# The Black Hole Spectrum in (Super)gravity 

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Michigan
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## Plan of the talk

- Motivations: Spectrum of near-extremal black holes
- BTZ black hole in $A d S_{3}$
- 4D charged black hole


Einstein Gravity

- Same examples in Supergravity


## Motivations, Near Extremal BH

## Near-Extremal Black Hole



- Large Charge and Large Extremal Bekenstein-Hawking Entropy/ Large Area
- Low Temperature/ Near-Extremal Limit

$$
E=M \rightarrow Q
$$

## Black Hole Thermo: Issues

- Bekenstein-Hawking Entropy vs (low) T

$$
S=\underset{\substack{\downarrow \\ S_{0}}}{\pi Q^{2}}+4 \pi^{2} Q^{3} T
$$

- Energy vs (low) T

$$
E=\underset{\substack{\downarrow \\ E_{0}}}{Q+2 \pi^{2} Q^{3} T^{2},{ }^{2}+r^{2}}
$$

- Gap-Scale: Thermodynamical description was thought to break down

[Preskill, Schwarz, Shapere,

$$
E_{\text {gap }} \sim T_{\text {gap }} \Rightarrow E_{\text {gap }} \sim \frac{1}{Q^{3}}
$$

# LIMITATIONS ON THE STATISTICAL DESCRIPTION OF BLACK HOLES 

JOHN PRESKILL* and PATRICIA SCHWARZ ${ }^{\dagger}$<br>Lauritsen Laboratory of High Energy Physics, California Institute of Technology, Pasadena, CA $91125, U S A$<br>and<br>ALFRED SHAPERE ${ }^{\ddagger}$, SANDIP TRIVEDI ${ }^{\S}$ and FRANK WILCZEK ${ }^{+}$<br>School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, NJ 08540, USA

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We argue that the description of a black hole as a statistical (thermal) object must break down as the extreme (zero-temperature) limit is approached, due to uncontrollable thermodynamic fluctuations. For the recently discovered charged dilaton black holes, the analysis is significantly different, but again indicates that a statistical description of the extreme hole is inappropriate. These holes invite a more normal elementary particle interpretation than is possible for Reissner-Nordström holes.

## Spectrum of near-extremal black holes??

- Statistical Description breaks down at low energies?
[Preskill et al 91]



## Spectrum of near-extremal black holes??

- Statistical Description breaks down at low energies?
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## Spectrum of near-extremal black holes??

- Proposal from microscopic models of BHs: gap
(No precise derivation of shape)
[Maldacena, Susskind 96]
[Maldacena, Strominger 97]



## Answer 1: Einstein Gravity

[lliesiu GJT 20]

[Ghosh Maxfield GJT 19]

- No Gap, quantum effects near the horizon become large and modify answer



## Answer 2: Supergravity

- Emergent, but broken, superconformal symmetry $\operatorname{PSU}(1,1 \mid 2)$. Precise derivation of the shape



# Thermodynamics of Near-Extreme Black Holes 

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#### Abstract

The thermodynamics of nearly-extreme charged black holes depends upon the number of ground states at fixed large charge and upon the distribution of excited energy states. Here three possibilities are examined: (1) Ground state highly degenerate (as suggested by the large semiclassical Hawking entropy of an extreme ReissnerNordstrom black hole), excited states not. (2) All energy levels highly degenerate, with macroscopic energy gaps between them. (3) All states nondegenerate (or with low degeneracy), separated by exponentially tiny energy gaps. I suggest that in our world with broken supersymmetry, this last possibility seems most plausible. An experiment is proposed to distinguish between these possibilities, but it would take a time that is here calculated to be more than about $10^{837}$ years.


## Einstein Gravity <br> Case 1: 3D Rotating BTZ

## 3D gravity and BTZ

- Consider 3D Einstein gravity (possibly coupled to matter)

$$
I_{E H}=-\frac{1}{16 \pi G_{N}} \int d^{3} x \sqrt{g_{3}}\left(R_{3}+\frac{2}{\ell_{3}^{2}}\right)
$$

- We will be interested in the rotating BTZ black hole solution near extremality with mass $E$ and angular momentum $J$

$$
\begin{aligned}
& d s^{2}=-f(r) d t^{2}+\frac{d r^{2}}{f(r)}+r^{2}\left(d \varphi-\frac{r_{-} r_{+}}{r^{2}} d t\right)^{2} \\
& f(r)=\frac{\left(r^{2}-r_{+}^{2}\right)\left(r^{2}-r_{-}^{2}\right)}{r^{2}}
\end{aligned}
$$

