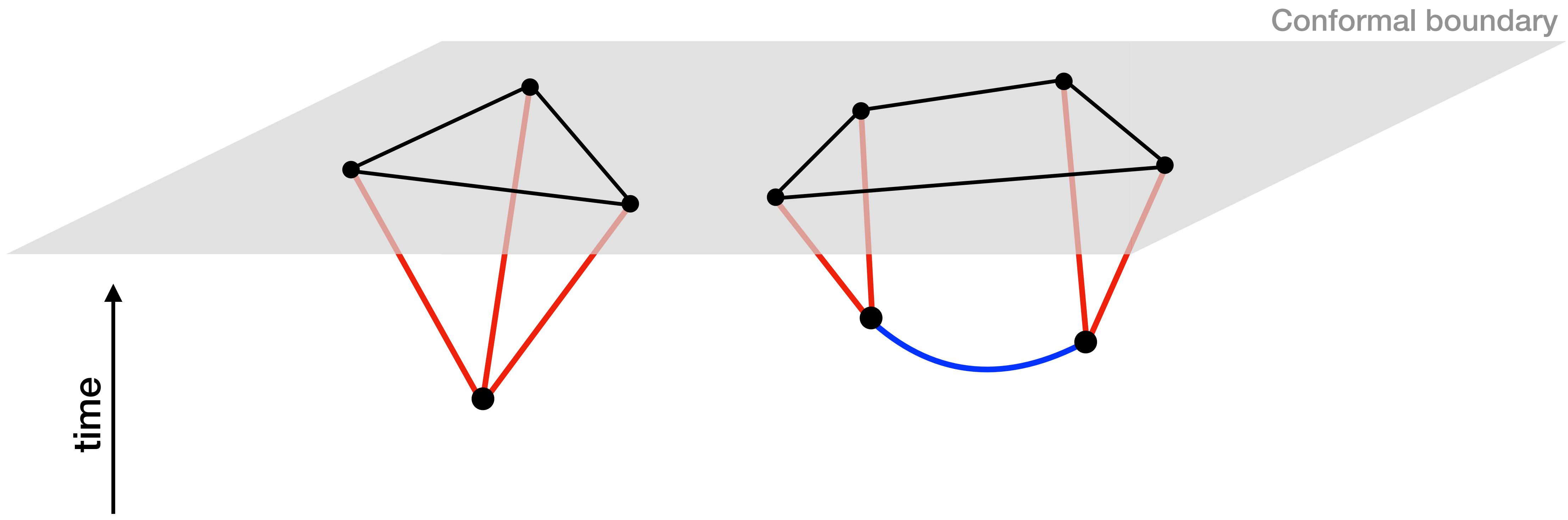
A Mellin Space Approach to Scattering in de Sitter

Charlotte Sleight

IAS

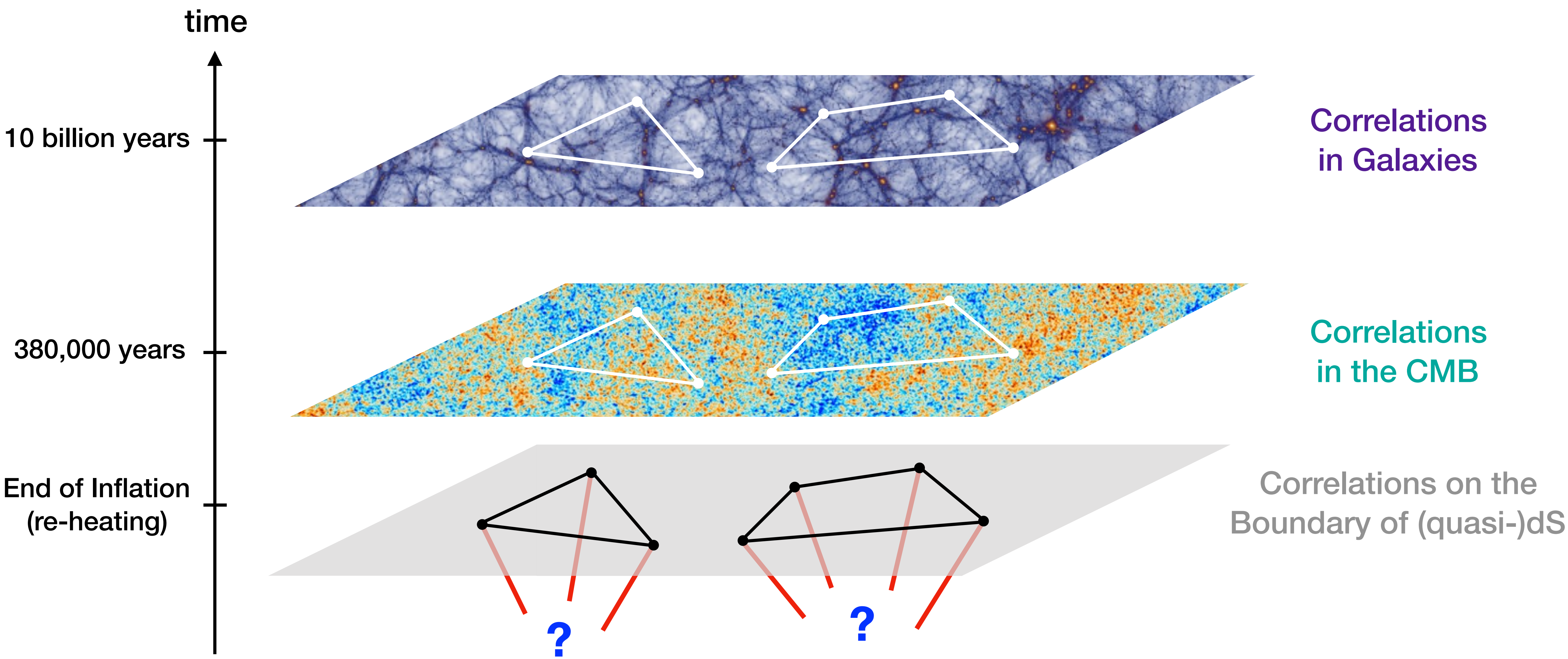
Mostly based on: 1906.12302, and 1907.01143 with M. Taronna

Boundary Correlators in de Sitter



Cosmological Correlators

In Cosmology we measure spatial correlations at late times



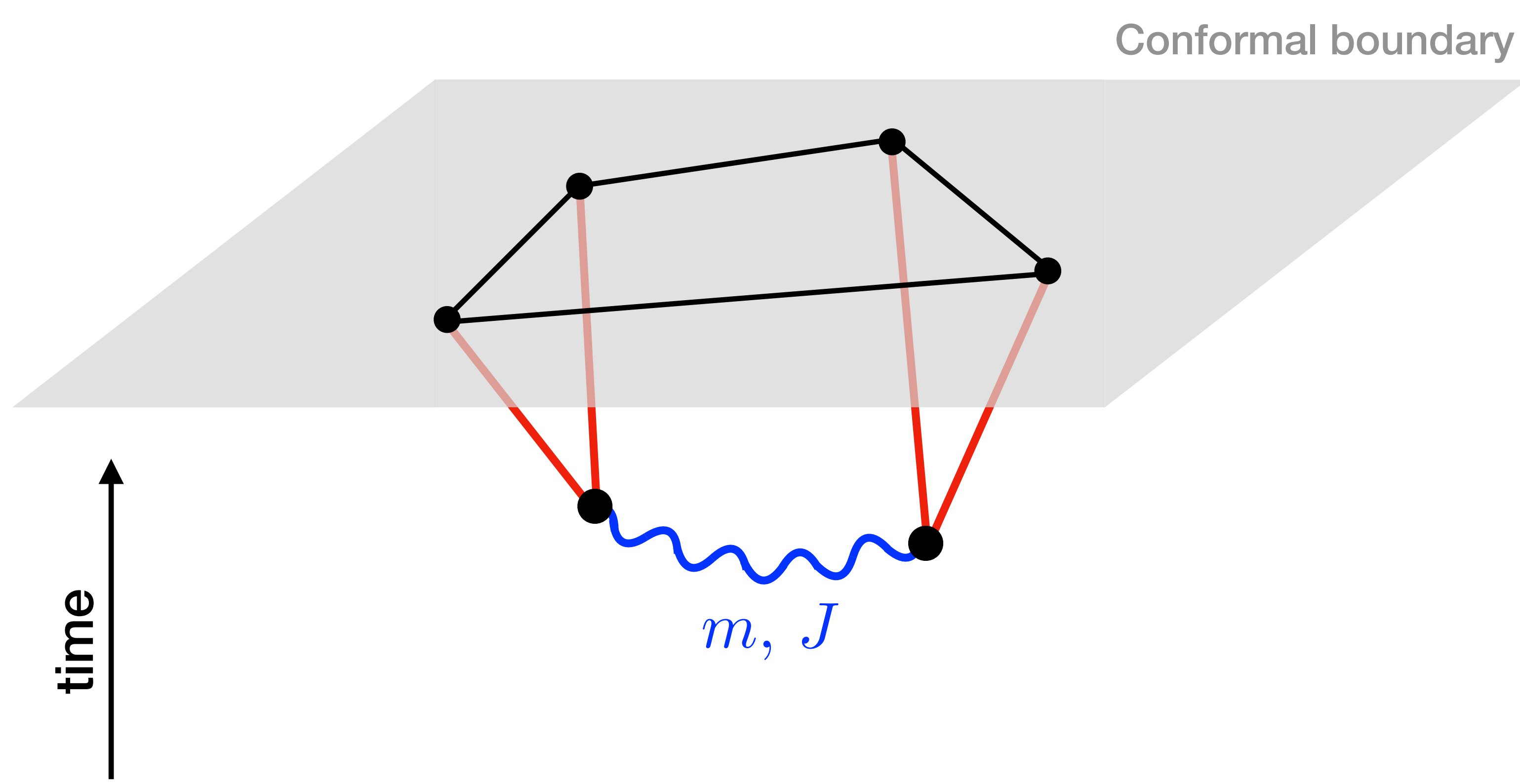
These can be traced back to the space-like boundary of the inflationary quasi-de Sitter spacetime.

