 Asymmetric Reheating	152
5.2.1 Introduction	152
5.2.2 Thermal History of the Mirror Twin	154
Twin Degrees of Freedom	154
Decoupling	155

Gravity demands EFT violation
Mostly my papers

New!

Backup Slides

Wilsonian Interpretation Redux

$$\Delta S_{1\text{PI}}(\Lambda) = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{g_1 g_2}{2\pi^2} \left(\Lambda_{\text{eff}}^2 - \frac{4}{p \circ p} \right) \varphi(p) \varphi(-p)$$

Then for any cutoff Λ , we have the behavior for small $p \circ p$

$$\Gamma_1^s(p) = -\frac{2g_1 g_2}{\pi^2 p \circ p} + \dots$$

Opposite sign from ϕ^4 theory!

Introduce auxiliary field, or just talk about a modified propagator after integrating it out

$$m^2 + (p_i + p_j)^2 - \frac{2g_1 g_2}{\pi^2} \frac{1}{(p_i + p_j) \circ (p_i + p_j)}$$

New light pole at $s = \frac{2g_1 g_2}{\pi^2} \frac{1 - \beta^2 \Lambda_\theta^4}{1 + \beta^2 m^2}$

 **CAUTION**

Lorentzian behavior with NC time involves some speculation

[but see e.g. Bozkaya et al '02]

Future Directions

In Wilsonian EFT, nonlocality for $p \gtrsim \Lambda \leftrightarrow x \lesssim 1/\Lambda$

Particles in NCFT are like rods of length $L \sim p\theta$ [Sheikh-Jabbari '99, Bigatti & Susskind '00, Seiberg, Susskind, Toumbas '00, ...]

So 'extra' nonlocality for $1/\Lambda^2 \lesssim p\theta^2 p$

Could this sort of nonlocality appear in other models?

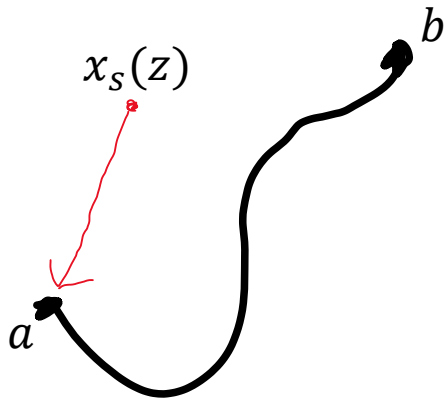
Generalized Uncertainty Principle: $\Delta x \gtrsim \frac{\hbar}{\Delta p} + \ell_p^2 \Delta p$

[originally Maggiore '93,
review Tawfik & Diab, '15]

Similar claims in String Theory as well [Gross & Mende '88, Konishi, Paffuti, Provero '90, Yoneya, '00, ...]

Future Directions

$$F(z) = \int_a^b dx f(x, z)$$



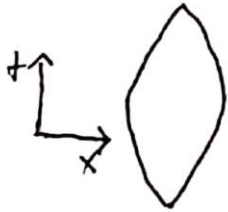
Alternatively, try to understand general nonlocal theories

[Tomboulis '15, Tomboulis & Chin '18] look carefully at a disjoint class of nonlocal theories

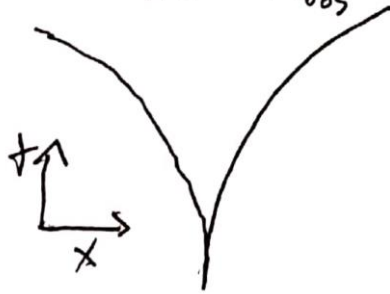
Presumably the magic of NCFT is associated with 'endpoint singularities'

Fine-tuning problems come from asking big, important questions

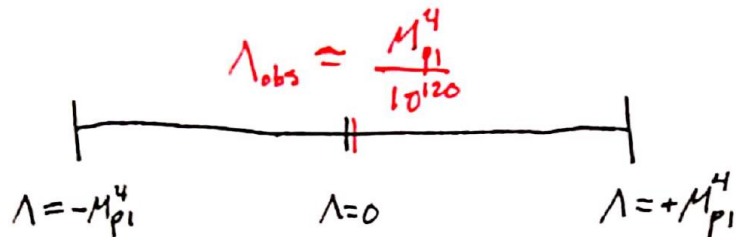
$$\Lambda > 0$$
$$|\Lambda| \gg \Lambda_{\text{obs}}$$



$$\Lambda < 0$$
$$|\Lambda| \gg \Lambda_{\text{obs}}$$



Why is there a macroscopic universe?