- Extremality bound: $E \geq|J|$



## The near extremal limit

- We will focus on states with angular momentum, at very low T. This implies that

$$
T \sim \frac{r_{+}-r_{-}}{\pi \ell_{3}} \rightarrow 0 \quad E-|J| \rightarrow 0 \quad r_{-} \sim r_{+} \sim r_{0}
$$

- The metric in the throat is approximately $A d S_{2} \times S^{1}$ in terms of
- In terms of

$$
r=\frac{r_{+}+r_{-}}{2}+\frac{r_{+}-r_{-}}{2} \rho
$$



## Semiclassical thermodynamics

- The extremal energy and Bekenstein-Hawking entropy are

$$
E_{0}=|J|, \quad S_{0}=2 \pi \sqrt{\frac{c|J|}{6}}
$$

- Thermodynamics at low temperatures

$$
E=E_{0}+2 \pi^{2} \Phi_{r} T^{2}, \quad S=S_{0}+4 \pi^{2} \Phi_{r} T
$$

- This will be the universal behavior, with a model dependent parameter $\Phi_{r}$, that in this case is

$$
\Phi_{r}=\frac{c}{24}
$$

## Dimensional reduction in throat

- In the gravitational path integral only some modes become relevant at low T

- Dimensional reduction of 3D Einstein action gives the Achucarro-Ortiz action

$$
I=-\frac{1}{8 G_{N}} \int \sqrt{g_{2}} \Phi\left(R_{2}-\frac{1}{4} \Phi^{2} F^{2}+\frac{2}{\ell_{3}^{2}}\right)
$$

## NHR: Two simplifications

- After integrating out the gauge field (with fixed charge boundary condition) we get a 2D dilaton-gravity theory

$$
I_{J}=-\frac{1}{8 G_{N}} \int \sqrt{g}\left(\Phi R-U_{J}(\Phi)\right), \quad U_{J}(\Phi)=\frac{1}{2} \frac{\left(8 G_{N} J\right)^{2}}{\Phi^{3}}-\frac{2}{\ell_{3}^{2}} \Phi
$$

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$$

- In the near horizon region $\Phi \approx \Phi_{0}$ and the EOM is $U\left(\Phi_{0}\right)=0$, determines extremal size. Expand for small fluctuations $\Phi=\Phi_{0}+4 G_{N} \phi$

$$
I_{J}=-S_{0} \chi-\frac{1}{2} \int \sqrt{g} \phi\left(R+\frac{2}{\ell_{2}^{2}}\right)
$$

$$
S_{0}=\frac{2 \pi \Phi_{0}}{4 G_{N}}=2 \pi \sqrt{\frac{c|J|}{6}}
$$

- Controls breaking of emergent conformal symmetry $\operatorname{SL}(2, \mathbb{R})$ in throat


## Near-extremal limit

- Final answer for gravitational path integral near extremality

$$
Z_{B T Z}[\beta, J]=e^{-\beta E_{0}} e^{S_{0}} \int \mathcal{D} g \mathcal{D} \phi e^{-I_{J T}[g, \phi]}
$$


[Moitra Sake Trivedi Vishal 19]
[Nayak Shukla Soni Trivedi Vishal 18]

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- Boundary conditions at the throat (from gluing with outside)

$$
\begin{gathered}
\left.\phi\right|_{\partial}=\frac{1}{\varepsilon} \frac{c}{24},\left.\quad L\right|_{\partial}=\frac{\ell_{2} \beta}{\varepsilon}, \quad \varepsilon \rightarrow 0 \\
\Phi_{r}: \text { Renormalized dilaton }
\end{gathered}
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- Contributions from KK modes are easy: interactions are suppressed and they give temperature independent corrections (shift $S_{0}$ and $E_{0}$ )
[Sen]

$$
I \rightarrow \Phi_{0} I_{2 \mathrm{D} \text { matter, KK modes }}
$$

## Jackiw-Teitelboim (JT) gravity

- Integrating out the (linear) dilaton first, the theory reduces to a boundary mode on rigid AdS2 [Almheiri, Polchinski] [Jensen] [Maldacena,Stanford, Yang] [Englesoy, Mertens Verlinde]...

$$
I_{J T}=\Phi_{r} \int_{0}^{\beta} d \tau\{f, \tau\} \quad f(\tau+\beta)=f(\tau)
$$

- This mode controls finite-temperature effects, breaks the emergent $S L(2, \mathbb{R})$ symmetry
- This theory can be quantized exactly to obtain the disk partition function
[Altland, Bagrets, Kamenev] [Stanford Witten] [Mertens GJT Verlinde]

$$
Z_{\mathrm{JT}}(\beta, J) \sim \frac{\Phi_{r}^{1 / 2}}{\sqrt{2 \pi} \beta^{3 / 2}} e^{\frac{2 \pi^{2} \Phi_{r}}{\beta}}
$$