Challenge: Classify the effects of new degrees of freedom

Boundary Correlators in de Sitter

The effects of particle exchanges are fixed by

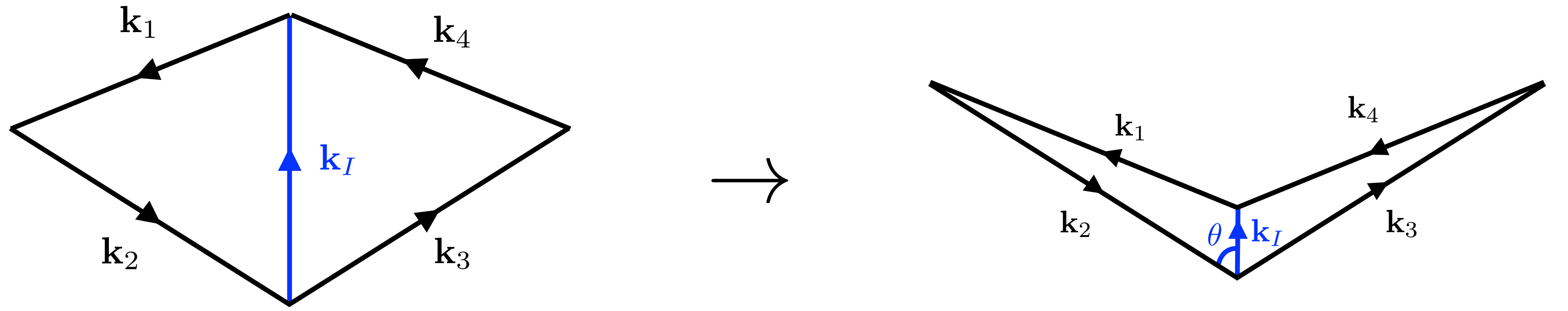
Conformal Symmetry + Singularities



Arkani-Hamed and Maldacena [hep-th] 1503.08043

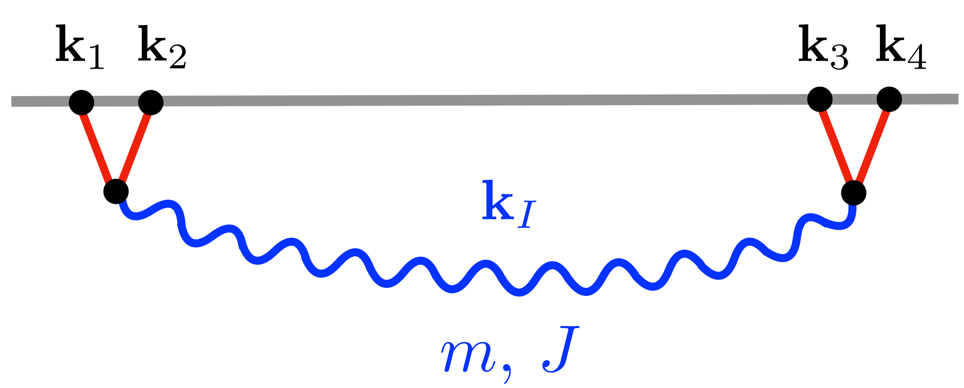
Boundary Correlators in de Sitter

In the soft limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|, \quad j = 1, 2, 3, 4$



particles with $m \sim O(H)$ leave a distinct oscillatory signature

Hubble scale



phase frequency Legendre polynomial

$$\sim \sin\left[\delta(m) + \frac{m}{H}\log k_I\right] P_J(\cos\theta)$$

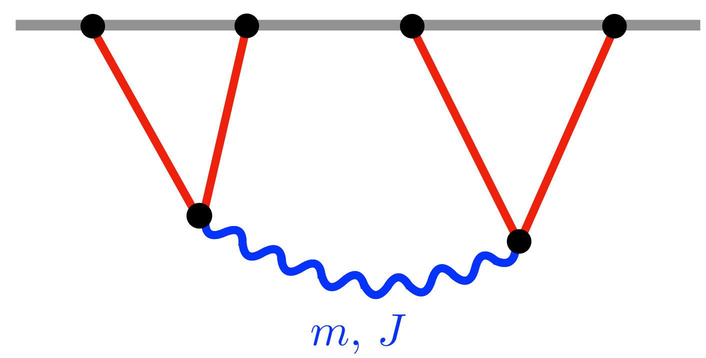
mass spin

$k_I = |\mathbf{k}_I|$

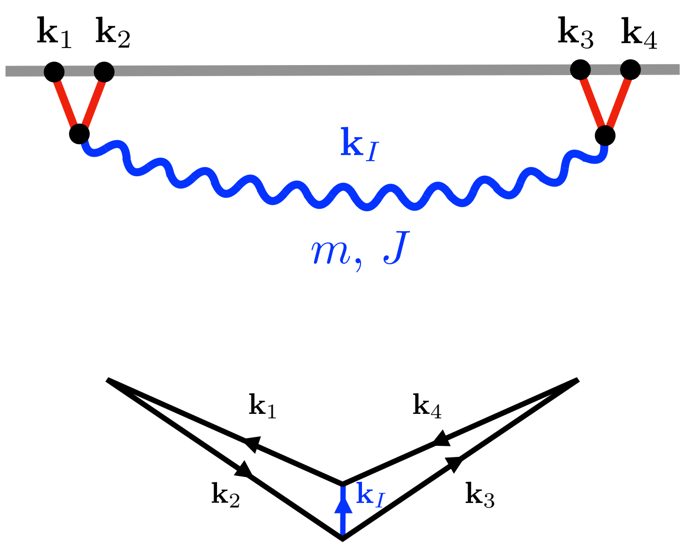
Arkani-Hamed and Maldacena [hep-th] 1503.08043

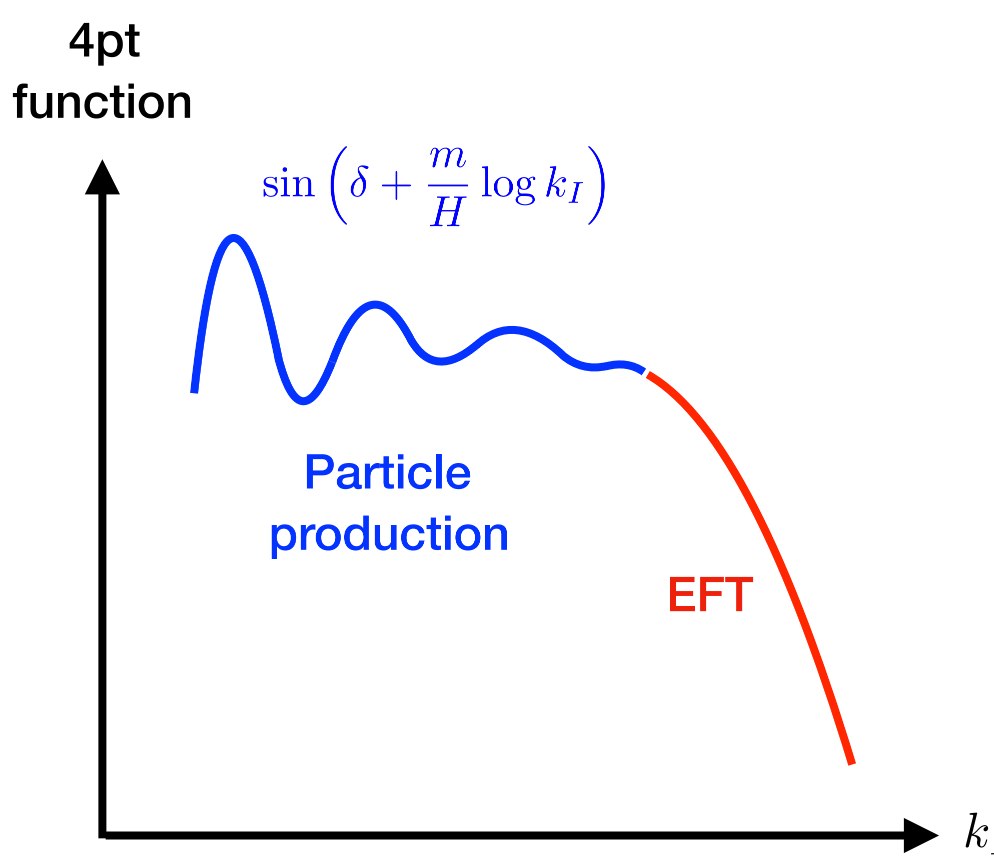
Boundary Correlators in de Sitter

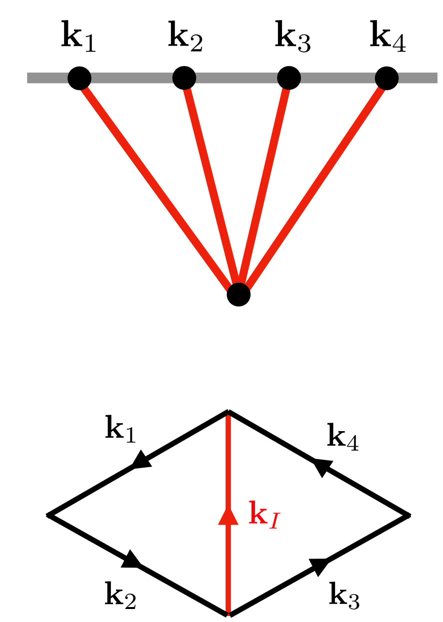
Only recently has Conformal symmetry been harnessed to obtain the **full exchange**



Away from the soft limit the dependence on k_I is smooth

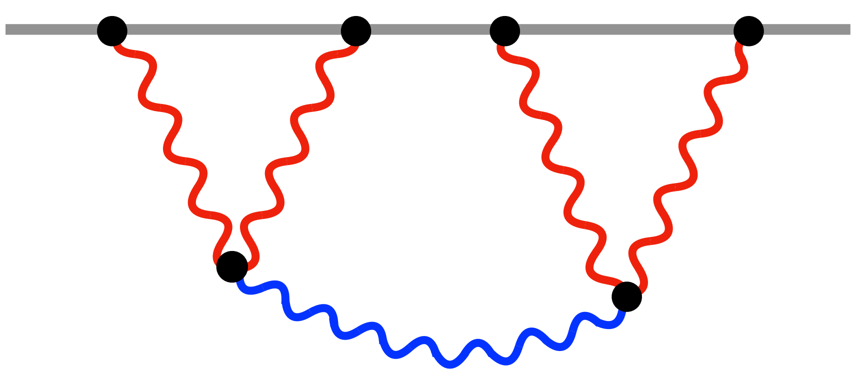




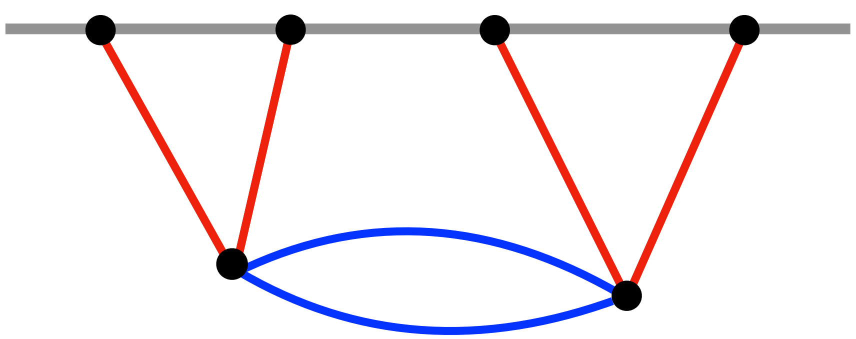


Arkani-Hamed and Maldacena [hep-th] 1503.08043; Arkani-Hamed, Baumann, Lee and Pimentel [hep-th] 1811.00024

Boundary Correlators in de Sitter



External Spins?

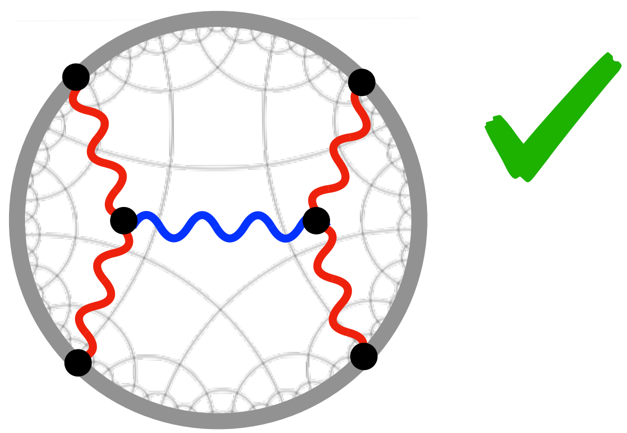


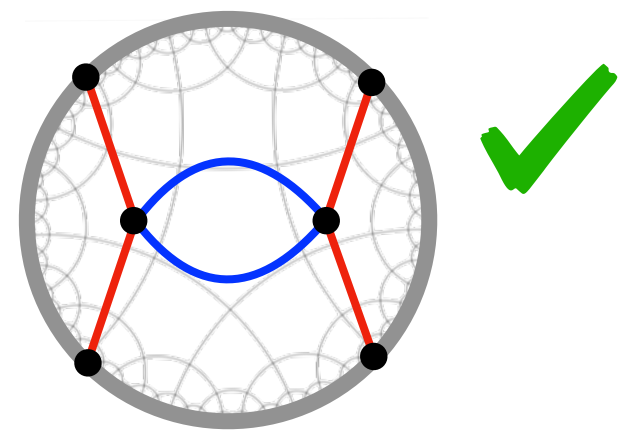
Beyond tree level?

