

Kaluza-Klein Spectrometry for String Theory Compactifications

Emanuel Malek

Humboldt-Universität zu Berlin



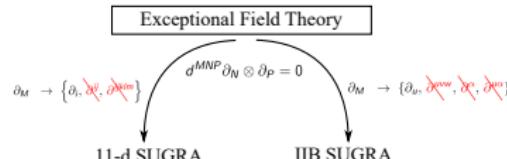
11th March 2021

Based on joint work with Giambrone, Guarino, Nicolai, Samtleben, Trigiante

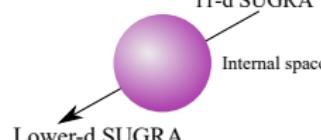
Motivation



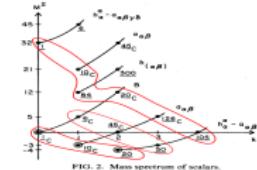
Exceptional Field Theory



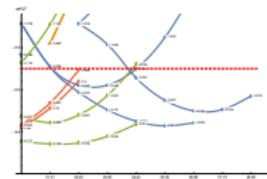
Consistent truncations



Kaluza-Klein Spectrometry



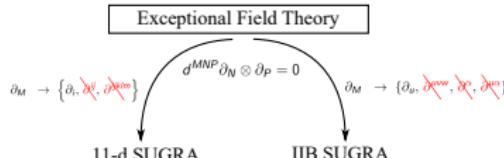
Applications



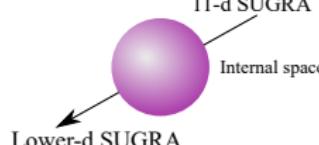
Motivation



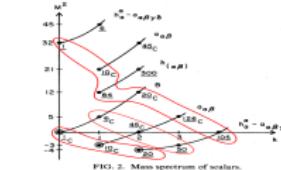
Exceptional Field Theory



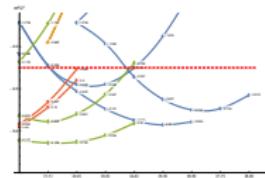
Consistent truncations



Kaluza-Klein Spectrometry



Applications



The importance of Kaluza-Klein spectra

- Compactification \Rightarrow massive Kaluza-Klein modes arise
- Kaluza-Klein spectrum important
 - AdS/CFT: conformal dimensions
 - Stability of non-SUSY vacua?

Computing Kaluza-Klein spectra is hard

- Free scalar on S^1 :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$

$$\phi(x, y) = \phi^{(k)}(x) e^{i k y / R}, \quad m^2 = \frac{k^2}{R^2}.$$

Computing Kaluza-Klein spectra is hard

- Free scalar on S^1 :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$

$$\phi(x, y) = \phi^{(k)}(x) e^{i k y/R}, \quad m^2 = \frac{k^2}{R^2}.$$

- SUGRA: (linearised) EoMs mix metric & fluxes \Rightarrow eigenmodes?

$$\nabla_Q f^{QMN P} + \frac{1}{2} F^{QMN P} \nabla_Q h_R{}^R - \nabla_Q \left(h^{QR} F_R{}^{MNP} \right) - 3 \nabla^Q \left(h^{S[M} F_{QS}{}^{NP]} \right) = -\frac{1}{288} \epsilon^{MNPQ_1 \dots Q_8} F_{Q_1 \dots Q_4} f_{Q_5 \dots Q_8}.$$

Computing Kaluza-Klein spectra is hard

- Free scalar on S^1 :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$

$$\phi(x, y) = \phi^{(k)}(x) e^{i k y/R}, \quad m^2 = \frac{k^2}{R^2}.$$

- SUGRA: (linearised) EoMs mix metric & fluxes \Rightarrow eigenmodes?

$$\nabla_Q f^{QMNP} + \frac{1}{2} F^{QMNP} \nabla_Q h_R{}^R - \nabla_Q \left(h^{QR} F_R{}^{MNP} \right) - 3 \nabla^Q \left(h^{S[M} F_{QS}{}^{NP]} \right) = -\frac{1}{288} \epsilon^{MNPQ_1 \dots Q_8} F_{Q_1 \dots Q_4} f_{Q_5 \dots Q_8}.$$

- Only two cases understood:

- Spin-2 fields [Bachas, Estes '11] ✓
- $M_{int} = \frac{G}{H}$ ✓

Another tool: Consistent truncations

- Non-linear truncation to subset of KK-modes
- Solutions are solutions to higher-dim theory
- Compute subset of masses for any vacuum!

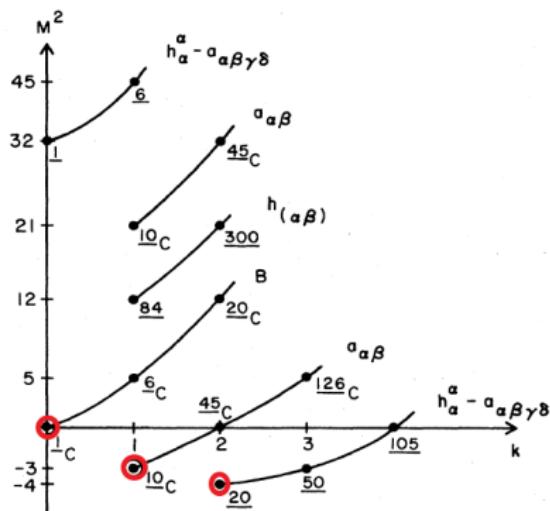
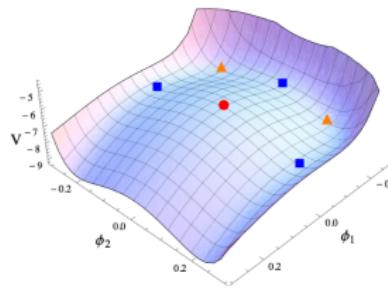


FIG. 2. Mass spectrum of scalars.



Another tool: Consistent truncations

- Non-linear truncation to subset of KK-modes
- Solutions are solutions to higher-dim theory
- Compute subset of masses for any vacuum!

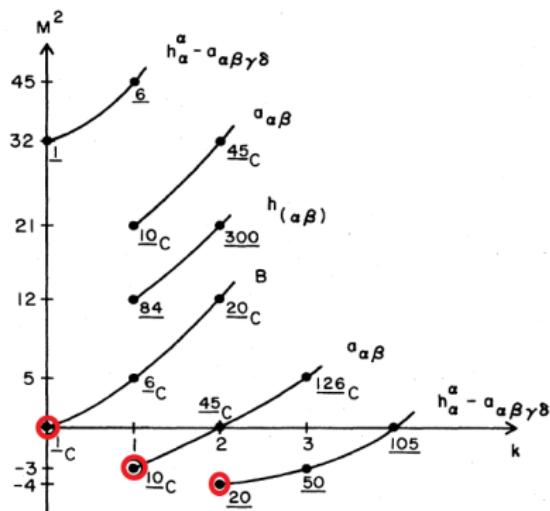
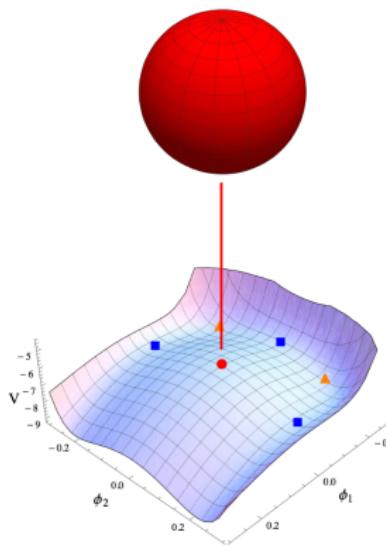
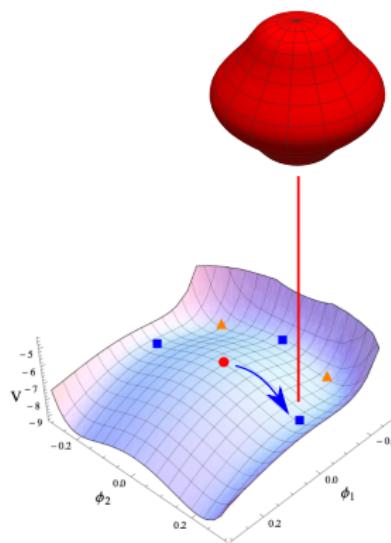
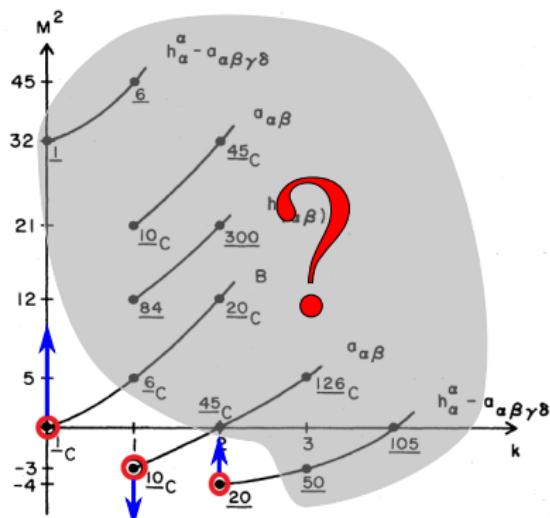


FIG. 2. Mass spectrum of scalars.



Another tool: Consistent truncations

- Non-linear truncation to subset of KK-modes
- Solutions are solutions to higher-dim theory
- Compute subset of masses for any vacuum!