- This theory is equivalent to an $\operatorname{SL}(2, \mathbb{R})$ BF theory


## A check: Pure 3D gravity

- The exact result in 3D pure gravity including perturbative quantum effects was computed by Maloney and Witten:

$$
Z_{\mathrm{BTZ}}=\chi_{\mathbf{1}}(-1 / \tau) \chi_{\mathbf{1}}(1 / \bar{\tau}) \quad \chi_{\mathbf{1}}(\tau)=\frac{(1-q) q^{-\frac{c-1}{24}}}{\eta(\tau)}
$$

- We can take a near extremal approximation of this formula, gives

$$
Z_{\mathrm{BTZ}}(\beta, J) \sim \frac{\Phi_{r}^{1 / 2}}{\sqrt{2 \pi} \beta^{3 / 2}} e^{S_{0}-\beta E_{0}+\frac{2 \pi^{2} \Phi_{r}}{\beta}}
$$

$$
\Phi_{r}=\frac{c}{24}
$$

- This is precisely the answer we expected from the reduction to JT gravity!


## Negrementernern

- Since the reduction to JT is valid beyond pure gravity, we have a universal spectrum near extremality

$$
\rho_{J}(E)=\frac{e^{S_{0}}}{2 \pi^{2}} \sinh \left(2 \pi \sqrt{2 \Phi_{r}\left(E-E_{0}\right)}\right)
$$



- According to [Preskill et al 91] the statistical mechanical description of NEBH was supposed to break down at $E \sim 1 / \Phi_{r}$, and it was believed to be a gap in their spectrum.
- Instead, there is no gap. A gravitational mode becomes strongly coupled and the density of states goes smoothly to zero


## Near-Extremal Spectrum

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$$



- Important that this effects break conformal invariance. If unbroken it would imply
[Jensen et al 11]

$$
\rho(E)=A \delta(E)+\frac{B}{E}
$$

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## Universal sector in 2D CFT

- Universal gravitational sector in $A d S_{3}$ when looking at near extremal states. Is there a universal sector of 2D CFTs?


Horizon

## Universal sector in 2D CFT

- Universal gravitational sector in $A d S_{3}$ when looking at near extremal states. Is there a universal sector of 2D CFTs?
$\checkmark$ Yes! Only assumptions: twist gap, large c and modular invariance
[Ghosh, Maxfield, GJT 19]
- 2D CFT description of the states:
- If we fix angular velocity: $\quad \beta_{L} \sim 2 \beta \quad \beta_{R} \sim 2 \pi \sqrt{\frac{c}{24 J}}$

$$
Z \approx \chi_{1}(-1 / \tau) \chi_{1}(-1 / \bar{\tau})+\ldots
$$

- Similar phenomena with correlators


## Pure 3D gravity

- Including only BTZ and the $S L(2, \mathbb{Z})$ black hole gives non-unitary partition function. Other interesting configurations in near-extremal limit are:

[Benjamin et al 19]



## Pure 3D gravity

- Including only BTZ and the $S L(2, \mathbb{Z})$ black hole gives non-unitary partition function. Other interesting configurations in near-extremal limit are:



## Einstein Gravity Case 2: 4D Charged BH

## Charged Black Hole in $A d S_{4}$

-4D action: $\quad I=-\int\left(R+\frac{6}{L^{2}}\right)-\frac{1}{e^{2}} \int F^{2}+I_{\text {Bdy }}$

- AdS RNBH

$$
\text { (metric) } \quad d s^{2}=f d \tau^{2}+\frac{d r^{2}}{f}+r^{2} d \Omega^{2}, \quad f=1-\frac{2 M}{r}+\frac{Q^{2}}{r^{2}}+\frac{r^{2}}{L^{2}}
$$

(gauge field) $\quad A=i \mu\left(1-\frac{r_{h}}{r}\right) d \tau \quad \mu=\frac{e}{4 \pi} \frac{Q}{r_{h}}$

- Large BH in AdS
- "Gap" scale

$$
Q^{2} \sim \frac{r_{0}^{4}}{L^{2}}, \quad E_{0} \sim \frac{r_{0}^{3}}{L^{2}} \sim Q^{3 / 2}, \quad S_{0} \sim Q \gg 1 \quad \Phi_{r}=r_{0} L
$$



## Charged Black Hole in $A d S_{4}$



- Throat (NHR)

- Matching surface:

$$
r_{c}-r_{0} \sim \frac{1}{\varepsilon} \gg L_{2}
$$

## Massless Sector at Iow T

- Reduction to 2D: we only want to keep massless modes
(metric)

$$
d s_{4 D}^{2}=\frac{r_{0}}{\Phi^{1 / 2}} d s_{2 D}^{2}+\Phi h_{m n}^{\left(S^{2}\right)}\left(d y^{m}+\mathbf{B}^{a} \xi_{a}^{m}\right)\left(d y^{n}+\mathbf{B}^{b} \xi_{b}^{n}\right)
$$