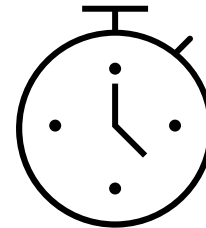
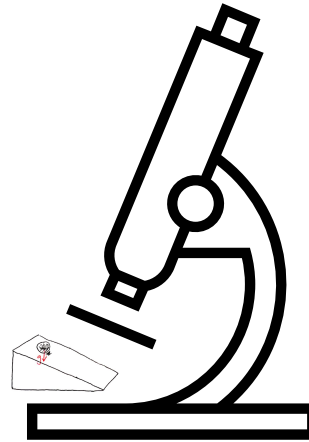
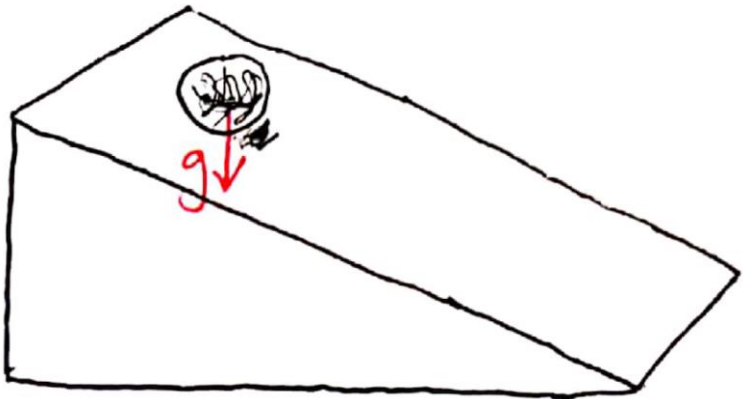