Another tool: Consistent truncations

- Non-linear truncation to subset of KK-modes

[EM, Samtleben 1911.12640]

Extend this to full KK spectrum!

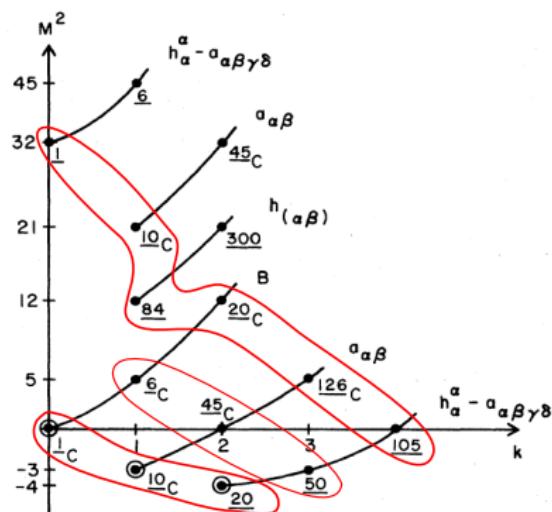
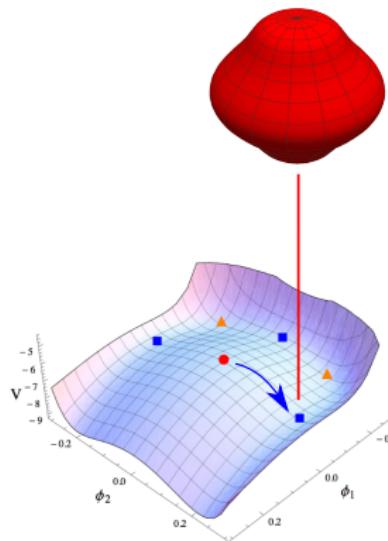


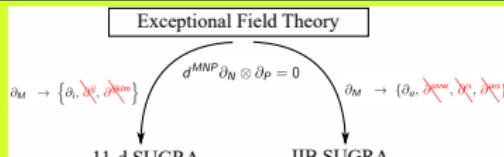
FIG. 2. Mass spectrum of scalars.



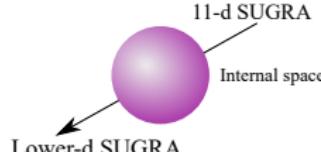
Motivation



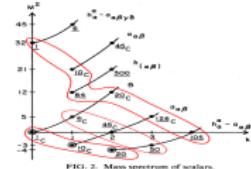
Exceptional Field Theory



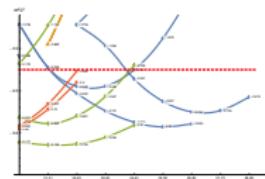
Consistent truncations



Kaluza-Klein Spectrometry



Applications



$E_{6(6)}$ Exceptional field theory

[Berman, Perry '10], [Berman, Godazgar², Perry '11], [Coimbra, Strickland-Constable, Waldram '11], [Berman, Cederwall, Kleinschmidt, Thompson '12], [Hohm, Samtleben '13] ...

- 11-d SUGRA: $(g, C_{(3)}, C_{(6)})$

$$M_{11} = M_5 \times M_{int}.$$

- Unify diffeos + gauge symmetries of 11-d SUGRA on M_{int}

$$\delta g = L_{\textcolor{red}{v}} g, \quad \delta C_{(3)} = L_{\textcolor{red}{v}} C_{(3)} + d\lambda_{(2)}, \quad \delta C_{(6)} = L_{\textcolor{red}{v}} C_{(6)} + d\sigma_{(5)} + \dots$$

- Generalised vector field

$$V^M = (\textcolor{red}{v}^i, \lambda_{ij}, \sigma_{ijklm}) \in \mathbf{27} \text{ rep of } E_{6(6)}$$

Generalised metric and other fields

- Internal bosonic fields on M_{int} :

$$\{g, C_{(3)}, C_{(6)}\} = \mathcal{M}_{MN} \in \frac{E_{6(6)}}{\mathrm{USp}(8)}.$$

- Fields with mixed legs:

$$\{g^{ij} g_{\mu j}, C_{\mu ij}, \dots\} = \mathcal{A}_\mu{}^M \in \mathbf{27} \text{ of } E_{6(6)},$$

$$\{C_{\mu\nu i}, C_{\mu\nu ijk l} \dots\} = \mathcal{B}_{\mu\nu M} \in \overline{\mathbf{27}} \text{ of } E_{6(6)}$$

- Spinors form reps of USp(8) [Coimbra, Strickland-Constable, Waldram '11]

Generalised Lie derivative

- Generalised Lie derivative: local $E_{6(6)}$ action

$$\begin{aligned}\mathcal{L}_V = V^M \partial_M - (\partial \times_{adj} V) &= \text{diffeo} + \text{gauge transf}, \\ \mathcal{L}_V d_{MNP} &= 0,\end{aligned}$$

with $\partial_M = (\partial_i, \partial^{ij}, \dots) = (\partial_i, 0, \dots, 0)$.

- E.g.

$$\mathcal{L}_V \mathcal{M}_{MN} = \left\{ L_v g, L_v C_{(3)} + d\lambda_{(2)}, L_v C_{(6)} + d\sigma_{(5)} + \dots \right\}.$$

- “Exceptional geometry”: generalised tensors, generalised connections, etc.

Section condition: Unifying 11-d & IIB

- “Section condition”

$$d^{MNP} \partial_N \otimes \partial_P = 0.$$

- Two inequivalent solutions

$$\begin{aligned} 11\text{-d: } \partial_M &\rightarrow \left\{ \partial_i, \cancel{\partial^{ij}}, \cancel{\partial^{ijklm}} \right\} \\ \text{IIB: } \partial_M &\rightarrow \left\{ \partial_u, \cancel{\partial^{uvw}}, \cancel{\partial^\alpha}, \cancel{\partial^{ua}} \right\} \end{aligned}$$

- Covariant restriction to 6 (11-d SUGRA) or 5 (IIB SUGRA) coordinates.

Rewriting the action

Full 10-d/11-d action: [Hohm, Samtleben '13]

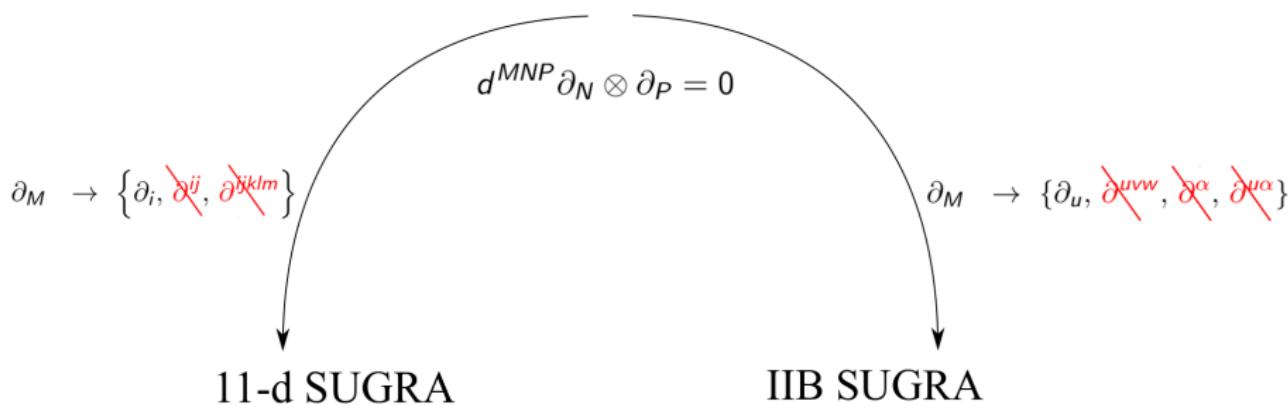
$$L = R_g + \frac{1}{24} g^{\mu\nu} \mathfrak{D}_\mu \mathcal{M}^{MN} \mathfrak{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{F}_{\mu\nu}^{ M} \mathcal{F}^{\mu\nu}{}^N \mathcal{M}_{MN} + L_{top} - V,$$

- “Like 5-d SUGRA”
- $\mathfrak{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$ “external covariant derivatives”
- $\mathcal{F}_{\mu\nu}^{ M} = 2\partial_{[\mu} A_{\nu]}^{ M} - 2\mathcal{L}_{A_{[\mu}} A_{\nu]}^{ M} + 10d^{MNP} \partial_N B_{\mu\nu P}$
- “Scalar potential”:

$$V = \mathcal{M}^{MN} \mathcal{R}_{MN} = \mathcal{M}^{MN} \partial_M \mathcal{M}^{PQ} \partial_N \mathcal{M}_{PQ} + \dots$$

$E_{6(6)}$ ExFT

$$L = R_g + \frac{1}{24} g^{\mu\nu} \mathfrak{D}_\mu \mathcal{M}^{MN} \mathfrak{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{F}_{\mu\nu}^{ M} \mathcal{F}^{\mu\nu}{}^N \mathcal{M}_{MN} + L_{top} - V$$

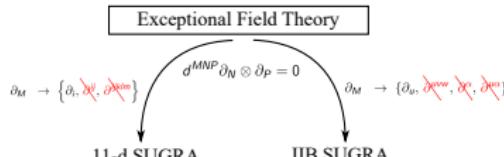


Similar story for $M_{10/11} = M_{int} \times M_D \rightarrow E_{11-D(11-D)} \text{ ExFT}$