SO(3) gauge field
 Killing vectors of sphere
(gauge field) $\quad A_{\mu}(x, y)=a_{\mu}(x) \longleftarrow \mathrm{U}(1)$ gauge field

- Action:

$$
I_{2 \mathrm{D}}=\int(\Phi R-2 U(\Phi))-\frac{1}{r_{0}} \int \Phi^{5 / 2} H^{2}-\frac{1}{e^{2} r_{0}} \int \Phi^{3 / 2} f^{2}
$$

$$
U(\chi)=r_{0}\left[-\frac{3}{L^{2}} \Phi^{1 / 2}-\frac{1}{\Phi^{1 / 2}}\right]
$$

## Effective 2D theory

- 2D Yang Mills: easy to integrate out fields (all Dirichlet)

$$
Z=\sum_{j, Q}(2 j+1)^{2} e^{\beta \mu \frac{Q}{e}} \int \mathcal{D} \Phi \mathcal{D} g e^{\int\left(\Phi R-2 U_{Q, j}(\Phi)\right)}
$$

- Charge dependent dilaton potential: $U_{Q, j}(\chi)=r_{0}\left[\frac{Q^{2}}{\Phi^{3 / 2}}+\frac{3 j(j+1)}{\Phi^{5 / 2}}-\frac{3}{L^{2}} \Phi^{1 / 2}-\frac{1}{\Phi^{1 / 2}}\right]$


## 

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- NHR: Linear dilaton approximation $\Phi=\Phi_{0}+\phi$

$$
U_{Q, j}\left(\Phi_{0}=r_{0}\right)=0, \quad \Rightarrow \quad U\left(\Phi_{0}+\phi\right) \approx-\frac{1}{L_{2}^{2}} \phi
$$

$$
Z_{\mathrm{JT}}=\int \mathcal{D} \phi \mathcal{D} g e^{\int_{\mathrm{NHR}} \phi\left(R+\frac{2}{L_{2}^{2}}\right)+I_{\mathrm{Bdy}}}
$$

$$
\longleftarrow\left[\begin{array}{l}
\left.\phi\right|_{\mathrm{Bdy}}=\frac{\Phi_{r}}{\varepsilon} \\
\left.L\right|_{\mathrm{Bdy}}=\beta \frac{L_{2}}{\varepsilon}
\end{array}\right.
$$

## Partition Function



NHR: Constant Dilaton
$Z=\sum_{j, Q}(2 j+1)^{2} \underbrace{e^{\beta \mu \frac{Q}{e}-\beta E_{0}}}_{\text {FAR: Classical fluctuations }} e^{S_{0}} Z_{\mathrm{JT}}\left[\Phi_{r}, \beta\right]$
NHR: Linear Dilaton / JT

## Partition Function: Corrections



- Heavy charged matter and KK modes

$$
\delta \log Z=\beta \delta E_{0}+\delta S_{0}+\mathcal{O}(\varepsilon)
$$

- Interactions: suppressed in $r_{0}$ and $\varepsilon$ (Log corrections $\delta S_{0} \sim \log L_{2}$ )
- Non-linear dilaton corrections: supp. in $r_{0}$ and $T$
- Non-perturbative: exponentially suppressed in $S_{0}$
$\Rightarrow$ To do: Add light charged matter (incorporate instability)


## Statistical Mechanics

- The 2D gauge modes are frozen when we fix charges. In a sector of fixed $Q$ and $J$ the spectrum is again:

$$
\rho_{Q, J}(E)=\frac{e^{S_{0}}}{2 \pi^{2}} \sinh \left(2 \pi \sqrt{2 \Phi_{r}\left(E-E_{0}\right)}\right)
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$$

- Fixed chemical potential and $\mathrm{j}=0$

$$
Z=e^{S_{0}-\beta E_{0}} Z_{\mathrm{JT}}\left(Q_{0}\right) \sum_{Q=Q_{0}+q} e^{2 \pi \mathcal{E} q-\beta \frac{q^{2}}{2 K}} \quad \longleftarrow\left[\begin{array}{ll}
K=\left.\frac{\partial Q}{\partial \mu}\right|_{T=0} & \text { (compressibility) } \\
\mathcal{E}=\left.\frac{\partial S_{0}}{\partial Q}\right|_{T=0} & \text { (electric field) }
\end{array}\right.
$$