It just is

The Cosmological Constant Problem

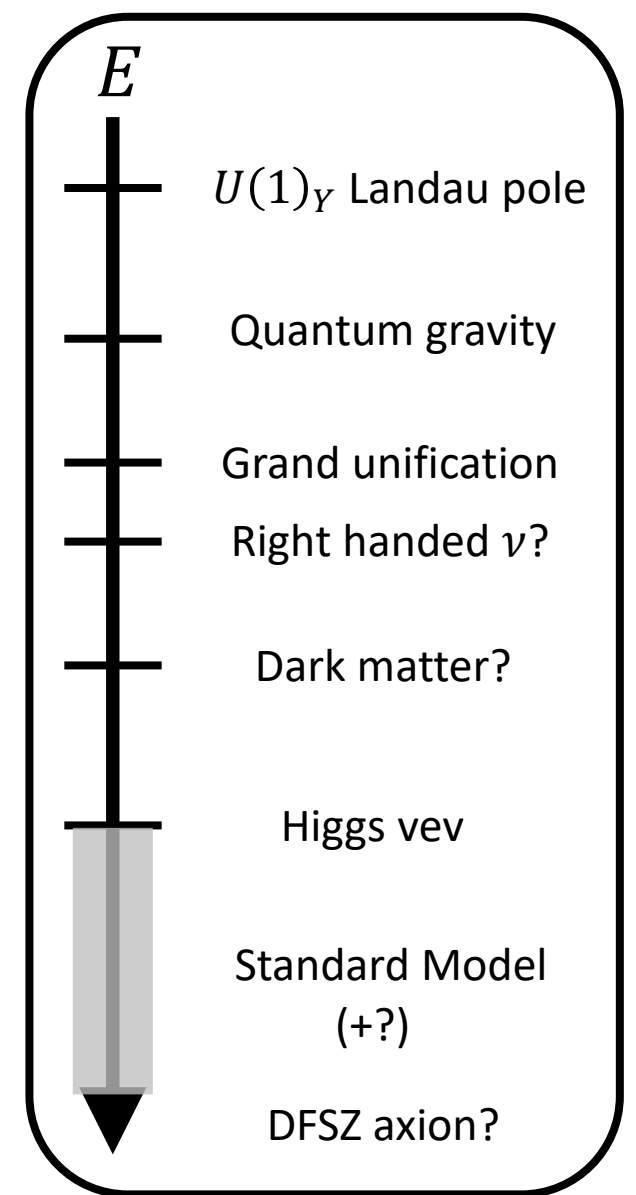
Effective Field Theory

Focus on the important degrees of freedom!



Correct your leading order description in a perturbative expansion depending on how much precision you want

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \approx m\mathbf{v} + \frac{1}{2}m \frac{v^2}{c^2} \mathbf{v} + \dots$$



$$\mathcal{L}(\phi_{SM}) = \mathcal{L}_{SM}(\phi_{SM}) + \frac{\mathcal{L}^{(5)}(\phi_{SM})}{\Lambda} + \frac{\mathcal{L}^{(6)}(\phi_{SM})}{\Lambda^2} + \dots$$

There is no hierarchy problem in the Standard Model

Our toy model of the SM – a single scalar whose mass is an input parameter

$$S = \int d^4x \left(-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_0^2 \phi^2 - V(\phi) - g \phi \mathcal{O}(\psi_i \psi_i) \right)$$

$$\Gamma_\phi^{(2)} = \text{---} + \sum \text{---} \bigcirc \text{---}$$

$$m_{\text{phys}}^2 = m_0^2 + \frac{g^2}{(4\pi)^2} \frac{1}{\epsilon} \sum_i M_i^2$$

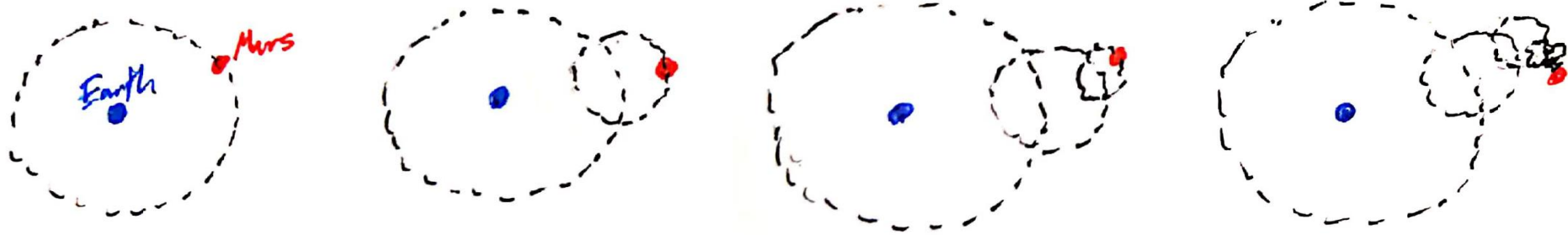
Mass not protected by a symmetry, so gets large corrections

The Higgs mass is an *input* so just choose the bare mass to give the right answer

Waiter, there's Philosophy in my Physics

“Who cares? We can fit the data by tuning those parameters.”

A model being not literally impossible is a **very low bar** for a scientific theory



$$r(\theta) = \sum_{i=1}^N r_n \sin \frac{\theta}{n}$$

A perfect fit, a perfect theory

To EFT or not to EFT?

Gravity very generally motivates looking at 'UV/IR mixing' effects!