… to tackle these cases we need to expand our toolkit

Boundary Correlators in de Sitter

...in AdS we have a pretty good understanding!





This talk: Can we adapt techniques for AdS Witten diagrams to de Sitter?

Boundary Correlators in de Sitter

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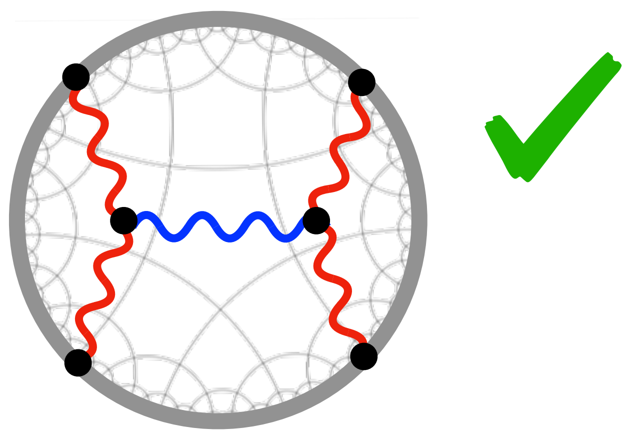
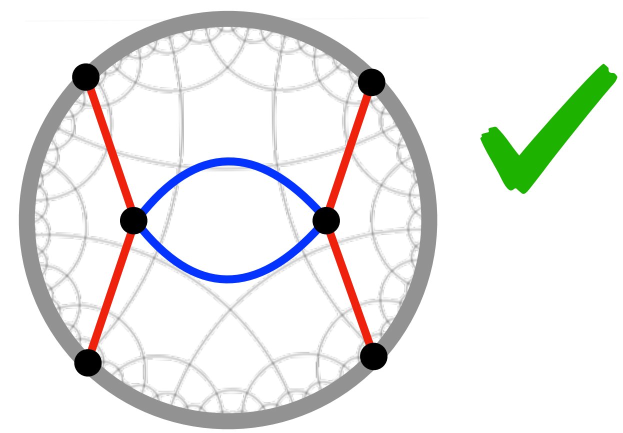


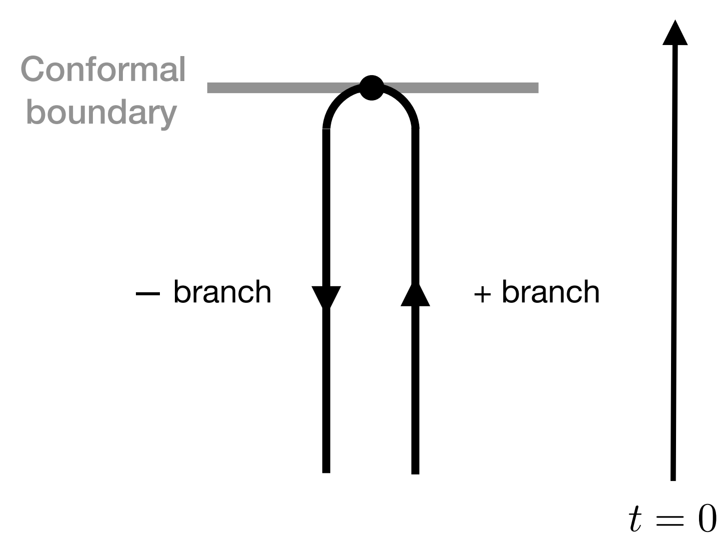


This talk: Can we adapt techniques for AdS Witten diagrams to de Sitter?

The **time-dependence** of the dS background makes them difficult to apply (at least directly)

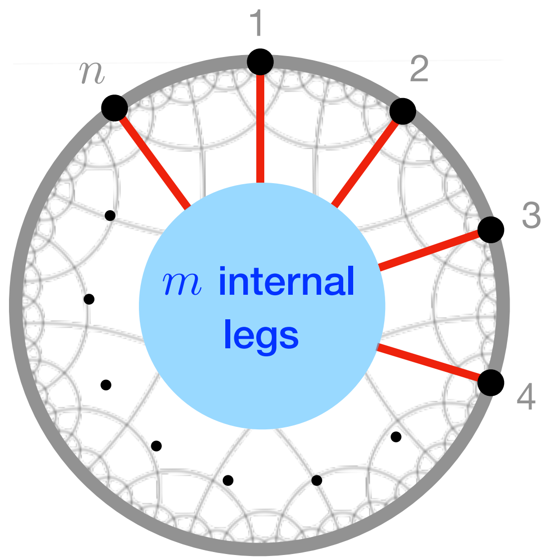
In dS typically use the **Schwinger-Keldysh (in-in) formalism** to compute fixed time correlators:

the bulk time integral
follows the in-in contour:



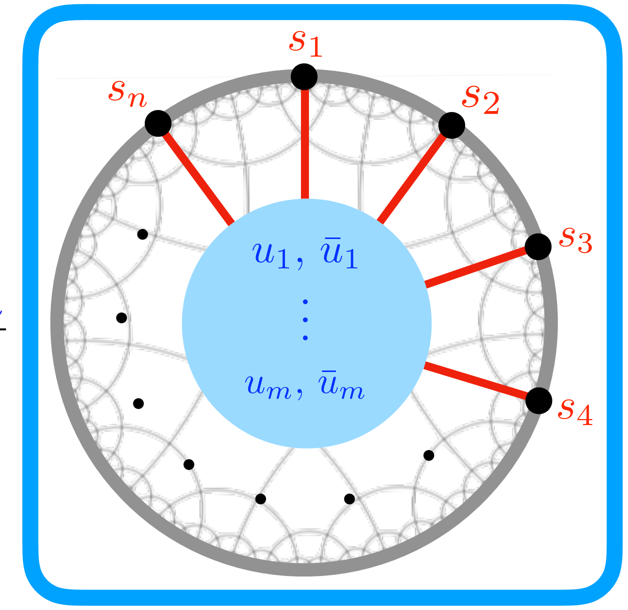
Bridging the Gap between AdS and dS

The key is to adopt a **Mellin-Barnes representation** in momentum space:



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i}\cdots\frac{\mathrm{d}s_n}{2\pi i}\frac{\mathrm{d}u_1}{2\pi i}\frac{\mathrm{d}\bar{u}_1}{2\pi i}\cdots\frac{\mathrm{d}u_m}{2\pi i}\frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

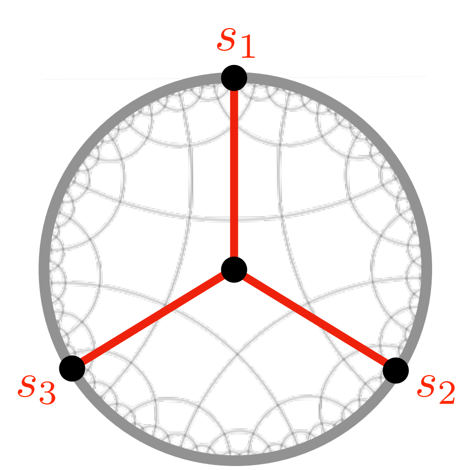
external legs   internal legs

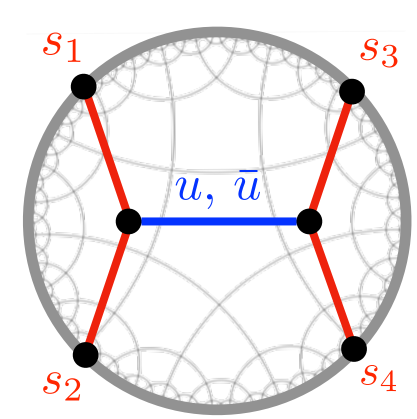


External leg $\longrightarrow$ **one** Mellin variable, s

Internal leg $\longrightarrow$ **two** Mellin variables, $u, \bar{u}$

For example:





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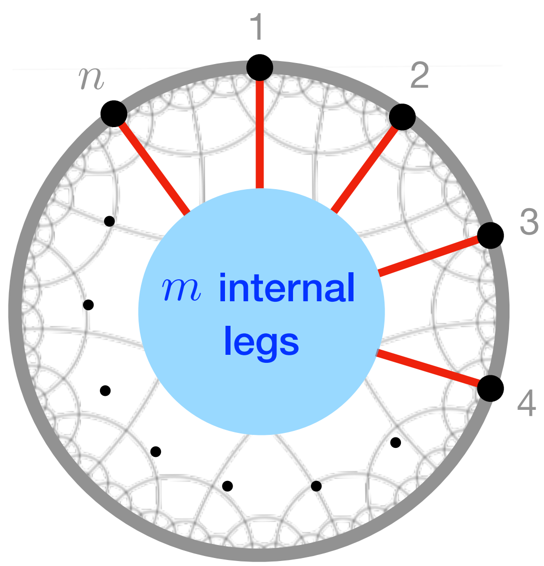
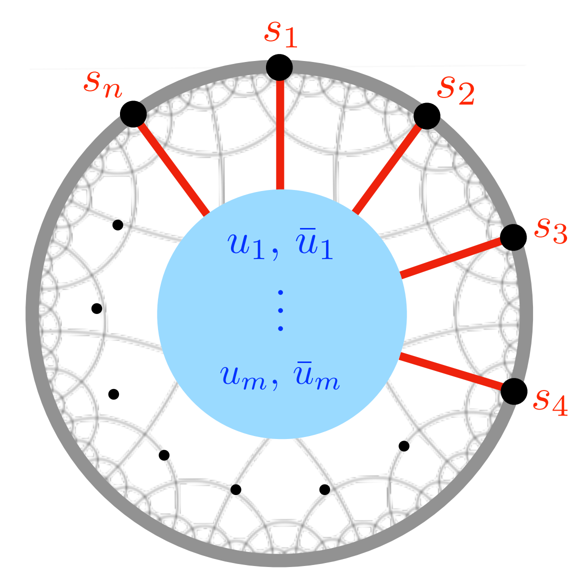
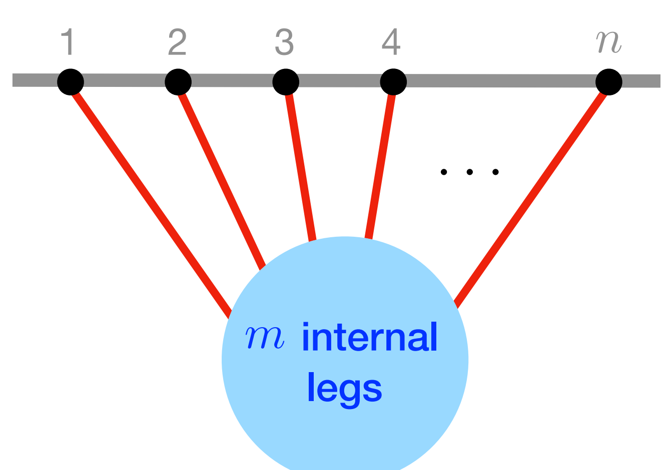
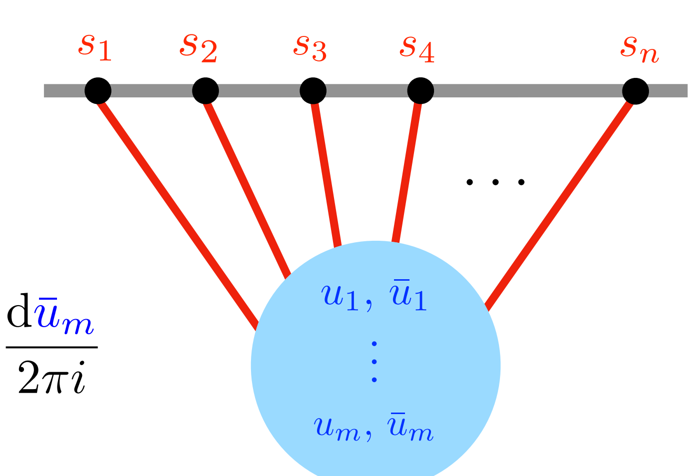