- Fixed charge and boundary metric (zero angular velocity)

$$
Z=e^{S_{0}-\beta E_{0}} Z_{\mathrm{JT}}\left(Q_{0}\right) \sum_{j}(2 j+1)^{2} e^{-\beta \frac{j(j+1)}{r_{0}^{3}}}
$$

## Black Hole Spectrum for Fixed Charges

- No Gap, quantum effects become large. Answer for non-SUSY theories



## Supergravity

## 4D $\mathcal{N}=2$ Supergravity $(\Lambda=0)$

- Fields: Metric $G_{M N}$, doublet of gravitinos $\Psi_{M}^{I}$ and $\mathrm{U}(1)$ gauge field $A_{M}$
- Lagrangian: $\quad E^{-1} \mathcal{L}=\kappa^{-2}\left(\frac{1}{2} R-\bar{\Psi}_{I M} \Gamma^{M N P} D_{N} \Psi_{P}^{I}-\frac{1}{4} F_{M N} F^{M N}\right.$

$$
\left.8 \pi G_{N}=\kappa^{2} \quad+\frac{\varepsilon^{I J}}{2 \sqrt{2}} \bar{\Psi}_{I}^{M}\left(F_{M N}+i \star F_{M N} \Gamma_{5}\right) \Psi_{J}^{N}+4 \text { gravitino }\right),
$$

- Local SUSY transformations: $\delta_{\epsilon} E_{M}^{A}=\frac{1}{2} \bar{\epsilon}^{I} \Gamma^{A} \Psi_{M I}+$ h.c.,

$$
\begin{aligned}
& \delta_{\epsilon} A_{M}=\frac{1}{\sqrt{2}} \varepsilon^{I J} \bar{\epsilon}_{I} \Psi_{M J}+\text { h.c. } \\
& \delta_{\epsilon} \Psi_{M}^{I}=\left(\partial_{M}+\frac{1}{4} \omega_{M}^{A B} \Gamma_{A B}\right) \epsilon^{I}-\frac{1}{4 \sqrt{2}} \Gamma^{A B} F_{A B} \Gamma_{M} \varepsilon^{I J} \epsilon_{J} .
\end{aligned}
$$

- In this theory, the Reissner-Nordstrom black hole is still a solution


## 4D $\mathcal{N}=2$ Supergravity

Far Region: Flat Minkowski SUSY Vacuum of $\mathcal{N}=2$ SUGRA with $\operatorname{ISO}(3,1 \mid 2)$


## 4D $\mathcal{N}=2$ Supergravity

Far Region: Flat Minkowski SUSY Vacuum of $\mathcal{N}=2$ SUGRA with $\operatorname{ISO}(3,1 \mid 2)$


This symmetry comes from the $S^{2}$ (NOT the outer $S U(2)$ which is usually broken)

## 4D $\mathcal{N}=2$ Supergravity

Far Region: Flat Minkowski
SUSY Vacuum of $\mathcal{N}=2$ SUGRA
with $\operatorname{ISO}(3,1 \mid 2)$


Near extremal spectrum comes from mode that breaks this symmetry

## Dimensional Reduction

- The emergent, broken, symmetry in the throat is now $\operatorname{PSU}(1,1 \mid 2)$, new fermionic modes from gravitino become relevant at low T [Michelson, Spradlin 99]


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- The reduction of the higher dimensional supergravity theory in the throat is $\mathcal{N}=4$ supersymmetric JT gravity. This can be rewritten as a $\mathfrak{p} \mathfrak{G} \mathfrak{t}(1,1 \mid 2)$ BF theory:
[Heydeman, Iliesiu, Zhao, GJT 20]

$$
I_{B F}=-i \int \operatorname{Str} \phi F, \quad F=d A-A \wedge A
$$

- Where $A$ is a $\mathfrak{p} \mathfrak{B u}(1,1 \mid 2)$ gauge field and $\phi$ is a zero-form in the adjoint of $\mathfrak{p} \mathfrak{B u}(1,1 \mid 2)$


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- Boundary conditions to glue to far-away region: $\left.\quad \delta\left(2 i \Phi_{r} A_{\tau}(\tau)+\phi(\tau)\right)\right|_{\partial \mathcal{M}}=0$

$$
I_{B F, \text { bdy. }}=\frac{i}{2} \int_{\partial \mathcal{M}} \operatorname{Str} \phi A=\Phi_{r} \int_{\partial \mathcal{M}} d \tau \operatorname{Str} A_{\tau}^{2}
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$$
\begin{aligned}
& I_{B F, \text { bdy. }}=\frac{i}{2} \int_{\partial \mathcal{M}} \operatorname{Str} \phi A=\Phi_{r} \int_{\partial \mathcal{M}} d \tau \operatorname{Str} A_{\tau}^{2} \\
& \mathscr{N}=4 \text { Schwarzian Theory }
\end{aligned}
$$