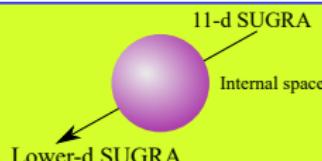
Motivation



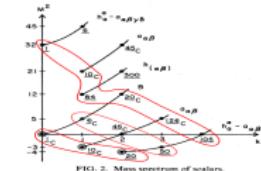
Exceptional Field Theory



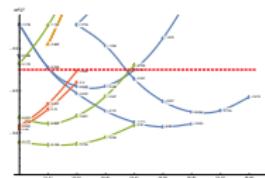
Consistent truncations



Kaluza-Klein Spectrometry



Applications



Consistent truncation

- Non-linear truncation to subset of KK modes
 - Described by lower-dim max SUGRA

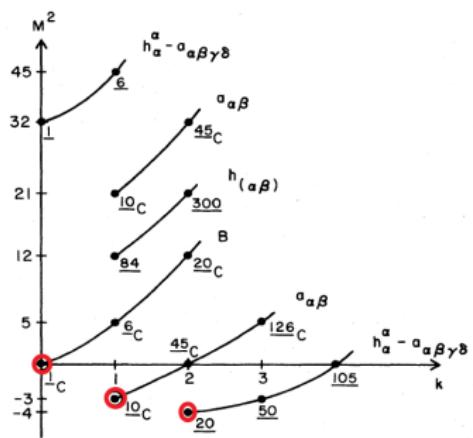
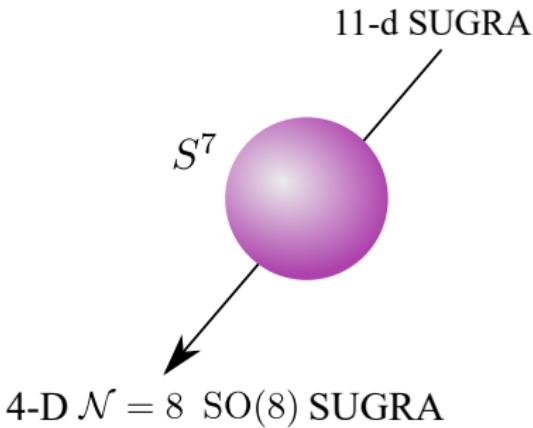


FIG. 2. Mass spectrum of scalars.



Consistent truncations = generalised Scherk-Schwarz

- Generalised Scherk-Schwarz Ansatz: $U_M{}^A(Y) \in E_{d(d)}$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x) U_M{}^A(Y) U_N{}^B(Y),$$

$$\mathcal{A}_\mu{}^M(x, Y) = A_\mu{}^A(x) U_A{}^M(Y)$$

- Consistency condition:

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C.$$

- Embedding tensor $X_{AB}{}^C \iff$ lower-dim gauge group.

Proof of consistency

- All Y -dependence in action & EoMs appears via \mathcal{L}

$$\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$$
$$\mathcal{F}_{\mu\nu}^M = 2\partial_{[\mu} A_{\nu]}^M - 2\mathcal{L}_{A_{[\mu}} A_{\nu]}^M + \dots$$

$$\mathcal{M}_{MN}(x, Y) = \mathcal{M}_{AB}(x) U_M^A(Y) U_N^B(Y),$$
$$A_\mu^M(x, Y) = A_\mu^A(x) U_A^M(Y),$$

$$D_\mu = \partial_\mu - A_\mu^A X_A.$$

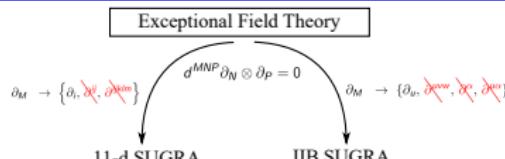
$$F_{\mu\nu}^A = 2\partial_{[\mu} A_{\nu]}^A - 2A_\mu^B A_\nu^C X_{BC}^A + \dots$$

- All Y -dependence factors out \Rightarrow consistent truncation

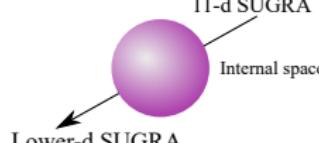
Motivation



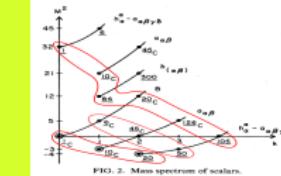
Exceptional Field Theory



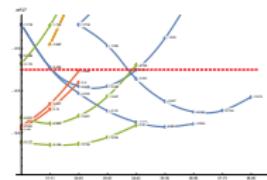
Consistent truncations



Kaluza-Klein Spectrometry

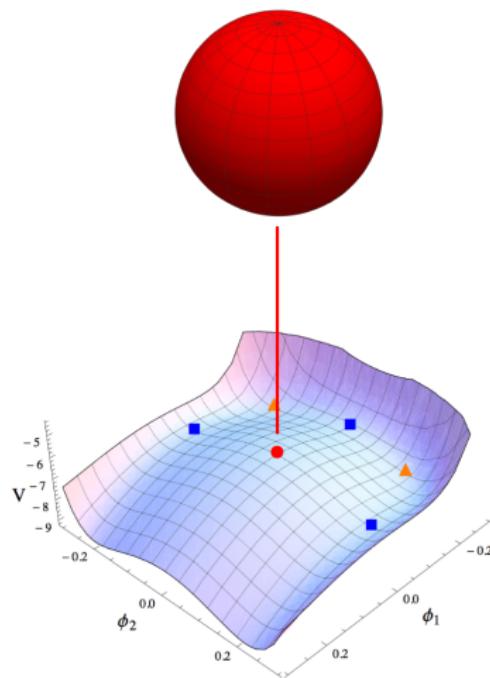


Applications



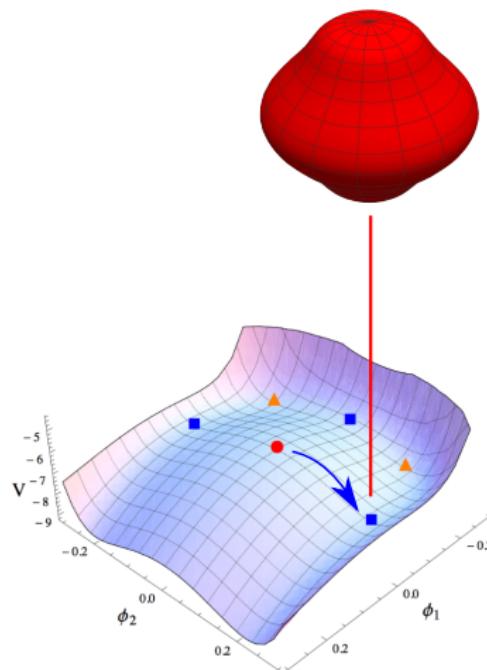
KK spectrometry

First at max symmetric point: $M_{AB}(x) = \delta_{AB}$



KK spectrometry

Then at less symmetric point: $M_{AB}(x) = \mathcal{V}_A^{\bar{A}} \mathcal{V}_B^{\bar{B}} \delta_{\bar{A}\bar{B}}$



KK Ansatz at symmetric point

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= M_{AB}(x) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^A(x) U_A{}^M(Y)\end{aligned}$$

- KK fluctuation: infinitesimal $M_{AB} = \delta_{AB} + j_{AB} \in E_{d(d)}/H_d$
- Expand in scalar harmonics \mathcal{Y}^Σ on M_{int}

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= \left(\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma(Y) \right) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{Y}_\Sigma(Y).\end{aligned}$$

- KK Ansatz = consistent truncation \otimes scalar harmonics

KK Ansatz at symmetric point

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= M_{AB}(x) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^A(x) U_A{}^M(Y)\end{aligned}$$

- KK fluctuation: infinitesimal $M_{AB} = \delta_{AB} + j_{AB} \in \mathfrak{e}_{d(d)} \ominus \mathfrak{h}_d$
- Expand in scalar harmonics \mathcal{Y}^Σ on M_{int}

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= \left(\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma(Y) \right) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{Y}_\Sigma(Y).\end{aligned}$$

- KK Ansatz = consistent truncation \otimes scalar harmonics

KK Ansatz at symmetric point

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= M_{AB}(x) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^A(x) U_A{}^M(Y)\end{aligned}$$

- KK fluctuation: infinitesimal $M_{AB} = \delta_{AB} + j_{AB} \in \mathfrak{e}_{d(d)} \ominus \mathfrak{h}_d$
- Expand in scalar harmonics \mathcal{Y}^Σ on M_{int}

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= \left(\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma(Y) \right) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^A{}^\Sigma(x) U_A{}^M(Y) \mathcal{Y}_\Sigma(Y).\end{aligned}$$

- KK Ansatz = consistent truncation \otimes scalar harmonics

c.f. $g_{ij}(x, y) = \sum_\ell g^{(\ell)}(x) \mathcal{Y}_{ij}^{(\ell)}(y)$, $C_{ijk}(x, y) = \sum_\ell C^{(\ell)}(x) \mathcal{Y}_{ijk}^{(\ell)}(y)$

$$g_{\mu i}(x, y) = \sum_\ell g_\mu^{(\ell)}(x) \mathcal{Y}_i^{(\ell)}(y)$$
, $C_{\mu ij}(x, y) = \sum_\ell C_\mu^{(\ell)}(x) \mathcal{Y}_{ij}^{(\ell)}(y)$

KK Ansatz at symmetric point

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= M_{AB}(x) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^A(x) U_A{}^M(Y)\end{aligned}$$

- KK fluctuation: infinitesimal $M_{AB} = \delta_{AB} + j_{AB} \in \mathfrak{e}_{d(d)} \ominus \mathfrak{h}_d$
- Expand in scalar harmonics \mathcal{Y}^Σ on M_{int}

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= \left(\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma(Y) \right) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_\mu{}^M(x, Y) &= A_\mu{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{Y}_\Sigma(Y).\end{aligned}$$

- KK Ansatz = consistent truncation \otimes scalar harmonics

Useful to write $j_{AB}^\Sigma = \mathcal{P}_{AB}{}^I j_I{}^\Sigma$, \rightarrow Projector onto $\mathfrak{e}_{d(d)} \ominus \mathfrak{h}_d$.