$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$



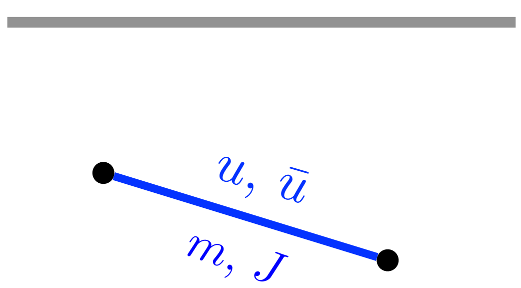


$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$



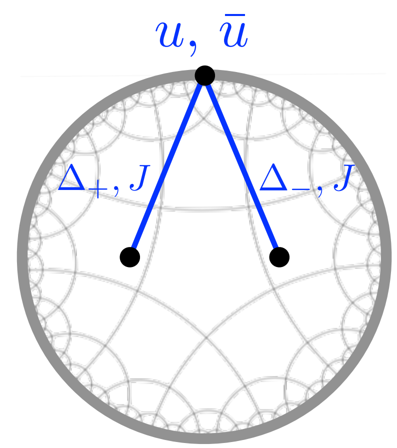
Bridging the Gap between AdS and dS

Propagators in dS are given by their counter-parts in EAdS up to a **phase**



dS bulk-bulk propagator

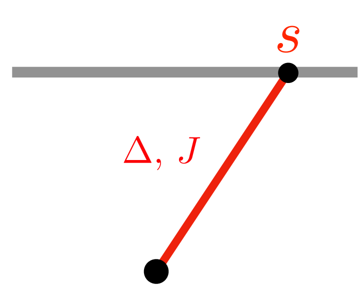
$$= \exp\left[\pm\left(u - \bar{u}\right)\pi i\right]$$



EAdS Harmonic function

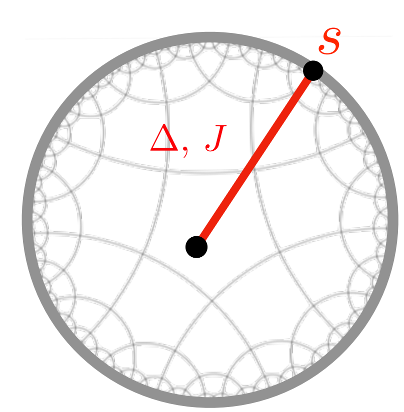
$$, \qquad m^2R^2 = \Delta_+\Delta_- + J$$

in-in contour branch



dS bulk-boundary propagator

$$= \exp\left[\pm\left(s + \tfrac{1}{2}\left(\Delta - \tfrac{d}{2}\right)\right)\pi i\right]$$



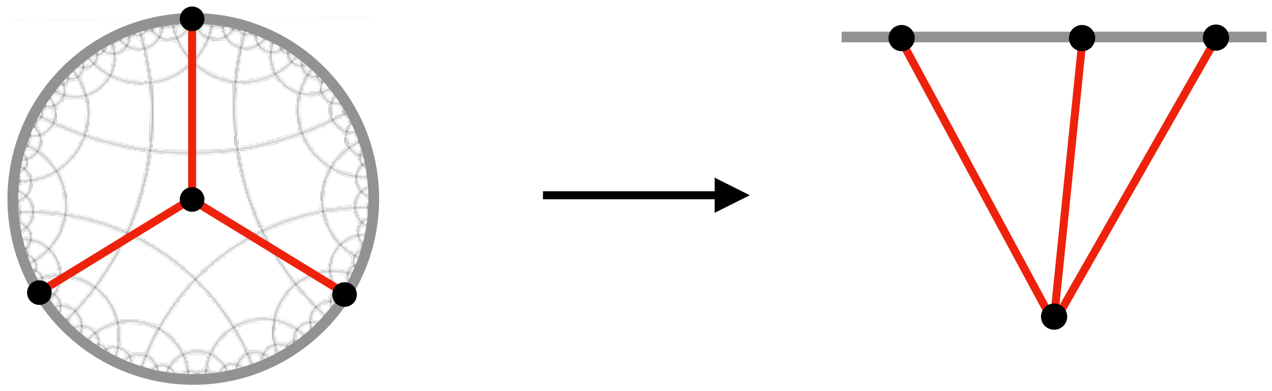
EAdS bulk-boundary propagator

$$, \qquad m^2R^2 = \Delta\left(d - \Delta\right) + J$$

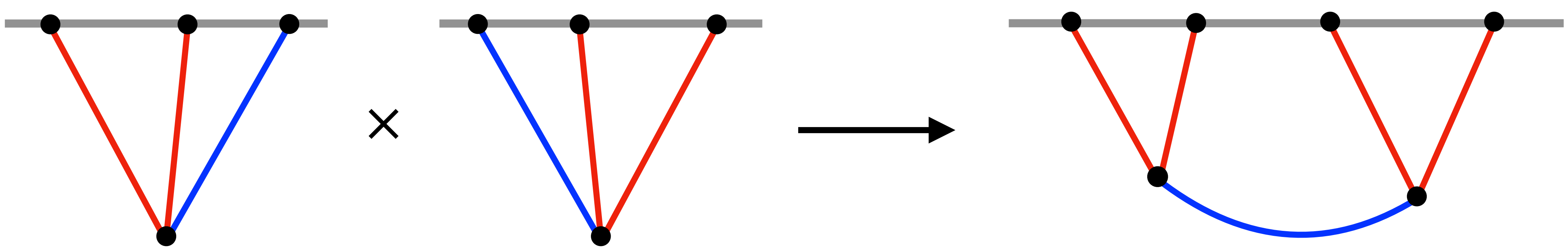
Provides a framework to extend existing techniques for Witten diagrams to de Sitter!

Outline

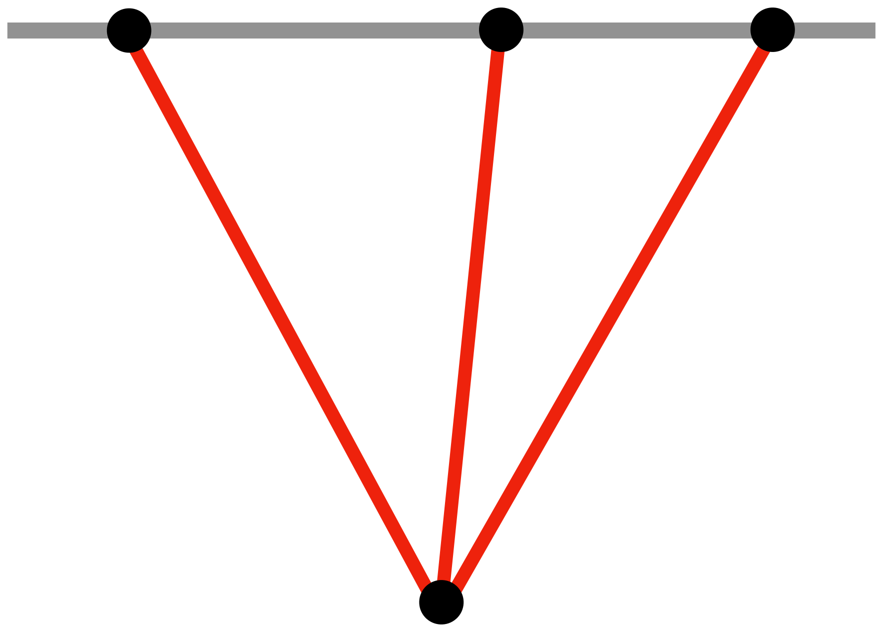
1.



2.

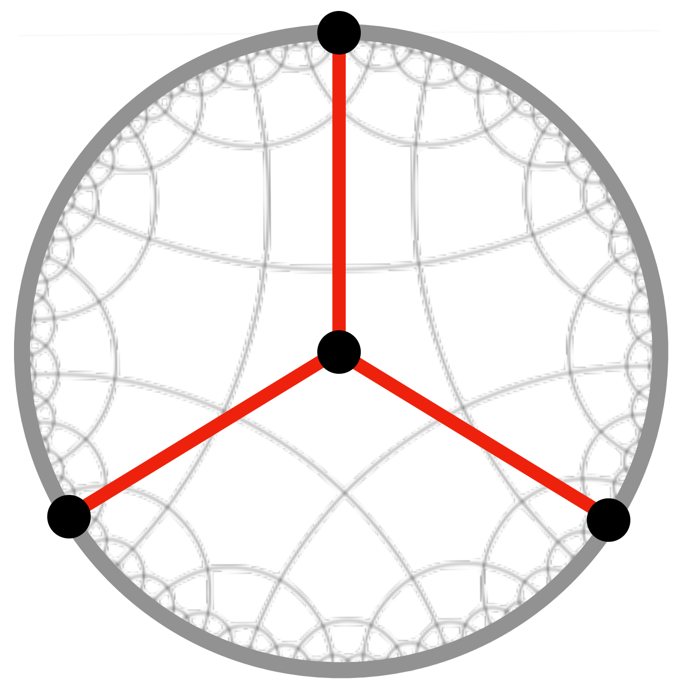


Contact Amplitudes



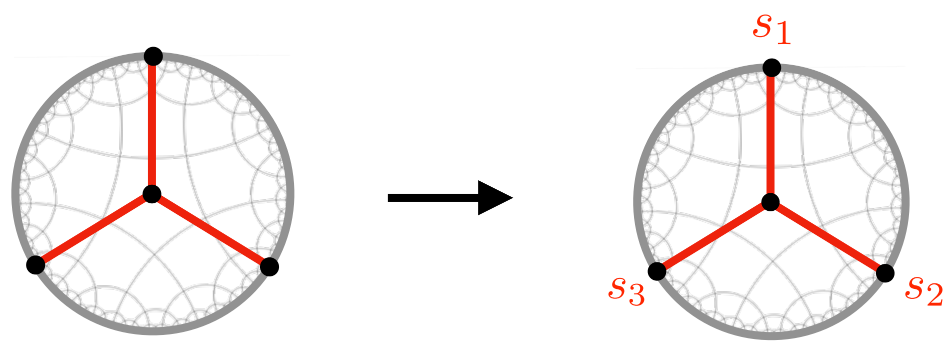
Contact Amplitudes

Starting point:

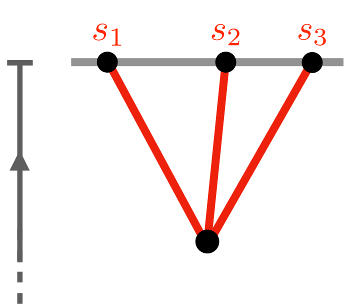


Contact Amplitudes: Strategy

1. Establish Mellin-Barnes representation in momentum space

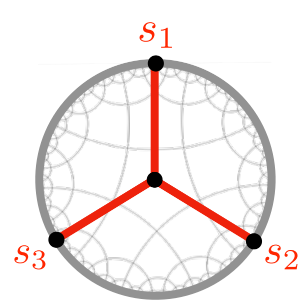


2. Convert each external leg to a propagator in dS by multiplying with the appropriate phase:

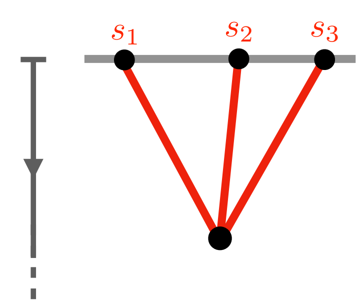


time-ordered
(+) branch

$$= +i\exp\left[-\pi i\sum_{j=1}^{3}\left(s_j+\tfrac{1}{2}\left(\Delta_j-\tfrac{d}{2}\right)\right)\right]$$