## $\mathcal{N}=4$ Schwarzian Theory

- Parametrization of the $\mathcal{N}=4$ superline $Z=\left(\tau, \theta^{a}, \bar{\theta}_{b}\right)$. The supercovariant derivatives are

$$
D_{a}=\frac{\partial}{\partial \theta^{a}}+\frac{1}{2} \bar{\theta}_{a} \partial_{\tau}, \quad \bar{D}^{a}=\frac{\partial}{\partial \bar{\theta}_{a}}+\frac{1}{2} \theta^{a} \partial_{\tau} .
$$

- Super-reparametrization: $\tau \rightarrow \tau^{\prime}(\tau, \theta, \bar{\theta}), \quad \theta^{a} \rightarrow \theta^{\prime a}(\tau, \theta, \bar{\theta}), \quad \bar{\theta}_{b} \rightarrow \bar{\theta}_{b}^{\prime}(\tau, \theta, \bar{\theta})$,

Satisfy the constrains [Matsuda Uematsu] [Schoutens]

$$
\begin{aligned}
D_{a} \bar{\theta}_{b}^{\prime} & =0 \quad, \quad \bar{D}^{a} \theta^{\prime b}=0, \\
D_{a} \tau^{\prime}-\frac{1}{2}\left(D_{a} \theta^{\prime b}\right) \bar{\theta}_{b}^{\prime} & =0 \quad, \quad \bar{D}^{a} \tau^{\prime}-\frac{1}{2}\left(\bar{D}^{a} \bar{\theta}_{b}^{\prime}\right) \theta^{\prime b}=0
\end{aligned}
$$

## $\mathcal{N}=4$ Schwarzian Theory

- We can parametrize the super-reparametrizations $\operatorname{Diff}\left(S^{1 \mid 4}\right)$ by the following functions:

$$
f(\tau) \in \operatorname{Diff}\left(S^{1}\right), \quad g(\tau) \in S U(2), \quad \eta^{a}(\tau), \quad \bar{\eta}_{a}(\tau) .
$$

- A finite reparam with all parameters turned on looks very complicated. A special subset are $\operatorname{PSU}(1,1 \mid 2)$ transformations. For example the bosonic subgroup $S L(2, \mathbb{R}) \times S U(2)$ is

$$
\begin{aligned}
\tau & \rightarrow \frac{a \tau+b}{c \tau+d}-\frac{c}{4(c \tau+d)^{3}}(\bar{\theta} \theta)^{2} \\
\theta^{a} & \rightarrow\left[e^{i \vec{t} \cdot \vec{\sigma}}\right]_{b}^{a} \theta^{b} \frac{1}{c\left(\tau-\frac{1}{2} \bar{\theta} \theta\right)+d} \\
\bar{\theta}_{a} & \rightarrow \bar{\theta}_{b}\left[e^{i \vec{t} \cdot \vec{\sigma}}\right]_{a}^{b} \frac{1}{c\left(\tau+\frac{1}{2} \bar{\theta} \theta\right)+d}
\end{aligned}
$$

## $\mathcal{N}=4$ Schwarzian Theory

- The N=4 Schwarzian derivative was defined by Matsuda and Uematsu

$$
S^{i}\left(Z ; Z^{\prime}\right)=-\frac{1}{6} D \sigma^{i} \bar{D} \log \left(\frac{1}{2}\left(D_{a} \theta^{b}\right)\left(\bar{D}^{a} \bar{\theta}_{b}^{\prime}\right)\right) .
$$

- The Schwarzian action corresponds to one component of this field
[Heydeman, Iliesiu, Zhao, GJT 20]

$$
I_{\mathcal{N}=4}=-\Phi_{r} \int d \tau S_{b}[f(\tau), g(\tau), \eta(\tau)]
$$

- More Explicitly:

$$
\begin{array}{cc}
I_{\mathcal{N}=4}=-\Phi_{r} \int_{0}^{\beta} d \tau\left[\operatorname{Sch}(f, \tau)+\operatorname{Tr}\left(g^{-1} \partial_{\tau} g\right)^{2}+(\text { fermions })\right] \\
\text { Bosonic } & \text { Particle moving } \\
\text { Schwarzian } & \text { on } S U(2)
\end{array}
$$

## Summary of Steps

4D $\mathcal{N}=2$ supergravity

Fixed $U(1)$ charge, Look at throat

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1D Boundary mode $\longleftrightarrow \mathcal{N}=4$ Super-Schwarzian
$\square$
Spectrum as function of energy and spin