Mass matrices at symmetric point

- Plug

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + j_{AB}^{\Sigma}(x) \mathcal{V}_{\Sigma}(Y)) U_M{}^A(Y) U_N{}^B(Y),$$

$$A_{\mu}{}^M(x, Y) = A_{\mu}{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{V}_{\Sigma}(Y).$$

into ExFT action

$$L = R_g + \frac{1}{24} g^{\mu\nu} \mathfrak{D}_{\mu} \mathcal{M}^{MN} \mathfrak{D}_{\nu} \mathcal{M}_{MN} - \frac{1}{4} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu}{}^N \mathcal{M}_{MN} + L_{top} - V,$$

and linearise \Rightarrow mass matrices

Mass matrices at symmetric point

- Plug

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= (\delta_{AB} + j_{AB}^{\Sigma}(x) \mathcal{Y}_{\Sigma}(Y)) U_M{}^A(Y) U_N{}^B(Y), \\ A_{\mu}{}^M(x, Y) &= A_{\mu}{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{Y}_{\Sigma}(Y).\end{aligned}$$

into ExFT action

$$L = R_g + \frac{1}{24} g^{\mu\nu} \mathfrak{D}_{\mu} \mathcal{M}^{MN} \mathfrak{D}_{\nu} \mathcal{M}_{MN} - \frac{1}{4} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu}{}^N \mathcal{M}_{MN} + L_{top} - V,$$

and linearise \Rightarrow mass matrices

- All internal dependence through:

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C,$$

$$\mathcal{L}_{U_A} \mathcal{Y}_{\Sigma} = L_{\mathcal{K}_A} \mathcal{Y}_{\Sigma} = \mathcal{T}_{A\Sigma}{}^{\Omega} \mathcal{Y}_{\Omega}.$$

- $\mathcal{T}_{A\Sigma}{}^{\Omega}$ generators of gauge group: $[\mathcal{T}_A, \mathcal{T}_B] = X_{AB}{}^C \mathcal{T}_C$

Mass matrices at symmetric point

- Plug

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= (\delta_{AB} + j_{AB}^{\Sigma}(x) \mathcal{Y}_{\Sigma}(Y)) U_M{}^A(Y) U_N{}^B(Y), \\ A_{\mu}{}^M(x, Y) &= A_{\mu}{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{Y}_{\Sigma}(Y).\end{aligned}$$

into ExFT action

$$L = R_g + \frac{1}{24} g^{\mu\nu} \mathfrak{D}_{\mu} \mathcal{M}^{MN} \mathfrak{D}_{\nu} \mathcal{M}_{MN} - \frac{1}{4} \mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu}{}^N \mathcal{M}_{MN} + L_{top} - V,$$

and linearise \Rightarrow mass matrices

- All internal dependence through:

$$\begin{aligned}\mathcal{L}_{U_A} U_B &= X_{AB}{}^C U_C, \\ \mathcal{L}_{U_A} \mathcal{Y}_{\Sigma} &= L_{\mathcal{K}_A} \mathcal{Y}_{\Sigma} = \mathcal{T}_{A\Sigma}{}^{\Omega} \mathcal{Y}_{\Omega}.\end{aligned}$$

- $\mathcal{T}_{A\Sigma}{}^{\Omega}$ generators of gauge group: $[\mathcal{T}_A, \mathcal{T}_B] = X_{AB}{}^C \mathcal{T}_C$
- \Rightarrow Mass² $\sim \mathcal{T}^2$, $\mathcal{T} \cdot X$ and X^2

Mass matrices at symmetric point

Mass matrices:

$$\mathbb{M}_{I\Sigma, J\Omega}^{(\text{scalar})} = \mathbb{M}_{IJ}^{(0)} \delta_{\Sigma\Omega} + \delta_{IJ} \mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} + \mathcal{N}_{IJ} \mathcal{C} \mathcal{T}_{C,\Omega\Sigma}$$

Mass matrices at symmetric point

Mass matrices:

$$\mathbb{M}_{I\Sigma, J\Omega}^{(\text{scalar})} = \mathbb{M}_{IJ}^{(0)} \delta_{\Sigma\Omega} + \delta_{IJ} \mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} + \mathcal{N}_{IJ} \mathcal{C} \mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}_{IJ}^{(0)}$

Mass matrices at symmetric point

Mass matrices:

$$\mathbb{M}_{I\Sigma, J\Omega}^{(\text{scalar})} = \mathbb{M}_{IJ}^{(0)} \delta_{\Sigma\Omega} + \delta_{IJ} \mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} + \mathcal{N}_{IJ}{}^C \mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}_{IJ}^{(0)}$

$$\begin{aligned}\mathbb{M}_{IJ}^{(0)} &= \frac{1}{7} \left(X_{AE}{}^F X_{BE}{}^F + X_{EA}{}^F X_{EB}{}^F + X_{EF}{}^A X_{EF}{}^B + 7 X_{AE}{}^F X_{BF}{}^E \right) \mathcal{P}_{AD}{}^I \mathcal{P}_{BD}{}^J \\ &+ \frac{2}{7} \left(X_{AC}{}^E X_{BD}{}^E - X_{AE}{}^C X_{BE}{}^D - X_{EA}{}^C X_{EB}{}^D \right) \mathcal{P}_{AB}{}^I \mathcal{P}_{CD}{}^J \\ &+ \frac{1}{6} \mathcal{P}_{AB}{}^I \mathcal{P}_{CD}{}^J X_{FA}{}^B X_{FC}{}^D.\end{aligned}$$

Mass matrices at symmetric point

Mass matrices:

$$\mathbb{M}_{I\Sigma, J\Omega}^{(\text{scalar})} = \mathbb{M}_{IJ}^{(0)} \delta_{\Sigma\Omega} + \delta_{IJ} \mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} + \mathcal{N}_{IJ} \mathcal{C} \mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}_{IJ}^{(0)}$
- Spin-2 mass matrix $\mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} = \mathcal{T}_{A,\Sigma\Lambda} \mathcal{T}_{A,\Lambda\Omega}$

Mass matrices at symmetric point

Mass matrices:

$$\mathbb{M}_{I\Sigma, J\Omega}^{(\text{scalar})} = \mathbb{M}_{IJ}^{(0)} \delta_{\Sigma\Omega} + \delta_{IJ} \mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} + \mathcal{N}_{IJ}{}^C \mathcal{T}_{C,\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}_{IJ}^{(0)}$
- Spin-2 mass matrix $\mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} = \mathcal{T}_{A,\Sigma\Lambda} \mathcal{T}_{A,\Lambda\Omega}$
- Key object:

$$\mathcal{N}_{IJ}{}^C = -4(X_{CA}{}^B + 12 X_{AB}{}^C) \mathcal{P}^{AD} [{}_I \mathcal{P}^{BD} {}_J].$$

KK spectrum on S^7

Compact (re-)derivation of KK spectrum on $\text{AdS}_4 \times S^7$:

- $\mathcal{Y}_a \mathcal{Y}^a = 1$
- Harmonics $\mathcal{Y}_{\Sigma} = \{1, \mathcal{Y}_a, \mathcal{Y}_{(a} \mathcal{Y}_{b)}, \dots\} \in [n, 0, 0, 0]$
- $j_{AB}^{\Sigma} \in ([2, 0, 0, 0] \oplus [0, 2, 0, 0]) \otimes [n, 0, 0, 0]$

$$L^2 \mathbb{M}_{(\text{scalar})}^2 = \frac{3}{4}(n+2)(n+4) + \mathcal{C}_{\text{SO}(8)}$$

- In terms of SO(8) Casimir $\longrightarrow \mathcal{C}_{\text{SO}(8)}$ & KK level n

KK spectrum on S^7

Compact (re-)derivation of KK spectrum on $\text{AdS}_4 \times S^7$:

- $\mathcal{Y}_a \mathcal{Y}^a = 1$
- Harmonics $\mathcal{Y}_\Sigma = \{1, \mathcal{Y}_a, \mathcal{Y}_{((a}\mathcal{Y}_{b))}, \dots\} \in [n, 0, 0, 0]$
- $j_{AB}^\Sigma \in ([2, 0, 0, 0] \oplus [0, 2, 0, 0]) \otimes [n, 0, 0, 0]$
- $[n, 0, 0, 0] \rightarrow 1 \text{ BPS multiplet: Mixing from } U_A{}^M \mathcal{Y}_\Sigma + \text{ExFT} \leftrightarrow 11\text{-d dictionary}$