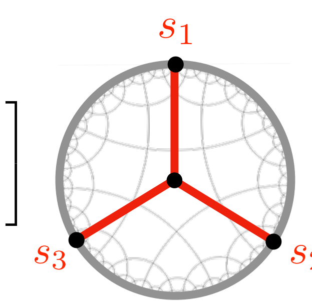


,

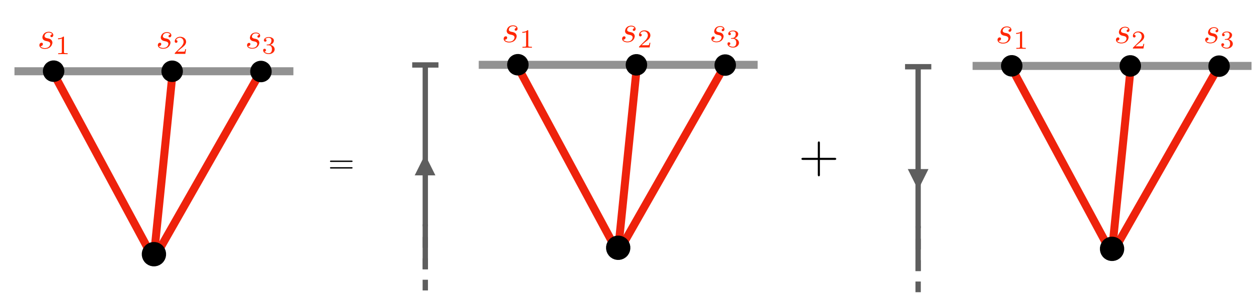


anti-time-ordered
(-) branch

$$= -i\exp\left[+\pi i\sum_{j=1}^{3}\left(s_j+\tfrac{1}{2}\left(\Delta_j-\tfrac{d}{2}\right)\right)\right]$$



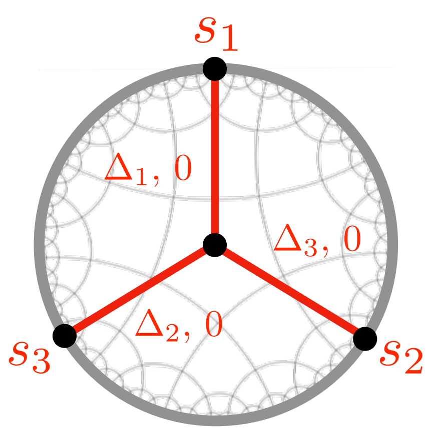
3. Full contact diagram:



These rules hold for particles of **any spin** and **any mass**, but for now we shall focus on scalars…

Contact Amplitudes

AdS 3pt contact amplitude of generic scalars



$$\propto \delta\left(\frac{d}{4}-(s_1+s_2+s_3)\right)\prod_{j=1}^{3}\Gamma\left(s_j+\frac{i\nu_j}{2}\right)\Gamma\left(s_j-\frac{i\nu_j}{2}\right)\left(\frac{k_j}{2}\right)^{-2s_j+i\nu_j}$$

bulk integration external legs

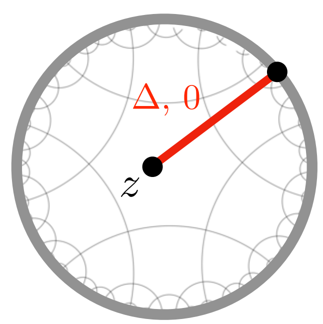
$$m_j^2R^2=\Delta_j\left(\Delta_j-d\right)$$

$$\Delta_j=\frac{d}{2}+i\nu_j$$

The connection to the bulk is manifest!

Bulk-boundary propagator in Poincaré coordinates:

$$\mathrm{d}s^2=R^2\frac{\mathrm{d}z^2+\mathrm{d}\mathbf{y}^2}{z^2}$$



$$=z^{\frac{d}{2}}\left(\frac{k}{2}\right)^{i\nu}K_{i\nu}\left(zk\right)=\int_{-i\infty}^{i\infty}\frac{\mathrm{d}s}{2\pi i}z^{\frac{d}{2}-2s}\,\Gamma\left(s+\tfrac{i\nu}{2}\right)\Gamma\left(s-\tfrac{i\nu}{2}\right)\left(\frac{k}{2}\right)^{-2s+i\nu}$$

Modified Bessel function of the second kind

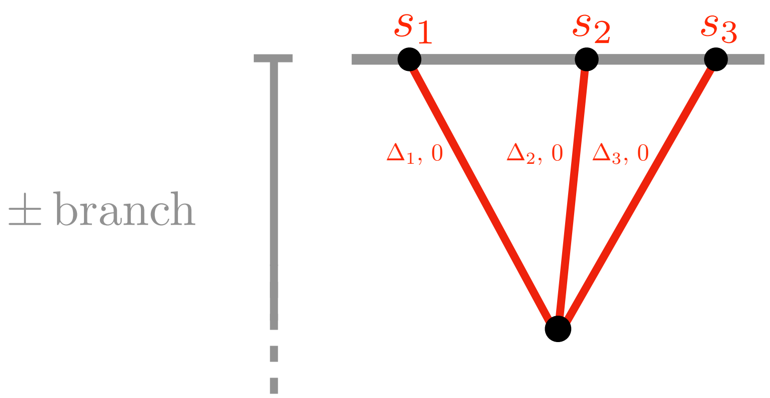
Bulk integration is encoded in a **Dirac delta function:**

$$\delta\left(\tfrac{d}{2}-2\left(s_1+s_2+s_3\right)\right)=\lim_{z_0\to 0}\int_{z_0}^{\infty}\frac{dz}{z^{d+1}}z^{\sum_{j=1}^{3}\left(\frac{d}{2}-2s_j\right)}$$

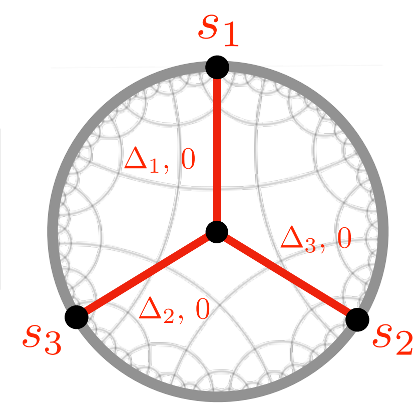
The sum of the Mellin variables is thus conserved: $s_1+s_2+s_3=\frac{d}{4}$

Contact Amplitudes

dS 3pt contact amplitude of generic scalars



$$= \pm i \exp\left[\mp \pi i \sum_{j=1}^{3} \left(s_j + \tfrac{1}{2} \left(\Delta_j - \tfrac{d}{2} \right) \right) \right]$$

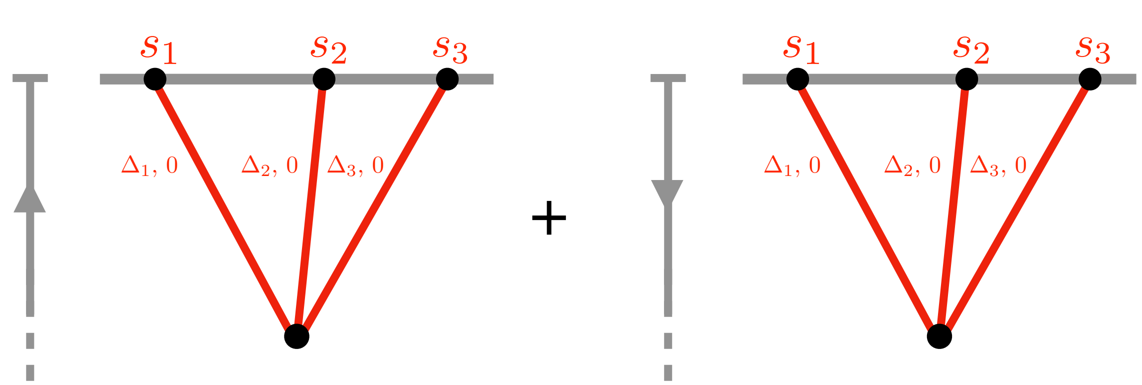


The **overall phase** for each branch is **constant**, since

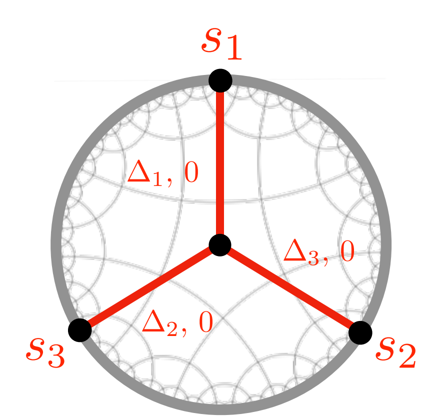
$$\frac{d}{2} - 2\left(s_1 + s_2 + s_3\right) = 0 \longrightarrow \mp \frac{i\pi}{2} \left(-d + \sum_{j=1}^{3} \Delta_j \right)$$

(bulk integration)

The **full amplitude** is the sum:



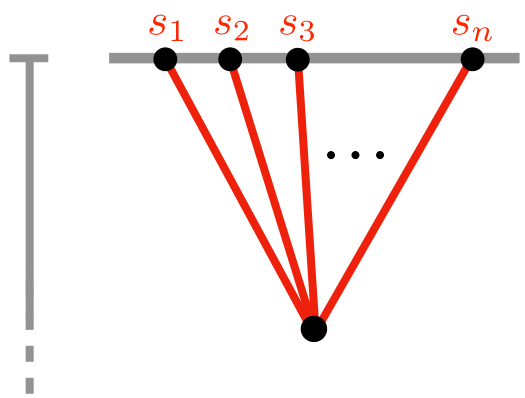
$$= \underbrace{\sin\left[\left(-d + \sum_{j=1}^{3} \Delta_j \right) \frac{\pi}{2} \right]}_{\text{interference factor}}$$



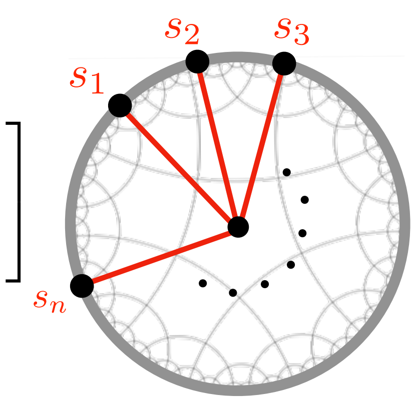
Contact Amplitudes

dS n ~~3~~pt contact amplitude of generic scalars

± branch



$$= \pm i \exp\left[\mp \pi i \sum_{j=1}^{\cancel{3}\, n} \left(s_j + \tfrac{1}{2}\left(\Delta_j - \tfrac{d}{2}\right)\right)\right]$$