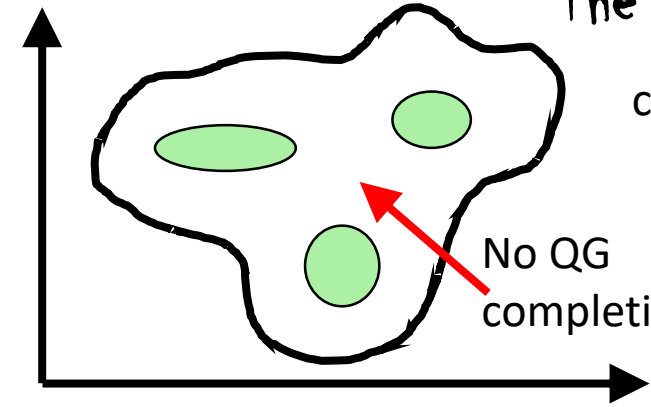
BH thermodynamics
c.f. Stephen

$$T_{BH} \propto 1/M_{BH}$$



EFT Theory Space

The Swampland

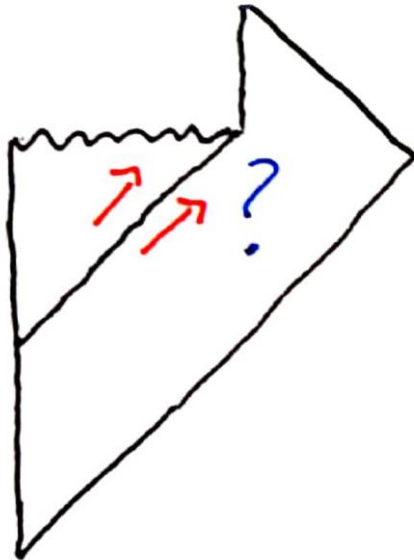


c.f. Cumrun

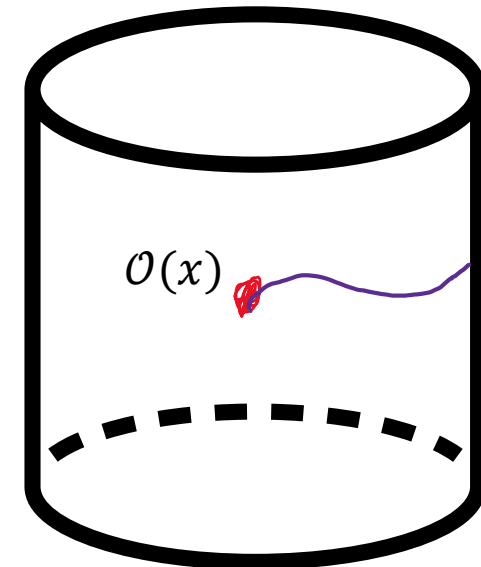
No QG completion

BH info and firewalls?!

c.f. Don



Necessity of nonlocality in gravitational observables



c.f. Steve

NC Quantization

$$\begin{array}{ccc}
 & \text{Alg}(\mathbb{R}^d[x], \cdot) & \\
 \hat{\mathcal{W}}: \mathbb{R}^d[x] \rightarrow \mathbb{R}_\theta^d[\hat{x}] & \swarrow & \nwarrow \text{Id}_{\mathbb{R}^d[x]} \\
 \text{Alg}(\mathbb{R}_\theta^d[\hat{x}], \cdot) & \xleftarrow{\hat{\mathcal{W}} \text{ an isomorphism of algebras}} & \text{Alg}(\mathbb{R}^d[x], \star_\theta)
 \end{array}$$

$$\hat{\mathcal{W}}[f \star_\theta g] = \hat{\mathcal{W}}[f] \cdot \hat{\mathcal{W}}[g]$$

$$\begin{aligned}
 e^{ikx} \star_\theta e^{ik'x} &= \hat{\mathcal{W}}^{-1} \left[\hat{\mathcal{W}}[e^{ikx}] \cdot \hat{\mathcal{W}}[e^{ik'x}] \right] \\
 &= \hat{\mathcal{W}}^{-1} \left[e^{-\frac{i}{2} \theta^{ij} k_i k'_j} e^{i(k+k') \cdot \hat{x}} \right]
 \end{aligned}$$

$$e^{ikx} \star_\theta e^{ik'x} \equiv e^{-\frac{i}{2} \theta^{\mu\nu} k_\mu k'_\nu} e^{i(k+k') \cdot x}$$