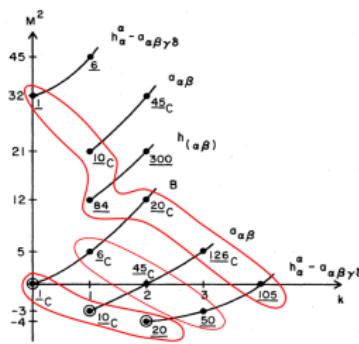
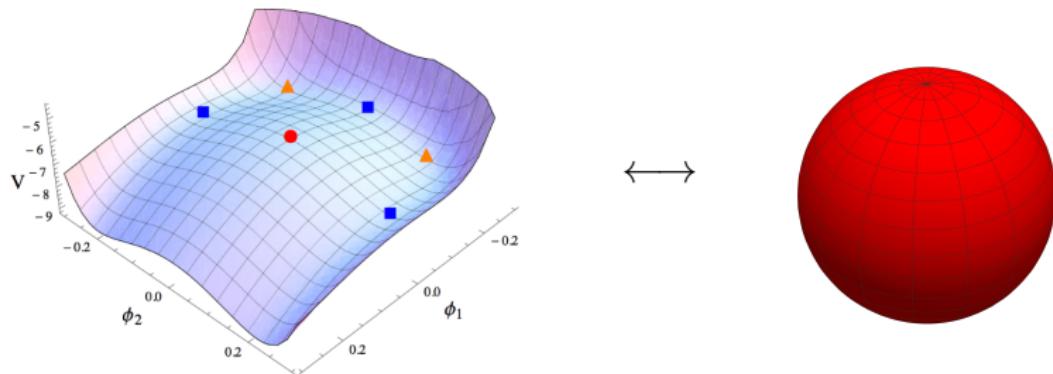


FIG. 2. Mass spectrum of scalars.

Moving to a less symmetric vacuum

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= \left(\delta_{AB} + j_{AB}^{\Sigma}(x) \mathcal{Y}_{\Sigma}(Y) \right) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_{\mu}{}^M(x, Y) &= A_{\mu}{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{Y}_{\Sigma}(Y).\end{aligned}$$

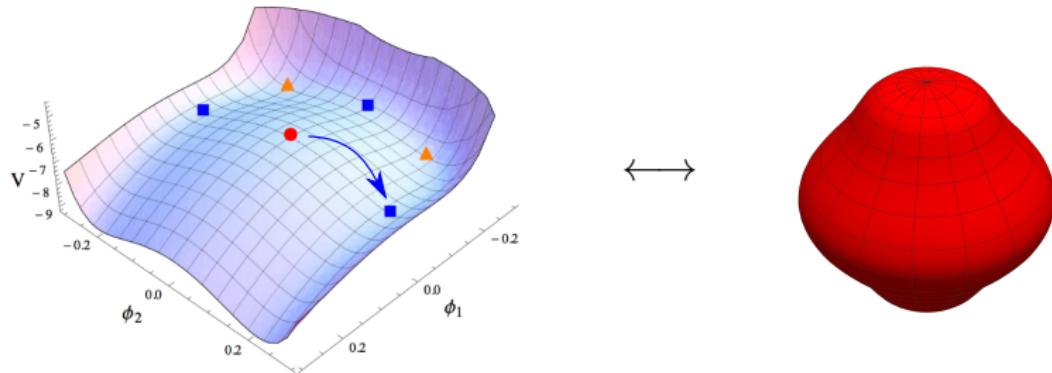
- KK Ansatz is exact in consistent truncation modes!
- Vacuum defined by scalars $\mathcal{V}^{\bar{A}}{}_A(x) \in E_{d(d)}/H_d$
- Use same harmonics \mathcal{Y}_{Σ} as max. symmetric vacuum.



Moving to a less symmetric vacuum

$$\begin{aligned}\mathcal{M}_{MN}(x, Y) &= (\delta_{AB} + j_{AB}^{\Sigma}(x) \mathcal{Y}_{\Sigma}(Y)) U_M{}^A(Y) U_N{}^B(Y), \\ \mathcal{A}_{\mu}{}^M(x, Y) &= A_{\mu}{}^{A\Sigma}(x) U_A{}^M(Y) \mathcal{Y}_{\Sigma}(Y).\end{aligned}$$

- KK Ansatz is exact in consistent truncation modes!
- Vacuum defined by scalars $\mathcal{V}^{\bar{A}}{}_A(x) \in E_{d(d)}/H_d$
- Use same harmonics \mathcal{Y}_{Σ} as max. symmetric vacuum.



Kaluza-Klein masses at less symmetric vacuum

Mass matrix at less symmetric vacuum

$$\mathbb{M}_{I\Sigma,J\Omega}^{(\text{scalar})} = \mathbb{M}_{IJ}^{(0)} \delta_{\Sigma\Omega} + \delta_{IJ} \bar{\mathbb{M}}_{\Sigma\Omega}^{(\text{spin}-2)} + \mathcal{N}_{IJ} \bar{C} T_{\bar{C},\Omega\Sigma}$$

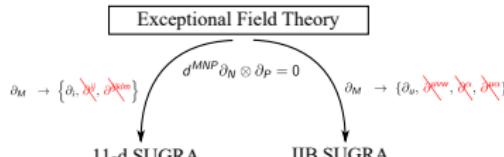
in terms of “dressed” quantities:

$$X_{\bar{A}\bar{B}}^{\bar{C}} = \mathcal{V}_{\bar{A}}^A \mathcal{V}_{\bar{B}}^B \mathcal{V}^{\bar{C}}_C X_{AB}^C, \quad T_{\bar{A},\Omega}^\Sigma = \mathcal{V}_{\bar{A}}^A T_{A\Omega}^\Sigma.$$

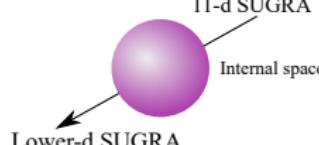
Motivation



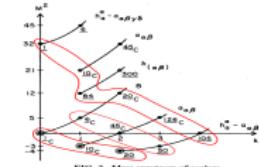
Exceptional Field Theory



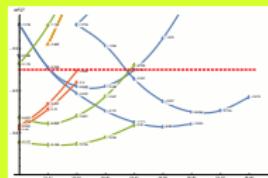
Consistent truncations



Kaluza-Klein Spectrometry



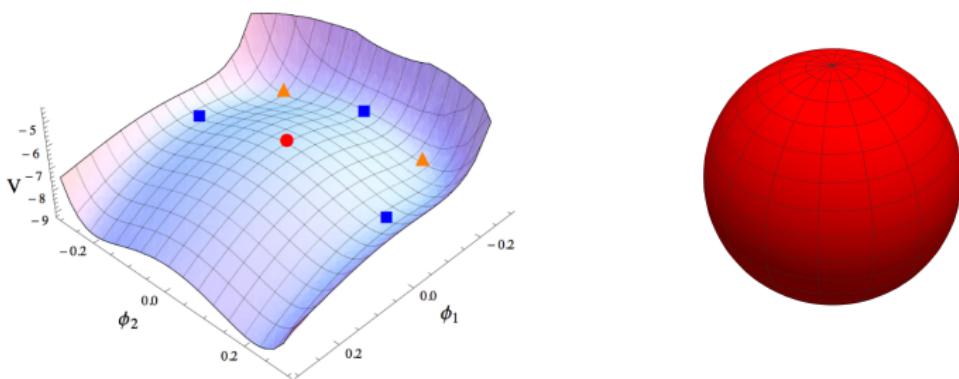
Applications



Non-supersymmetric $\text{SO}(3) \times \text{SO}(3)$ AdS₄

4-D SO(8) gSUGRA

- SO(3) \times SO(3)-invariant $\mathcal{N} = 0$ AdS₄ vacuum [Warner '83]

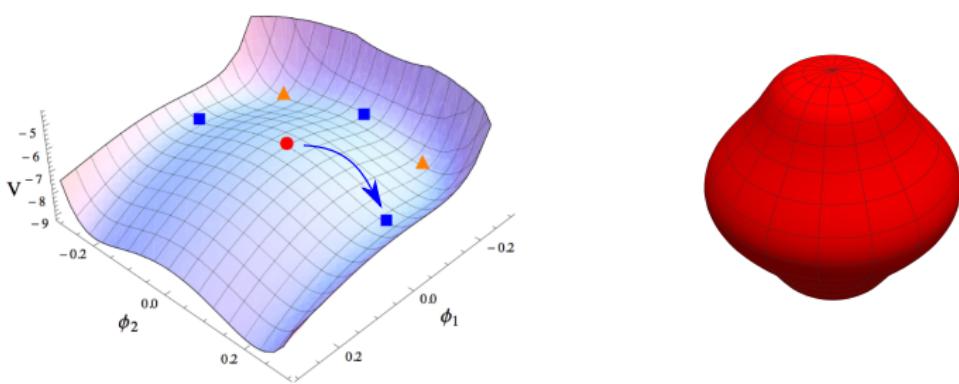


- Uniquely fully stable in 4-D! [Fishbacher, Pilch, Warner '10]
- Uplift to 11-d [Godazgar, Godazgar, Kruger, Nicolai, Pilch '14]

Non-supersymmetric $\text{SO}(3) \times \text{SO}(3)$ AdS₄

4-D SO(8) gSUGRA

- SO(3) \times SO(3)-invariant $\mathcal{N} = 0$ AdS₄ vacuum [Warner '83]



- Uniquely fully stable in 4-D! [Fishbacher, Pilch, Warner '10]
- Uplift to 11-d [Godazgar, Godazgar, Kruger, Nicolai, Pilch '14]

Stability in 11-d?