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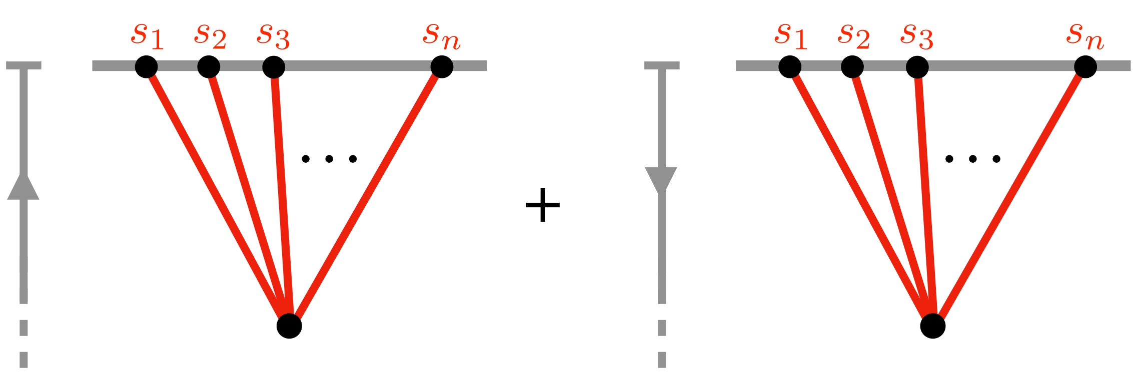

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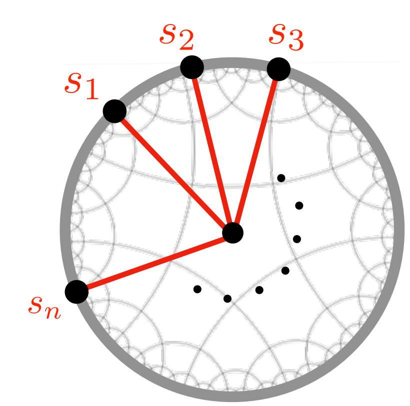
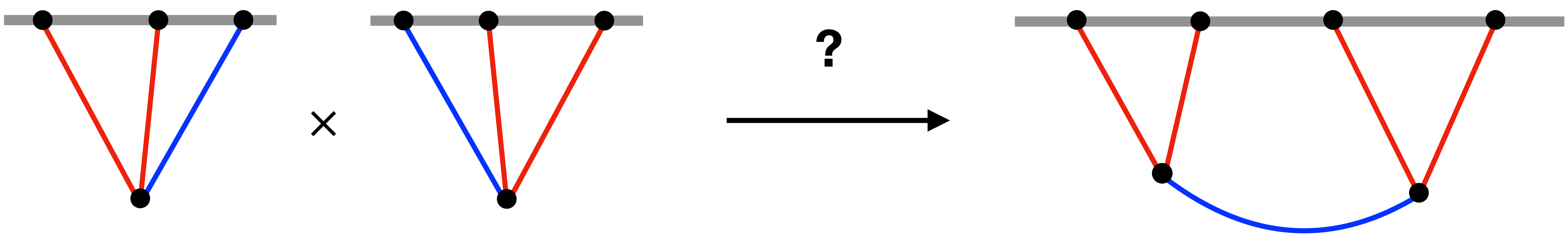


Exchange Amplitudes

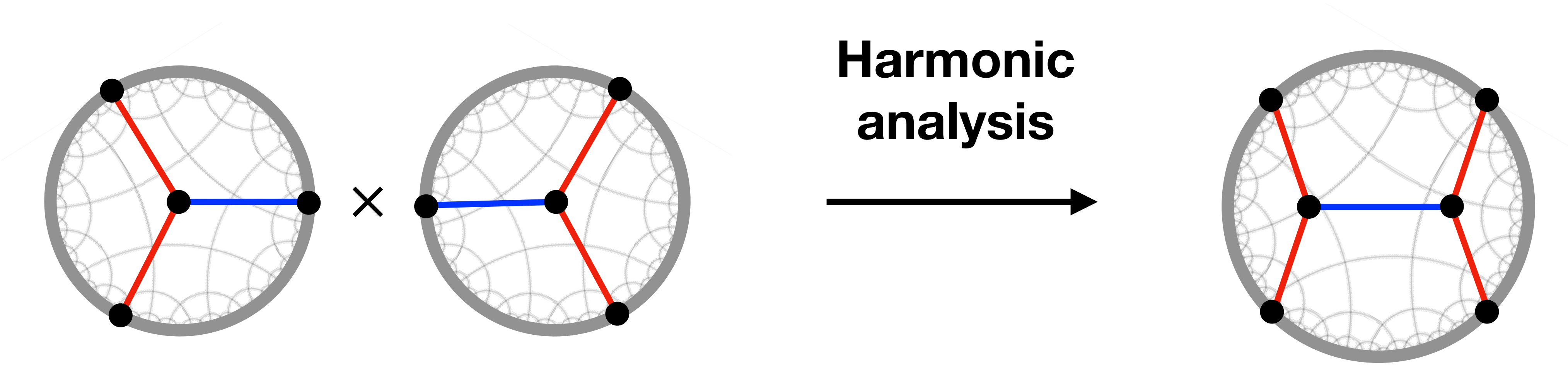
3pt diagrams are the **basic building blocks** from which we construct 4pt exchanges



… we just need to figure out the rules to glue them together.

Exchange Amplitudes

3pt diagrams are the **basic building blocks** from which we construct 4pt exchanges



Through the Mellin-Barnes representation this can be adapted to dS!

Harmonic Analysis in EAdS_{d+1}

Expand in complete basis of orthogonal Eigenfunctions of the AdS_{d+1} Laplacian

$$\left[R^2\nabla^2+\left(\tfrac{d}{2}+i\nu\right)\left(\tfrac{d}{2}-i\nu\right)+J\right]\Omega_{\nu,J}=0$$

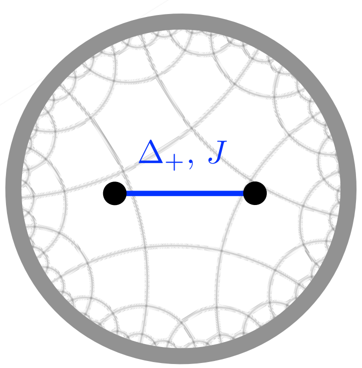
AdS radius

spectral parameter

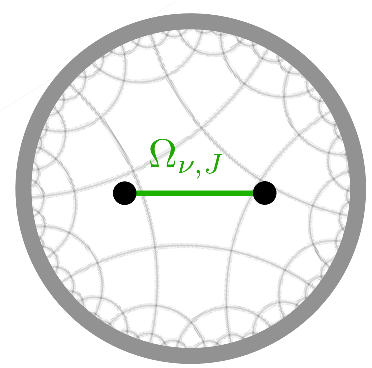
spin

bulk-to-bulk propagator for a spin-J field φ, generic mass $m^2R^2=-\left(\Delta_+\Delta_-+J\right)$:

$$\Delta_++\Delta_-=d,\quad \Delta_+\geq\Delta_-$$



$$=\int_{-\infty}^{\infty}d\nu\,\frac{1}{\left(\frac{d}{2}+i\nu\right)\left(\frac{d}{2}-i\nu\right)-\Delta_+\Delta_-}$$



$+\ \ldots\ .$

Contact terms

(harmonic functions of spin < J)

The **Spectral integral** implements the **Dirichlet boundary condition**:

$$\lim_{z\to 0}\varphi\left(z,\mathbf{y}\right)=O_{\Delta_+,J}\left(\mathbf{y}\right)z^{\Delta_+-J},$$

which is violated by a single harmonic function:

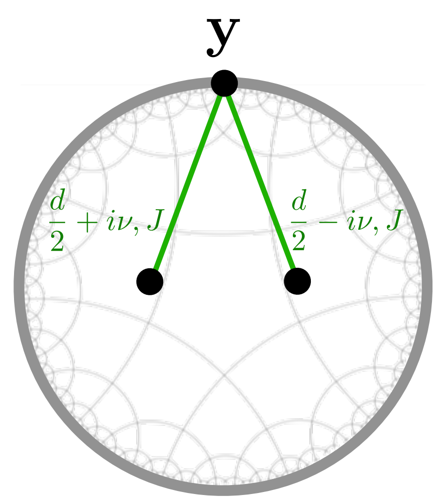
shadow operator

$$\Omega_{i\left(\frac{d}{2}-\Delta_+\right),J}\quad\longrightarrow\quad\lim_{z\to 0}\varphi\left(z,\mathbf{y}\right)=O_{\Delta_+,J}\left(\mathbf{y}\right)z^{\Delta_+-J}+O_{\Delta_-,J}\left(\mathbf{y}\right)z^{\Delta_--J}$$

Harmonic Analysis in EAdS$_{d+1}$

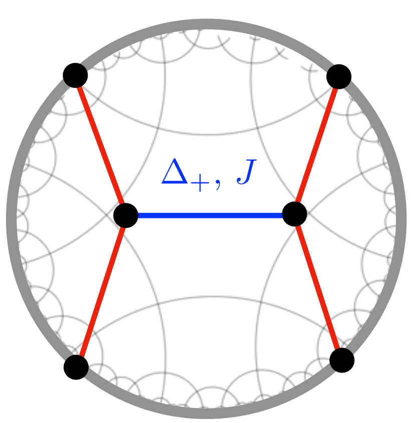
Harmonic functions admit a "split representation":

$$\Omega_{\nu,J} = \int \mathrm{d}^d\mathbf{y}$$

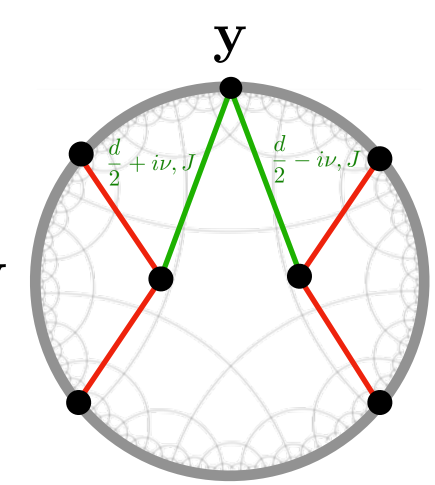


Integrated product of bulk-boundary propagators

Exchange diagrams reduce to **integrated products of 3pt diagrams**



$$= \int_{-\infty}^{\infty} d\nu \frac{1}{\left(\frac{d}{2}+i\nu\right)\left(\frac{d}{2}-i\nu\right) - \Delta_+\Delta_-} \int \mathrm{d}^d\mathbf{y}$$



$$+ \ \dots,$$

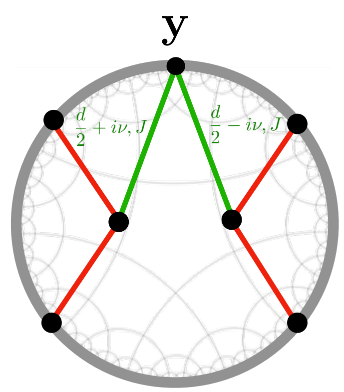
"Conformal Partial Wave" $\longleftrightarrow$ $\Omega_{\nu,J}$

e.g. Costa, Gonçalves and Penedones [hep-th] 1404.5625

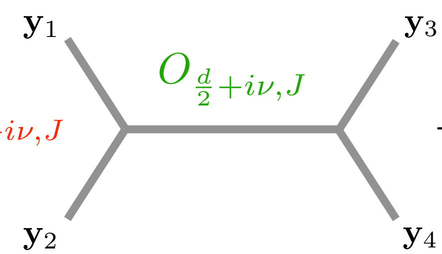
Harmonic Analysis in EAdS$_{d+1}$

A Conformal Partial Wave is a linear combination of two **conformal blocks**:

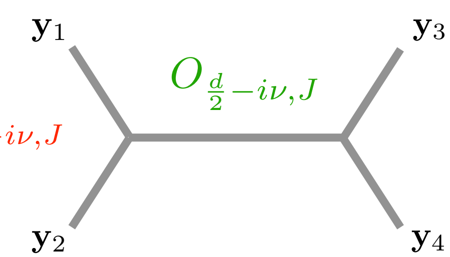
$\int \mathrm{d}^d\mathbf{y}$



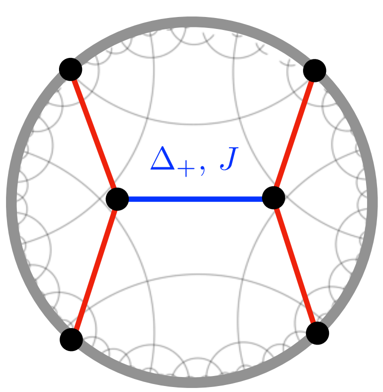
$= \kappa_{\frac{d}{2}+i\nu,J}$



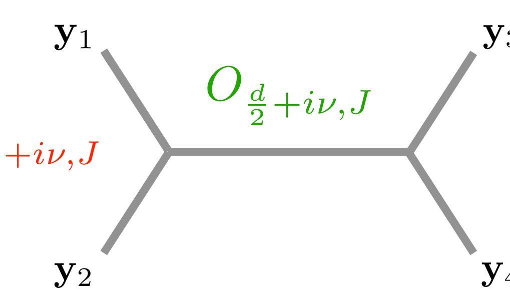
$+ \kappa_{\frac{d}{2}-i\nu,J}$



When $\nu \in \mathbb{R}$ these correspond to contributions from Principal Series representations, which are non-unitary in AdS.



$$= 2\int_{-\infty}^{\infty} d\nu \, \frac{1}{\left(\frac{d}{2}+i\nu\right)\left(\frac{d}{2}-i\nu\right)-\Delta_+\Delta_-} \, \kappa_{\frac{d}{2}+i\nu,J}$$

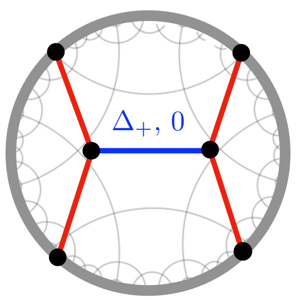


$+ \ldots$

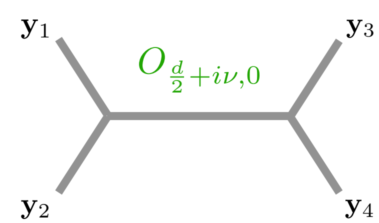
For exchange Witten diagrams, contributions from **unitary representations** are encoded in poles at $\nu \in i\mathbb{R}$!

Harmonic Analysis in $EAdS_{d+1}$

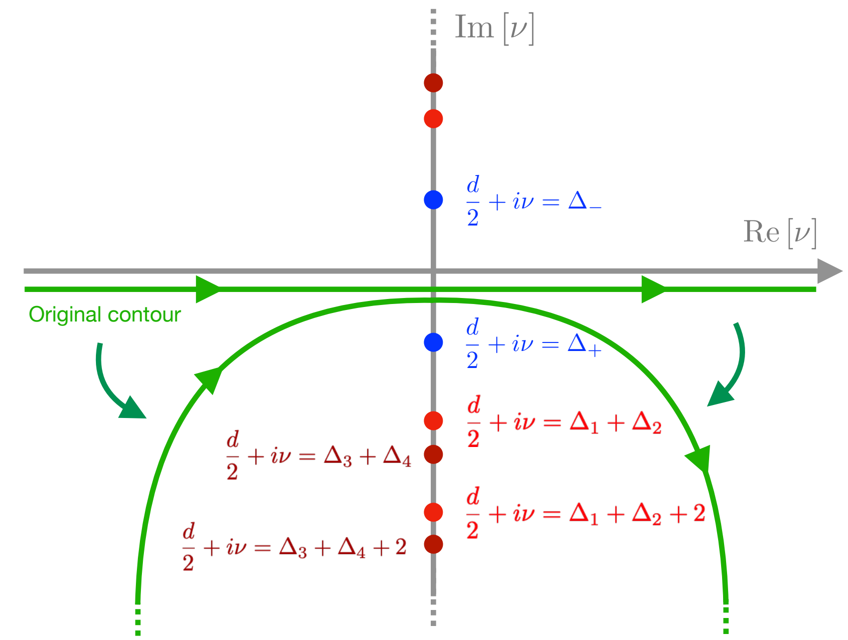
Example: four-point exchange for scalar fields



$$= 2\int_{-\infty}^{\infty} d\nu\, \underbrace{\frac{1}{\left(\frac{d}{2}+i\nu\right)\left(\frac{d}{2}-i\nu\right)-\Delta_+\Delta_-}}_{1}\ \underbrace{\kappa_{\frac{d}{2}+i\nu,0}}_{2}$$



,



contour can only be closed
on the **negative** imaginary axis

Two types of contributions:

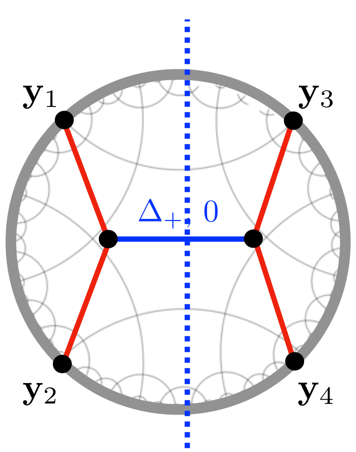
As required by the boundary condition

$$\lim_{z\to 0}\varphi\left(z,\mathbf{y}\right)=O_{\Delta_+,0}\left(\mathbf{y}\right)z^{\Delta_+}$$

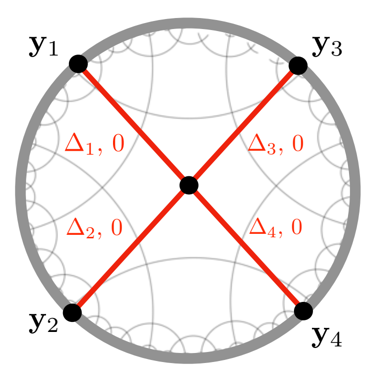
1 $\quad \frac{d}{2}+i\nu=\Delta_+$

2 $\quad \frac{d}{2}+i\nu=\Delta_1+\Delta_2+2n\,,\quad \frac{d}{2}+i\nu=\Delta_3+\Delta_4+2m\,,\quad n,\,m\in\mathbb{N}$

Exchanged single particle state



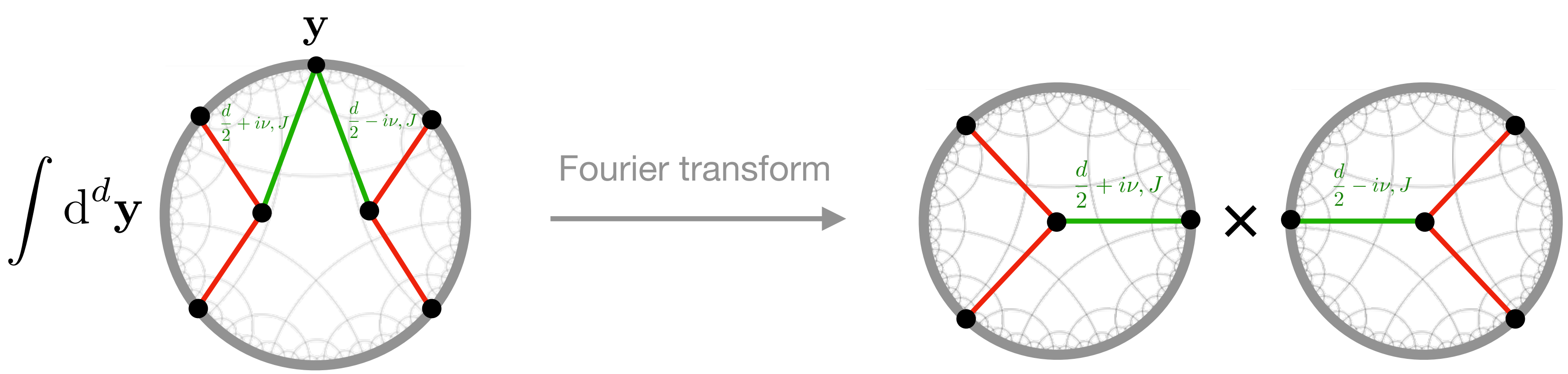
Two-particle states
bulk “contact" terms



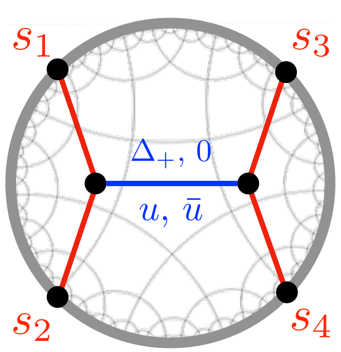
Harmonic Analysis in EAdS_{d+1}

In the view of extending to dS, we turn to the Mellin-Barnes representation in momentum space

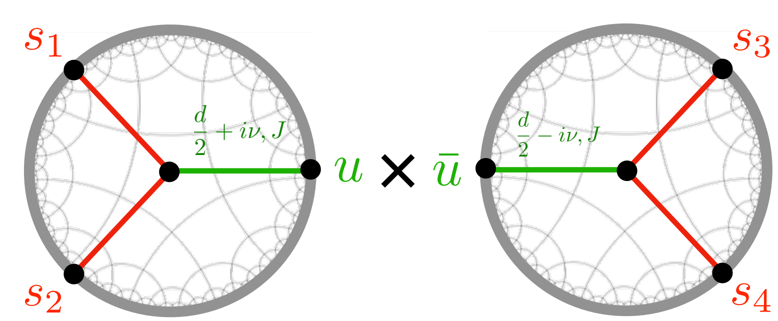
In momentum space, Conformal Partial Waves **factorise**:



The **Mellin-Barnes representation** for the exchange is **inherited** from the 3pt factors:

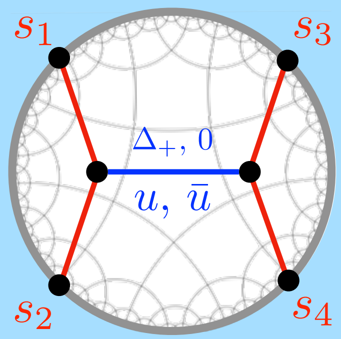


$$= \int_{-\infty}^{\infty} d\nu \frac{1}{\left(\frac{d}{2}+i\nu\right)\left(\frac{d}{2}-i\nu\right)-\Delta_+\Delta_-}$$

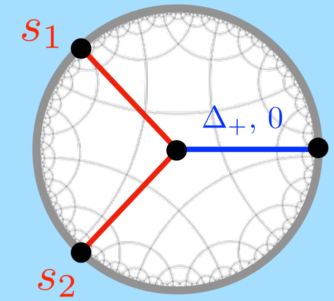


Key point: At the level of the Mellin representation, the spectral integral can be lifted!