## $\mathcal{N}=4$ Schwarzian Theory

- The spectrum of this two-dimensional black hole is extracted from the partition function:

$$
Z(\beta, \alpha)=\int \frac{\mathcal{D} f \mathcal{D} g \mathcal{D} \eta \mathcal{D} \bar{\eta}}{\operatorname{PSU}(1,1 \mid 2)} \exp \left(\Phi_{r} \int d \tau S_{b}[f, g, \eta, \bar{\eta}]\right)
$$

- Boundary conditions:



## $\mathcal{N}=4$ Schwarzian Theory

- The partition function can be computed exactly using localization [Stanford Witten] or canonical methods [Mertens GJT Verlinde]. The answer is:

- Also obtained from a limit of $\mathcal{N}=4$ Virasoro characters of Eguchi and Taormina


## $\mathcal{N}=4$ Spectrum

- The $\operatorname{PSU}(1,1 \mid 2)$ symmetry is broken, as much as the conformal symmetry in the bosonic case. There is a global super-translation group which survives with four supercharges. This organizes spectrum is supermultiplets

$$
\begin{aligned}
Z(\beta, \alpha)= & \sum_{J} \chi_{J}(\alpha) \rho_{\mathrm{ext}}(J)+\int d E e^{-\beta E}\left(\chi_{1 / 2}(\alpha)+2 \chi_{0}(\alpha)\right) \rho_{\mathrm{cont}}(1 / 2, E) \\
& +\sum_{J \geq 1} \int d E e^{-\beta E}\left(\chi_{J}(\alpha)+2 \chi_{J-\frac{1}{2}}(\alpha)+\chi_{J-1}(\alpha)\right) \rho_{\mathrm{cont}}(J, E),
\end{aligned}
$$

$S U(2)$ characters

$$
\chi_{J}(\alpha) \equiv \sum_{m=-J}^{J} e^{4 \pi i \alpha m}=\frac{\sin (2 J+1) 2 \pi \alpha}{\sin 2 \pi \alpha}
$$

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\end{aligned}
$$

Non-zero index: - $E=0$ : Supermultiplet ( $J$ )
Zero index: $-E \neq 0$ : Supermultiplet $\mathbf{1 / 2}=(1 / 2) \oplus 2(0)$

## $\mathcal{N}=4$ Spectrum

- Density of states at fixed $S U(2)$ charge $J$, where $E_{0}(J)=J^{2} / 2 \Phi_{r}$

$$
\begin{aligned}
\rho_{\mathrm{ext}}(J) & =e^{S_{0}} \delta_{J, 0} \\
\rho_{\text {cont }}(J, E) & =\frac{e^{S_{0}} J}{4 \pi^{2} \Phi_{r} E^{2}} \sinh \left(2 \pi \sqrt{2 \Phi_{r}\left(E-E_{0}(J)\right)}\right) \Theta\left(E-E_{0}(J)\right), \text { for } J \geq \frac{1}{2},
\end{aligned}
$$




Figure 3: Left: Density of supermultiplets labeled by the highest spin J. We show 0, which is simply a delta function at $E=0 ; \mathbf{1} / \mathbf{2}$ which is continuous but starts at $E_{\text {gap }} \equiv E_{0}(1 / 2)$; and $\mathbf{1}$ which is also continuous starting at $E_{0}(1)$. Right: Degeneracy for all states with $J=0$. These come from $\mathbf{0}$, the delta function at $E=0, \mathbf{1} / \mathbf{2}$,starting at $E_{\text {gap }}$, and $\mathbf{1}$, starting at $E_{0}(1)$. All other supermultiplets do not have a $J=0$ component.


Figure 2: Schematic shape of the black hole spectrum at fixed $S U(2)$ charge as a function of energy above extremality $E$. We show the semiclassical answer (red dahsed) and the solution including quantum effects (purple). (a) Answer for Einstein gravity. We see there is no gap at scale $E \sim 1 / \Phi_{r}$ and the extremal entropy goes to zero. (b) Answer for supergravity (either $\mathcal{N}=2$ in 4 D or $\mathcal{N}=(4,4)$ in 3 D ). We find a gap at the scale $E_{g a p}=\frac{1}{8 \Phi_{r}}$ and a number $e^{S_{0}}$ of extremal states, consistent with string theory expectations. (c) Einstein gravity spectrum for $J \neq 0$. (d) Supergravity spectrum for $J \neq 0$, the jumps indicate contributions from different supermultiplets $\mathbf{J}, \mathbf{J}+\mathbf{1} / \mathbf{2}$ and $\mathbf{J}+\mathbf{1}$.