- Uplift to 11-d stable? Higher KK modes?
- AdS swampland conjecture [Ooguri, Vafa '16]
- Brane-Jet instability [Bena, Pilch, Warner '20]

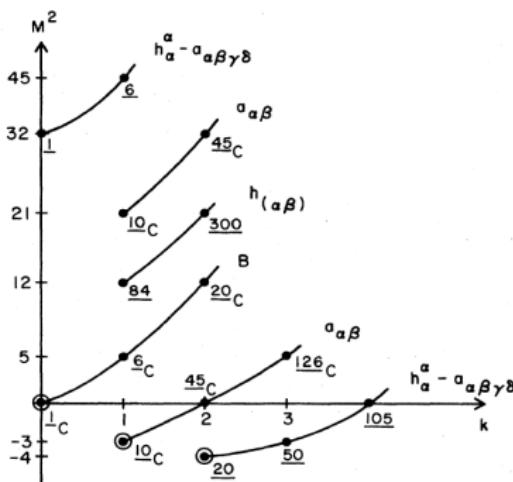
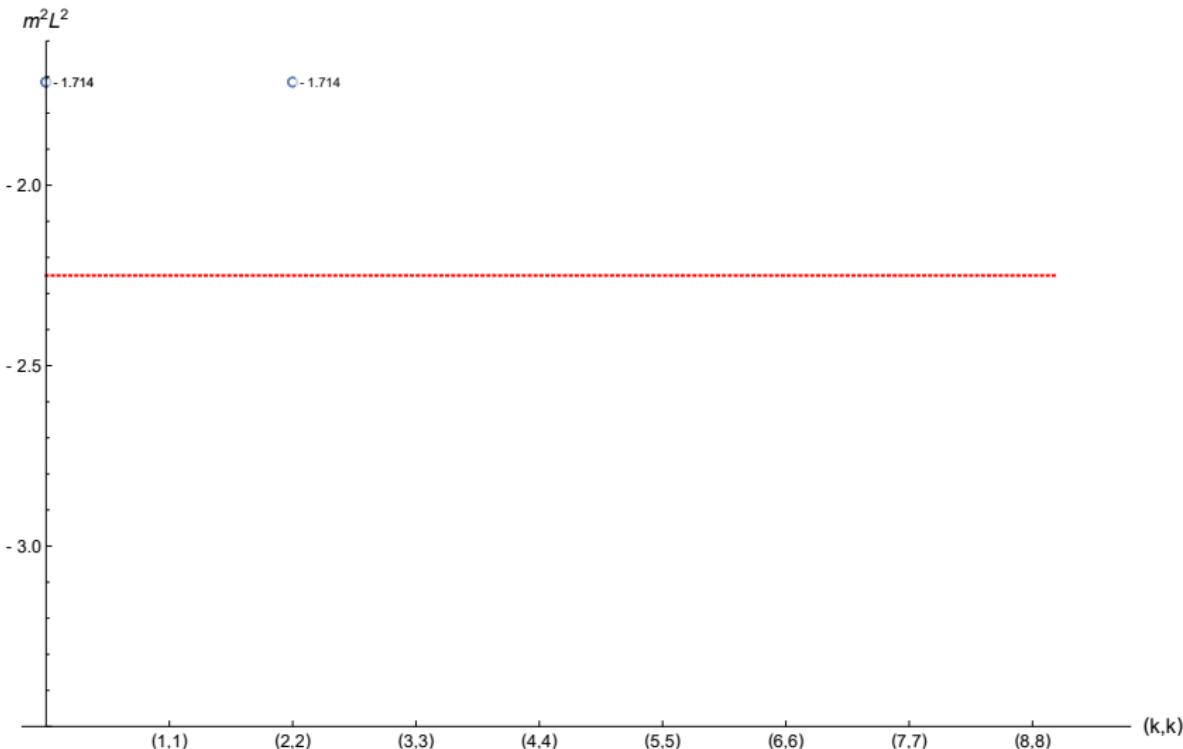


FIG. 2. Mass spectrum of scalars.

Tachyonic KK modes

Level 0: $\mathcal{N} = 8$ supergravity multiplets

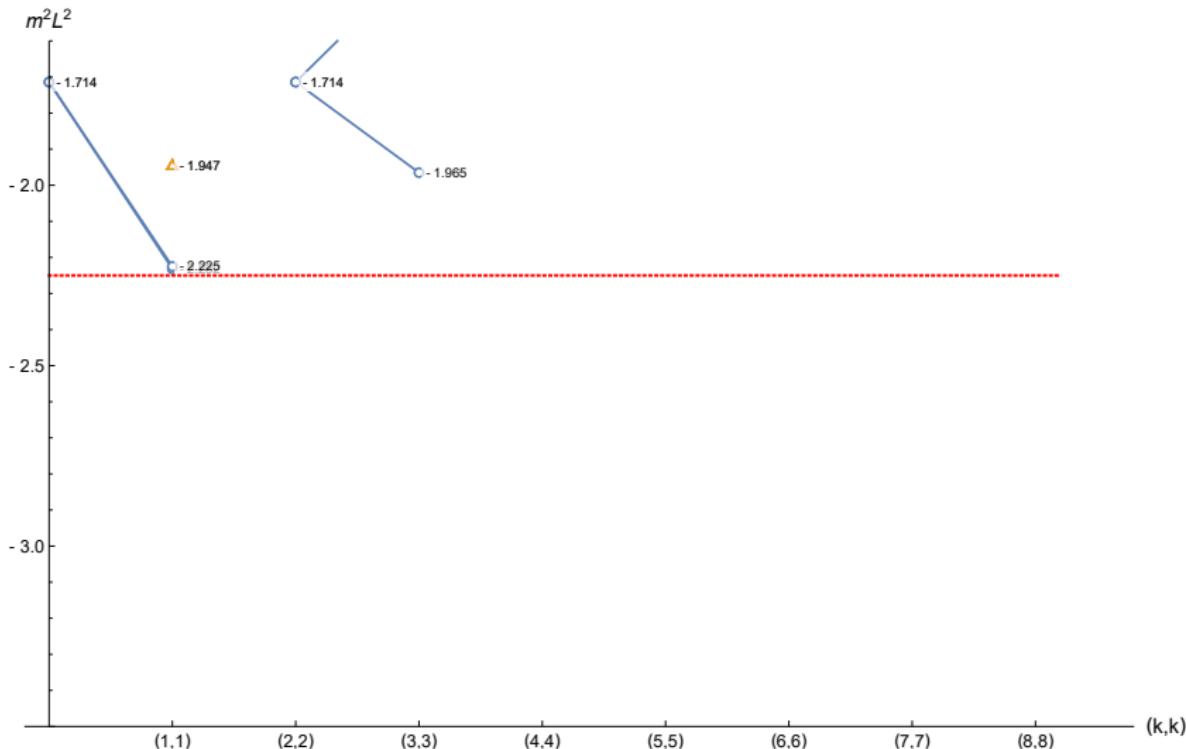
[Fischbacher, Pilch, Warner '10]



Tachyonic KK modes

Level 1: still stable!

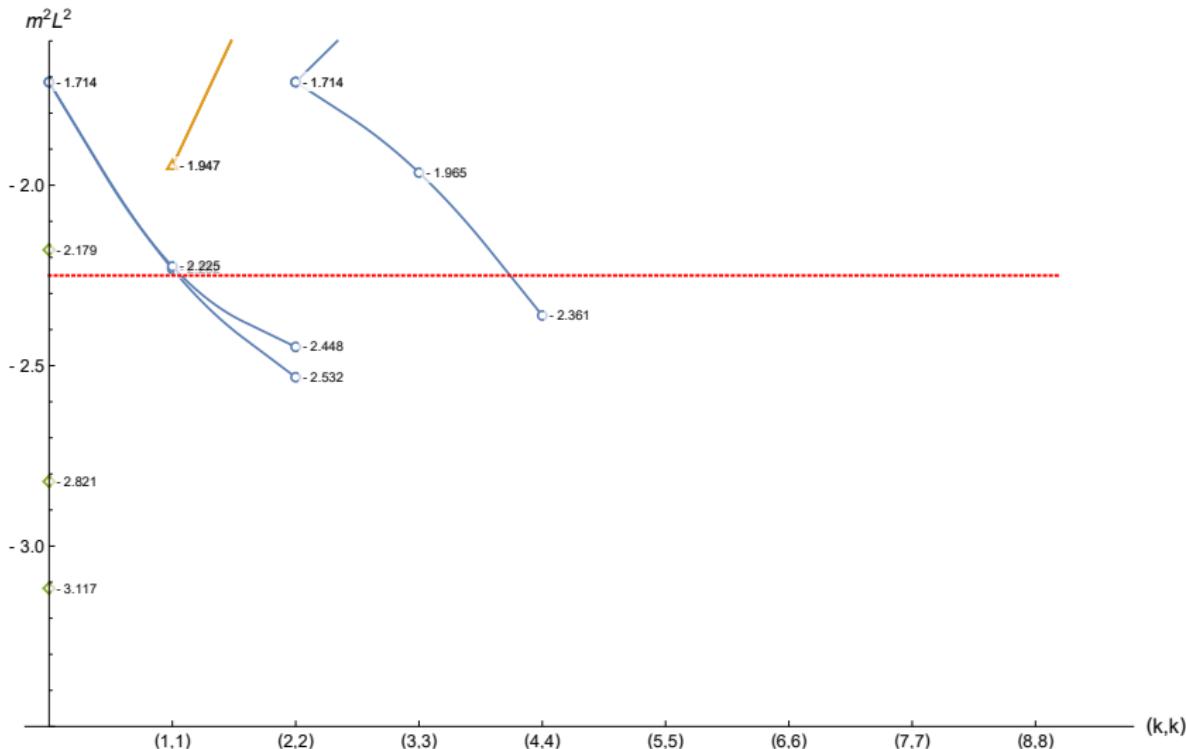
[EM, Nicolai, Samtleben: 2005.07713]



Tachyonic KK modes

Level 2: tachyons!