Correspondence Principle

⁵We note here that the failure of a ‘correspondence principle’ between commutative and noncommutative theories as $\theta^{\mu\nu} \rightarrow 0$ is clearly intrinsically linked to the appearance of UV/IR mixing. This failure doesn’t violate Kontsevich’s proof of the existence of deformation quantization for any symplectic manifold [81], as that is confined solely to ‘formal’ deformation quantization — that is, the production of a formal power series expansion of the algebra of observables in terms of the deformation parameter. As was noted in Section 2 and is now on prime display, the physics of the theory with nonperturbative θ -dependence is starkly different from that of any truncation.

Renormalizability

⁷There has been much work on understanding renormalizability of NCFTs, especially with an eye toward finding a mathematically well-defined four-dimensional quantum field theory with a non-trivial continuum limit. Renormalizability has been proven for modifications of NCFTs where the free action is supplemented by an additional term which adjusts its long-distance behavior. Such an action is manufactured either by requiring it manifest ‘Langman-Szabo’ duality [82] $p_\mu \leftrightarrow 2(\theta^{-1})_{\mu\nu}x^\nu$ [83, 84] or by adding a $1/p \circ p$ term to the free Lagrangian [85], the latter of which directly has the interpretation of adding ‘somewhere to put the $1/p \circ p$ counterterm’. For recent reviews of these and related efforts we refer the reader to [86, 87]. It would be interesting to understand fully the extent to which the physics of these schemes agrees with the interpretation of the IR effects as coming from auxiliary fields [37, 61].

Pole Accessibility

¹¹Note that this peculiar connection regarding (in)accessibility is due to the Lorentz violation. While the normal pole which is inaccessible in Euclidean signature becomes accessible for timelike momenta in Lorentzian signature, the Wick rotation affects the noncommutative momentum contraction differently. When taking $x_4 \rightarrow -ix_0$, one also rotates $\theta_{4\nu} \rightarrow -i\theta_{0\nu}$ such that Equation 2.1 continues to hold for the same numerical $\theta_{\mu\nu}$. For the simplest configuration of full-rank noncommutativity with $\theta_{\mu\nu}$ block-off-diagonal and only one eigenvalue $1/\Lambda_\theta^2$, the Euclidean $p \circ p = p^2/\Lambda_\theta^4$ becomes a Lorentzian $p \circ p = (p_0^2 - p_1^2 + p_2^2 + p_3^2)/\Lambda_\theta^4$. So a noncommutative pole which is inaccessible in the Euclidean theory becomes accessible in the Lorentzian theory for spacelike momenta, while a noncommutative pole which can be accessed in the Euclidean theory becomes accessible in the s -channel in Lorentzian signature.

Start in Euclidean space

$$X_a = (X_0, X_1, X_2, X_3) \quad [X_a, X_b] = i\theta_{ab}, \quad \theta_{ab} = \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad g_{ab} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = g^{ab}$$

$$p_a = (p_0, p_1, p_2, p_3) \rightarrow p^a = g^{ab} p_b = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

$$\theta_a^b = \theta_{ac} g^{bc} = \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = \theta_a^b, \quad \theta^{ab} = \theta_c^b g^{ac} = \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

If you tried
 $\theta_a^b = \theta_{ac} g^{bc}$
 you'd get the
 wrong sign for

$$\theta^{2a}{}_b = \theta^{ac} \theta_{cb} = \theta \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

pop Euclidean > 0 in agreement
 w/ everything

$$\text{pop}_{\text{Euclidean}} = \theta^2 (p_0, p_1, p_2, p_3) \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \theta^2 (p_0^2 + p_1^2 + p_2^2 + p_3^2) = p_{\text{Euclidean}}^2 \theta^2$$