## $(4,4)$ Supergravity in $A d S_{3}$

- The gravity sector is described by CS with super-group $\operatorname{PSU}(1,1 \mid 2)_{L} \otimes \operatorname{PSU}(1,1 \mid 2)_{R}$ at level $k$
- The bosonic sector is 3D Einstein gravity coupled to CS $S U(2)_{L} \otimes S U(2)_{R}$ at level $k$, with charges $J_{L}, J_{R}$
- Extremal-BPS State: Large momentum $P$ and $S U(2)$ charges $J_{L} \neq 0, J_{R}=0$. Near-extremal states described by same spectrum as before, with parameters:
$S_{0}=2 \pi \sqrt{k P-J_{L}^{2}}$

$$
\Phi_{r}=\frac{k}{4} \quad J \rightarrow J_{R}
$$

- The gap in the spectrum for $J_{R}=0$ is $E_{\text {gap }}=\frac{1}{2 k}$

(b) $J_{R}=0$

(d) $J_{R} \neq 0$


## Gap in the D1-D5 system

- The near-extremal black hole appearing in the D1-D5 system has a geometry $A d S_{3} \times S^{3}$. This is described by $(4,4)$ sugra with level $k=Q_{1} Q_{5}$
- The near-extremal spectrum predicts the index matches with the BekensteinHawking formula and the gap at $J_{R}=0$ is given by

$$
E_{\text {gap }}=\frac{1}{2 Q_{1} Q_{5}}
$$

- This is controlled by the 2D Virasoro algebra but also has a stringy origin explained by Maldacena and Susskind.


## Conclusions

## Conclusions and Open Questions



- Black hole spectrum near extremality fixed by pattern of symmetry breaking. Another important example is $S U(1,1 \mid 1)$.
- Prediction for behavior of higher-dimensional CFTs at low temperature/large charge, emergence of "local criticality". Can we show this using a bootstrap argument?
- A gravitational calculation of the index?
- Matrix dual to pure $\mathscr{N}=4$ Super-JT?
- Another application: Hartle-Hawking wavefunction of a $S^{1} \times S^{2}$ universe


## Extra

## BHs with emergent $S U(1,1 \mid 1)$

- Examples: Near-BPS black holes in 4D and 5D gauged SUGRA with negative cosmological constant.
- Low-temperature thermodynamics captured by the $\mathcal{N}=2$ Schwarzian
[Fu Gaiotto Maldacena Sachdev]

$$
I_{\mathcal{N}=2}=\Phi_{r} \int \operatorname{Sch}(f, \tau)+2\left(\partial_{\tau} \sigma\right)^{2}+(\text { fermions }),
$$

- Density of states from exact quantization:
[Stanford Witten][Mertens GJT Verlinde]

$$
\begin{aligned}
Z_{D}(\alpha, \beta) & =\sum_{Q \in \mathbb{Z}+\frac{\nu}{2},|Q|<\frac{\hat{q}}{2}} e^{2 \pi i \alpha Q} e^{S_{0}} \frac{\pi}{\hat{q}} \cos \left(\frac{\pi Q}{\hat{q}}\right) \\
& +e^{S_{0}} \sum_{Q \in \mathbb{Z}+\frac{\nu}{2}}\left(e^{2 \pi i \alpha Q}+e^{2 \pi i \alpha(Q-\hat{q})}\right) \int_{E_{0}(Q)}^{\infty} d E e^{-\beta E} \frac{\sinh \left(2 \pi \sqrt{2 \Phi_{r}\left(E-E_{0}(Q)\right)}\right)}{2 \hat{q} E} .
\end{aligned}
$$

where $E_{0}(Q) \equiv \frac{1}{8 \Phi_{r}}\left(\frac{Q}{\hat{q}}-\frac{1}{2}\right)^{2}$.

- Besides $S_{0}$ and $\Phi_{r}$, also depends on two discrete parameters $\hat{q} \in \mathbb{Z}, \nu=0,1$


## BHs with emergent $S U(1,1 \mid 1)$



No anomaly


Anomaly

Figure 5: Density of supermultiplets as a function of energy $E$ and charge $Q$. Left: Odd $\widehat{q}$ and no anomaly. The delta function at $E=0$ involves charges in the range $|Q|<1 / 2$. The supermultiplet with the lowest gap has $Q_{0}=1 / 2 \pm 1 /(2 \widehat{q})$ with $E_{\text {gap }}=E_{0}\left(Q_{0}\right)$. Other supermultiplets labeled by $Q$ start at higher energies as shown. Right: Odd $\widehat{q}$ and anomaly. The delta function at $E=0$ involves charges in the range $|Q|<1 / 2$. The supermultiplet with $Q=1 / 2$ has no gap. Other supermultiplets have a gap, as shown.

- Q: How to extract the discrete parameters $\hat{q} \in \mathbb{Z}, \nu=0,1$ from the higher dimensional black hole?