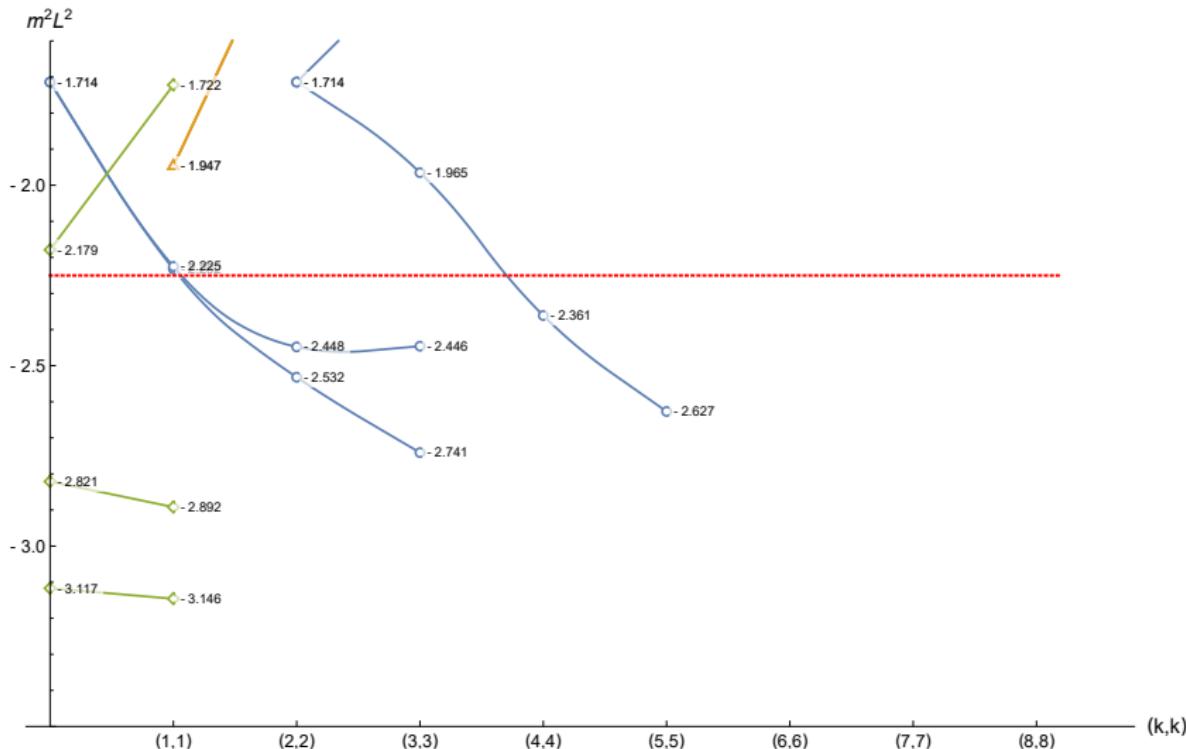
[EM, Nicolai, Samtleben: 2005.07713]



Tachyonic KK modes

Level 3

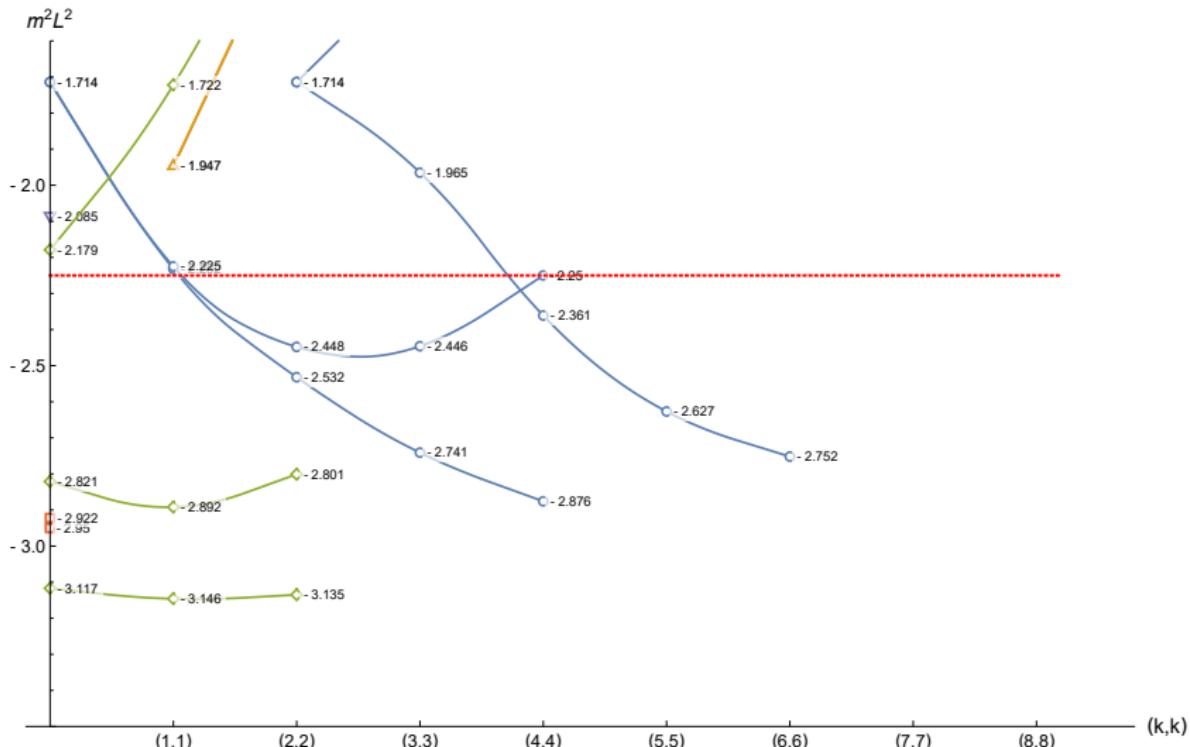
[EM, Nicolai, Samtleben: 2005.07713]



Tachyonic KK modes

Level 4

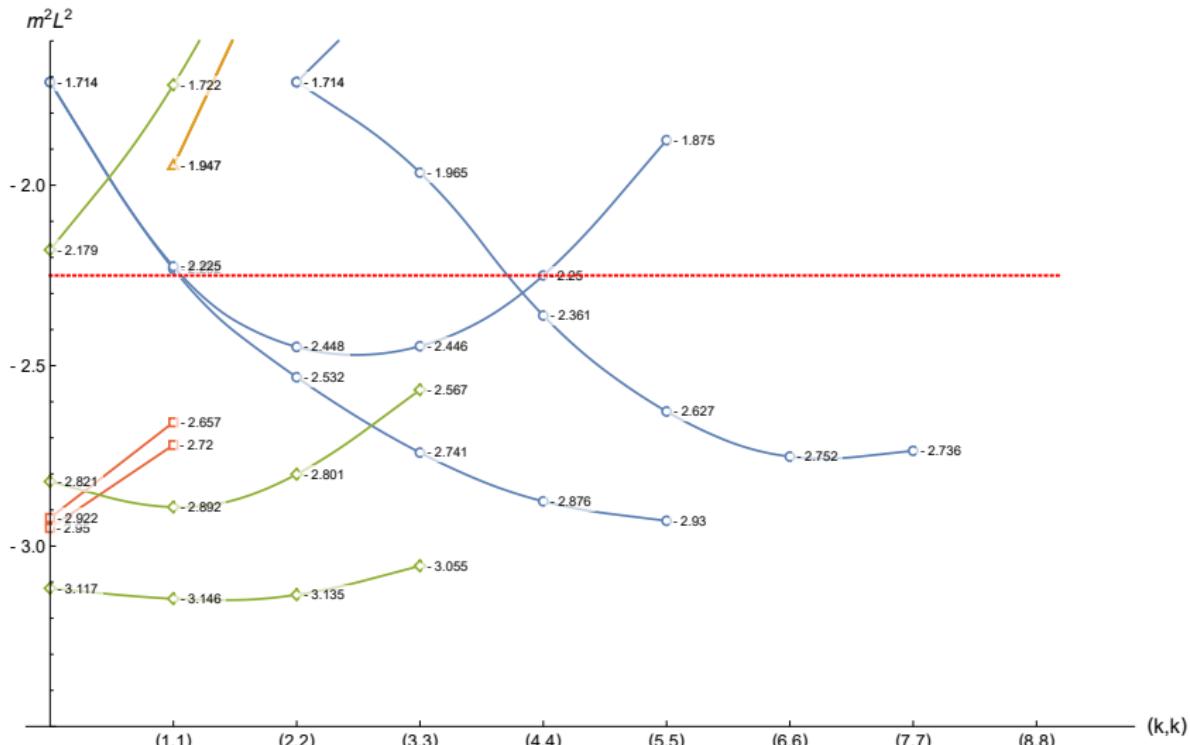
[EM, Nicolai, Samtleben: 2005.07713]