Now Wick rotate in coords & metric & KK-tensor

$$X_4 \rightarrow -iX_0, \quad \theta_{4a} \rightarrow -i\theta_{0a}, \quad g_{4a} \rightarrow -g_{0a}$$

So that now

$$[X_\mu, X_\nu] = i\theta_{\mu\nu}, \quad \theta_{\mu\nu} = \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ still real}, \quad g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = g^{\mu\nu}$$

$$\text{Now } p_\mu = (p_0, p_1, p_2, p_3) \rightarrow p^\mu = g^{\mu\nu} p_\nu = \begin{pmatrix} -p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

$$p_{\text{Mink}}^2 = p_\mu p^\mu = -p_0^2 + p_1^2 + p_2^2 + p_3^2 \text{ in agreement w/ } p_3 \text{ Srednicki conventions}$$

$$\theta_\mu{}^\nu = \theta_{\mu\sigma} g^{\nu\sigma} = \theta \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \theta^{\mu\nu} = \theta \begin{pmatrix} 0 & -1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \theta^{\mu\nu} = \theta^{\nu\mu}$$

$$\theta^{2\mu}{}_\nu = \theta^{\mu\sigma} \theta_{\sigma\nu} = \theta^2 \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\text{pop}_{\text{Mink}} = -\theta^2 (p_0, p_1, p_2, p_3) \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} -p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \theta^2 (p_0^2 - p_1^2 - p_2^2 - p_3^2)$$

This agrees w/ Gomis, Mehen Eq 2.11

and with what I have in Footnote 9.

For timelike momenta $p_{\text{Mink}}^2 = -m^2$, $p_0 > |p|$ so $\text{pop}_{\text{Mink}} > 0$

Prop $m^2 + p^2 + \sum \frac{1}{\text{pop}}$. If $\sum > 0$ inaccessible in Euclidean & for timelike mom
 and the converse

Dim Reg

$$\Gamma_{1,\text{planar}}^{(2)} = \frac{\tilde{g}^2 \tilde{\mu}^\epsilon}{3 (4\pi)^{d/2}} (m^2)^{\frac{d}{2}-1} \Gamma\left(1 - \frac{d}{2}\right)$$

$$\Gamma_{1,\text{nonplanar}}^{(2)} = \frac{\tilde{g}^2 \tilde{\mu}^\epsilon}{6 (4\pi)^{d/2}} 2^{\frac{d}{2}} (m^2)^{\frac{1}{2}(\frac{d}{2}-1)} (\sqrt{p \circ p})^{1-\frac{d}{2}} K_{\frac{d}{2}-1}(m\sqrt{p \circ p})$$

$$\Gamma_{1,\text{planar}}^{(2)} = -\frac{\tilde{g}^2 m^2}{3(4\pi)^2} \left[\frac{2}{\epsilon} + \ln \frac{\mu^2}{m^2} \right]$$

UV Limit First ->

$$\Gamma_{1,\text{nonplanar}}^{(2)} = \frac{g^2 m^2}{6(4\pi)^2} \left[\frac{4}{m^2 p \circ p} - \ln \frac{4}{m^2 p \circ p} - 1 + 2\gamma \right]$$

IR Limit First ->

$$\Gamma_{1,\text{nonplanar}}^{(2)} = -\frac{\tilde{g}^2 m^2}{6(4\pi)^2} \left[\frac{2}{\epsilon} + \ln \frac{\mu^2}{m^2} \right]$$

Strong UV/IR Duality

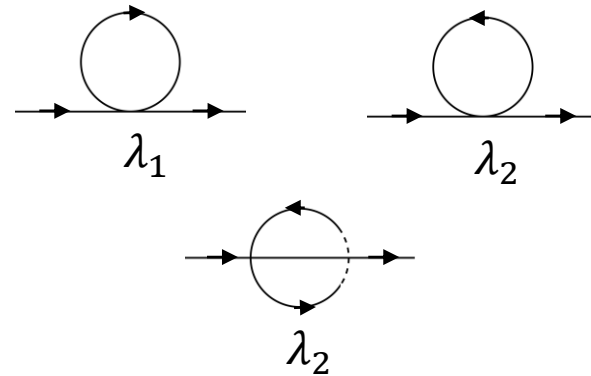
$$\text{Commutative } \Lambda^2 \rightarrow \text{Noncommutative } \frac{1}{p \circ p}$$

Is this always a necessary relationship? Not quite.