Tachyonic KK modes

Level 5

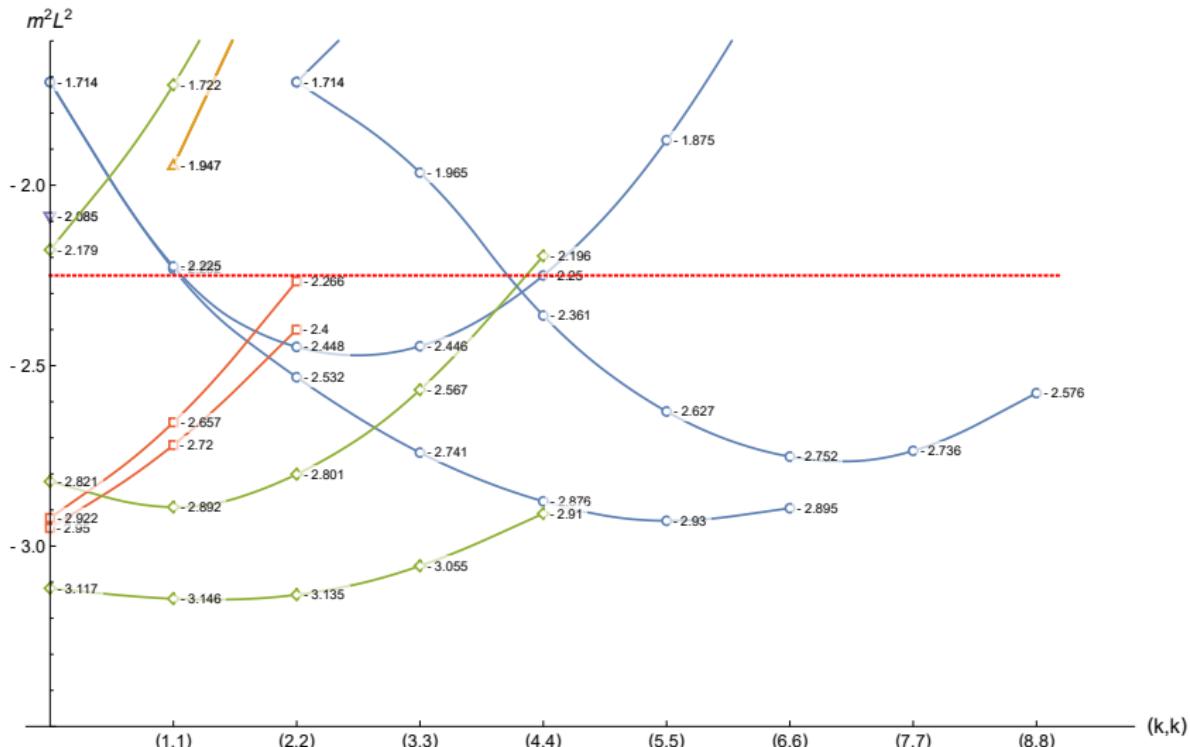
[EM, Nicolai, Samtleben: 2005.07713]



Tachyonic KK modes

Level 6

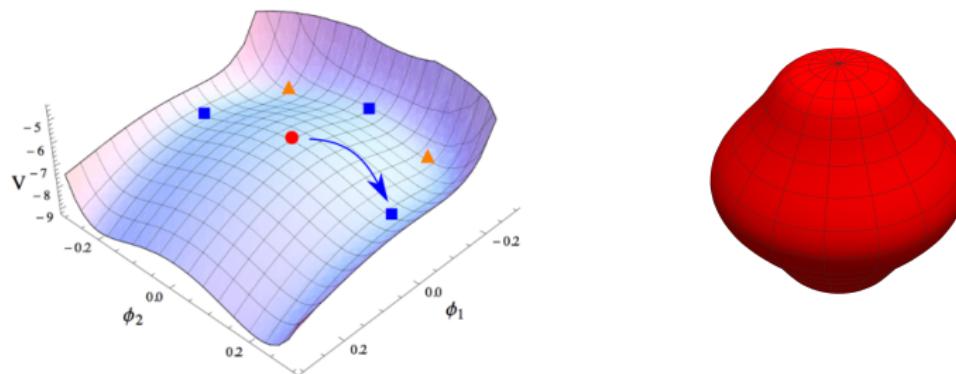
[EM, Nicolai, Samtleben: 2005.07713]



Non-supersymmetric AdS₄ in ISO(7) SUGRA

4-D ISO(7) gSUGRA

- G_2 -invariant $\mathcal{N} = 0$ AdS₄ vacuum + 6 other
- Consistent truncation of massive IIA on S^6



- Fully stable in 4-D! [Guarino, Tarrio, Varela '20], [Fischbacher, Pilch, Warner '10]
- No non-perturbative instabilities found (e.g. brane-jet instabilities, bubbles of nothing)

Perturbative stability of $G_2 \mathcal{N} = 0$ AdS₄ vacuum

[Guarino, EM, Samtleben: 2011.06600]

- For G_2 invariant vacuum, analytic KK spectrum:

$$L^2 \mathbb{M}_{(\text{scalar})}^2 = (n+2)(n+3) - \frac{3}{2} \mathcal{C}_{G_2} .$$

- In terms of: S^6 KK level n , G_2 Casimir $\leftrightarrow \mathcal{C}_{G_2}$
- Positive mass spectrum!
- AdS Swampland Conjecture?!

Perturbative stability in 10 dimensions

[Guarino, EM, Samtleben: 2011.06600]

- 6 other non-SUSY AdS₄ vacua
- Numerical evaluation up to level $n = 4$:
 - no tachyons
 - lowest-lying masses increase monotonically with level
- Evidence for perturbative stability in 10-d.

AdS₄ S-fold vacua

4-D $[\text{SO}(6) \times \text{SO}(1, 1)] \ltimes \mathbb{R}^{12}$ gSUGRA

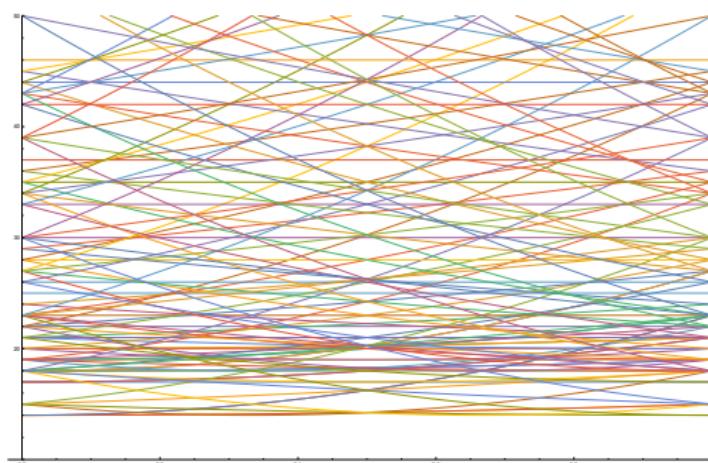
- Family of $\mathcal{N} = 2$ AdS₄ vacua, parameterised by $\chi \in \mathbb{R}$
[\[Guarino, Sterckx, Trigiante '20\]](#)
- In 4-D:
 - Unique special point: $\chi = 0 \rightarrow \text{SU}(2) \times \text{U}(1)_R$ symmetry
 - $\chi \neq 0$ distinct AdS₄ vacua with $\text{U}(1) \times \text{U}(1)_R$ symmetry
- Uplift via consistent truncation: [\[Inverso, Samtleben, Trigiante '16\]](#)

$$\text{AdS}_4 \times S^5 \times S_T^1 \quad \text{S-fold of IIB}$$

Global properties of the conformal manifold

KK spectrum as function of χ [Giambrone, EM, Samtleben, Trigiante: 2003.xxxxx]

- $\chi \sim \chi + \frac{2\pi}{T}$ periodicity, T radius of S_T^1



- At $\chi = \frac{p\pi}{T}$, $p \in \mathbb{Z}$, KK modes at level p on S_T^1 become massless
- Space invaders \rightarrow enhancement to $SU(2) \times U(1)_R$ symmetry

Global properties of the conformal manifold

[Giambrone, EM, Samtleben, Trigiante: 2003.xxxxx]

- $\chi \in \mathbb{R}^+$ is a 4-D artefact
- $\chi \in \left[0, \frac{2\pi}{T}\right)$ in 10 dimensions
- Periodicity \leftrightarrow complex structure of internal space

$$S^5 \times S^1 \sim (S^3 \times S^1) \times S^2$$

- $\chi = \frac{\pi}{T}$ is distinct but also $SU(2) \times U(1)_R$ symmetry
- Generic χ has $U(1) \times U(1)_R$ symmetry

Conclusions

- Powerful new method for computing Kaluza-Klein spectra
- KK spectrum of less symmetric vacua
- Non-SUSY $SO(3) \times SO(3)$ AdS₄ vacuum is unstable
c.f. Brane-Jet instability [[Bena, Pilch, Warner '20](#)]
- 7 perturbatively stable AdS₄ vacua of massive IIA supergravity \leftrightarrow AdS Swampland Conjecture?!
- Global properties of conformal manifold

Outlook

- Non-SUSY vacua
 - Other examples in 3-D [[Fischbacher '02](#)], [[Fischbacher, Nicolai, Samtleben '02](#)]
 - Generic requirements for (in)stability?
- KK spectrum of SUSY AdS vacua: Predictions and checks of AdS/CFT
- Extension to consistent truncations with less SUSY
⇒ KK spectrum of 1/2-max AdS vacua, c.f. spin-2 [[Chen, Gutperle, Uhlemann '19](#)], [[Gutperle, Uhlemann, Varela '18](#)]
- Beyond AdS?