E.g. Self-interaction of complex scalar with global U(1)

[Ruiz Ruiz, '02]

$$V = m^2 |\phi|^2 + \frac{\lambda_1}{4} \phi^* \star \phi \star \phi^* \star \phi + \frac{\lambda_2}{4} \phi^* \star \phi^* \star \phi \star \phi$$



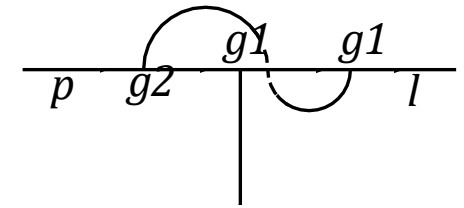
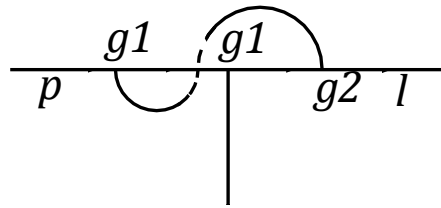
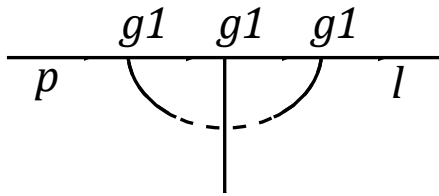
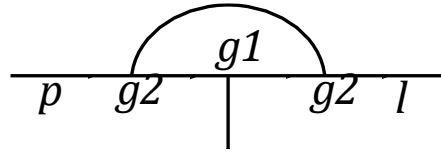
So are there other examples where it does occur?

CPT

¹⁷We note that while the CPT theorem has only been proven in NCFT without space-time noncommutativity [109–112], the difficulty in the general case is related to the issues with unitarity discussed in Section 2, and we expect it should hold in a sensible formulation of the space-time case as well.

¹⁸We should note that in the construction of noncommutative QED it has been argued that it is sensible to assign θ the anomalous charge conjugation transformation $C : \theta^{\mu\nu} \rightarrow -\theta^{\mu\nu}$ ([113] and many others since). The argument is that charged particles in noncommutative space act in some senses like dipoles whose dipole moment is proportional to θ , and so charge conjugation should naturally reverse these dipole moments. Here, however, our particles are uncharged, and thus we have no basis for arguing in this manner. Furthermore, such an anomalous transformation makes charge conjugation relate theories on *different* noncommutative spaces $\mathcal{M}_\theta \rightarrow \mathcal{M}_{-\theta}$. The heuristic picture of the CPT theorem (that is, the reason we care about CPT being a symmetry of our physical theories) is that after Wick rotating to Euclidean space, such a transformation belongs to the connected component of the Euclidean rotation group [114], and so is effectively a symmetry of spacetime. So it is at the least not clear that defining a CPT transformation that takes one to a different space accords with the reason CPT should be satisfied in the first place.

Three Point Function



$$\lim_{p, \ell \rightarrow 0} \Gamma_{3,np}^{\varphi\bar{\psi}\psi}(p, \ell) = \frac{g_1^2(g_1 + 2g_2)}{16\pi^2} \log(\Lambda^2) + \text{finite}$$

$$\lim_{\Lambda \rightarrow \infty} \Gamma_{3,np}^{\varphi\bar{\psi}\psi}(p, \ell) = \frac{g_1^2}{16\pi^2} \left[g_1 \log \left(\frac{4}{(p + \ell) \circ (p + \ell)} \right) + g_2 \log \left(\frac{4}{p \circ p} \right) + g_2 \log \left(\frac{4}{\ell \circ \ell} \right) \right] + \text{finite}$$