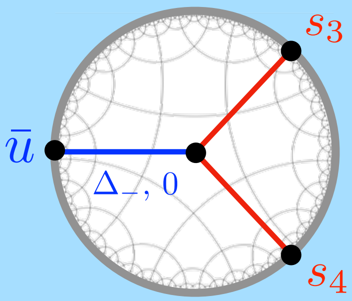
Mellin representation of AdS exchanges



$$= \operatorname{cosec}\left(\pi\left(u+\bar{u}\right)\right)\delta_{\mathrm{AdS}}\left(u,\bar{u}\right)$$



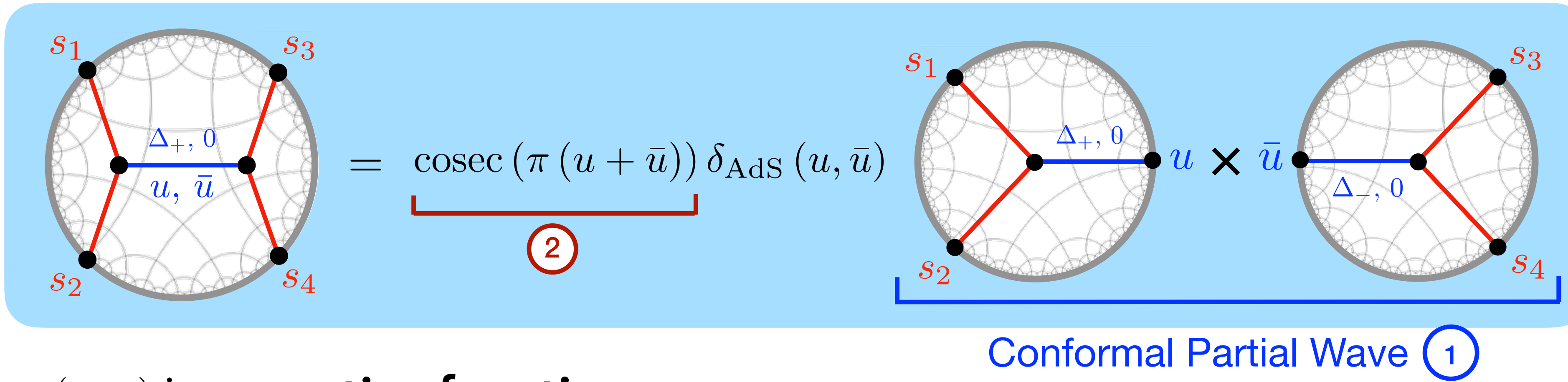
$$u \times \bar{u}$$



The factor $\delta_{\mathrm{AdS}}\left(u,\bar{u}\right)$ is an **entire function**:

$$\delta_{\mathrm{AdS}}\left(u,\bar{u}\right)=\frac{1}{2}\sin\left(\pi\left(u+\tfrac{1}{2}\left(\Delta_{-}-\tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u}+\tfrac{1}{2}\left(\Delta_{-}-\tfrac{d}{2}\right)\right)\right)$$

Mellin representation of AdS exchanges

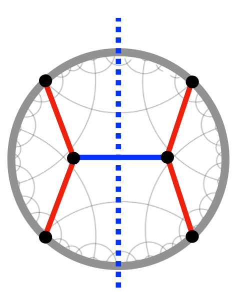


The factor $\delta_{\text{AdS}}(u,\bar{u})$ is an **entire function**:

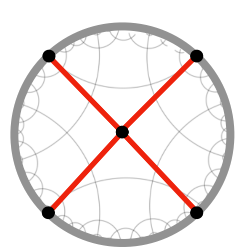
$$\delta_{\text{AdS}}(u,\bar{u}) = \frac{1}{2}\sin\left(\pi\left(u+\tfrac{1}{2}\left(\Delta_{-}-\tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u}+\tfrac{1}{2}\left(\Delta_{-}-\tfrac{d}{2}\right)\right)\right)$$

The poles in $u,\ \bar{u}$ are associated to **states exchanged** in the direct channel!

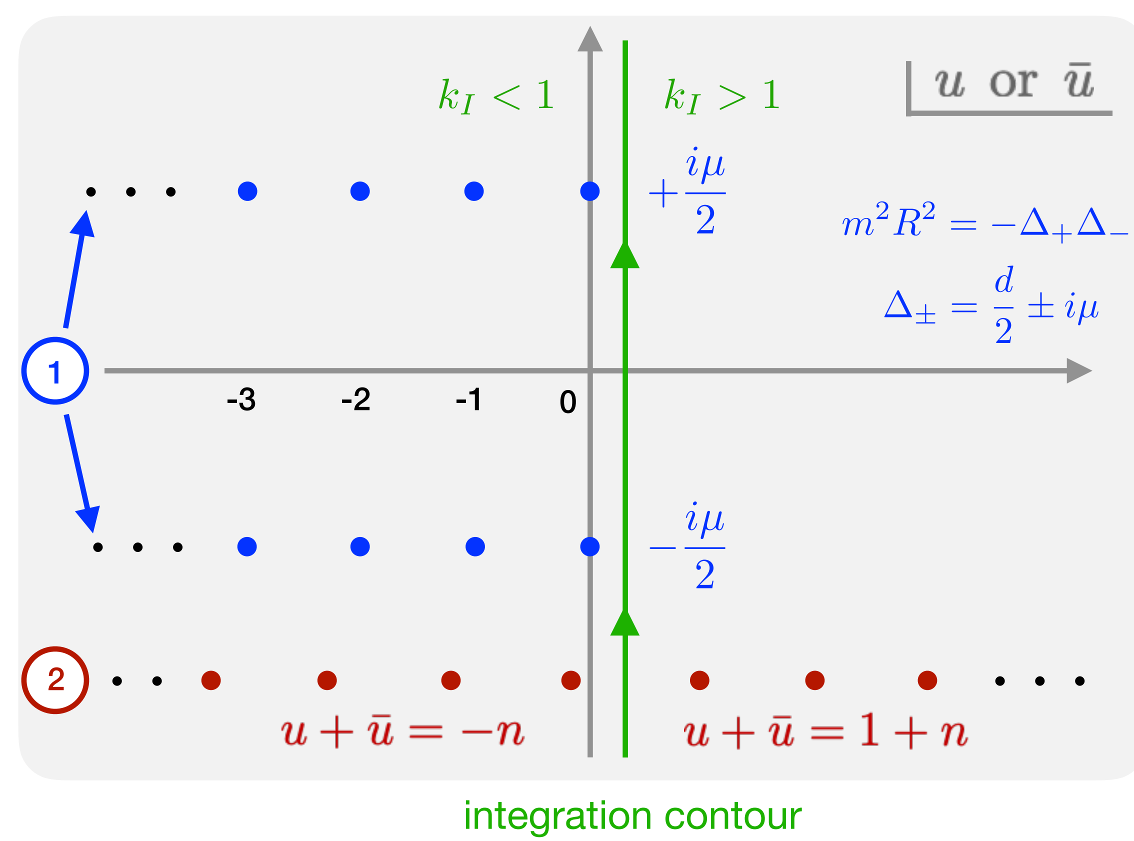
Two types of poles contribute to the **direct channel expansion** in the limit $k_I \to 0$:



Exchanged single-particle state:

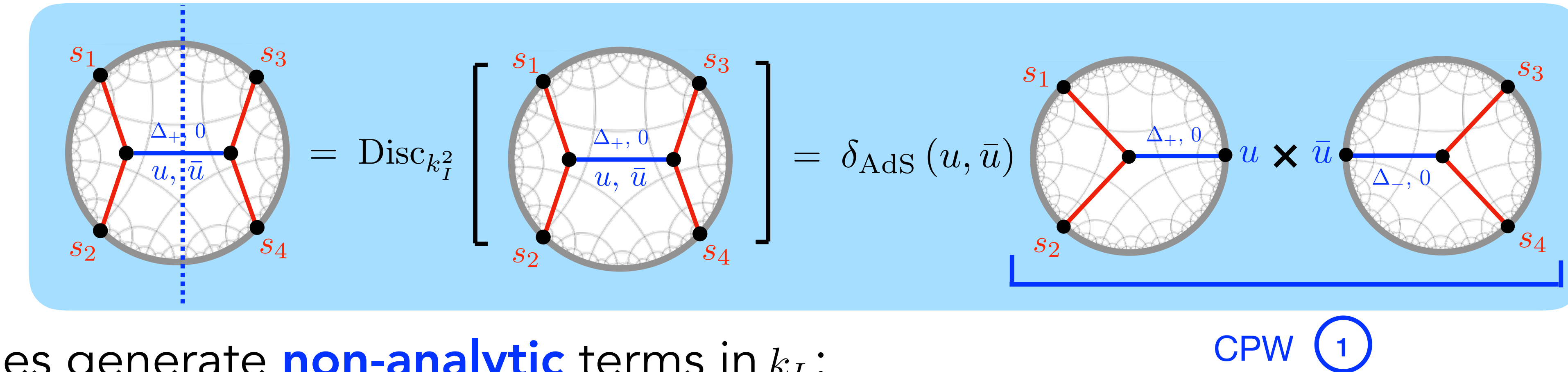


Bulk contact terms:



Mellin representation of AdS exchanges

On type 1 poles the exchange **factorises**



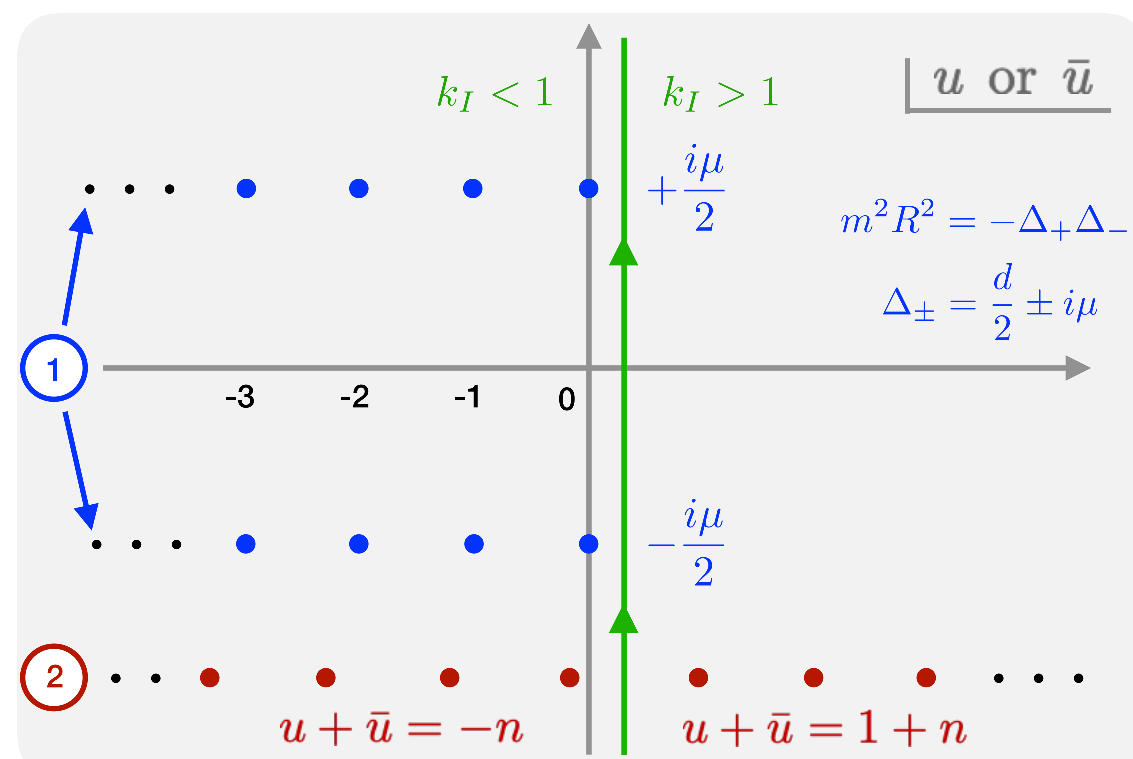
Their residues generate **non-analytic** terms in k_I :

$$\sim \# \left(k_I^2\right)^{\Delta_+ - \frac{d}{2}} \left(1 + k_I^2 + \left(k_I^2\right)^2 + \ldots\right) + \text{analytic}$$

Energy of exchanged single-particle state

descendent contributions

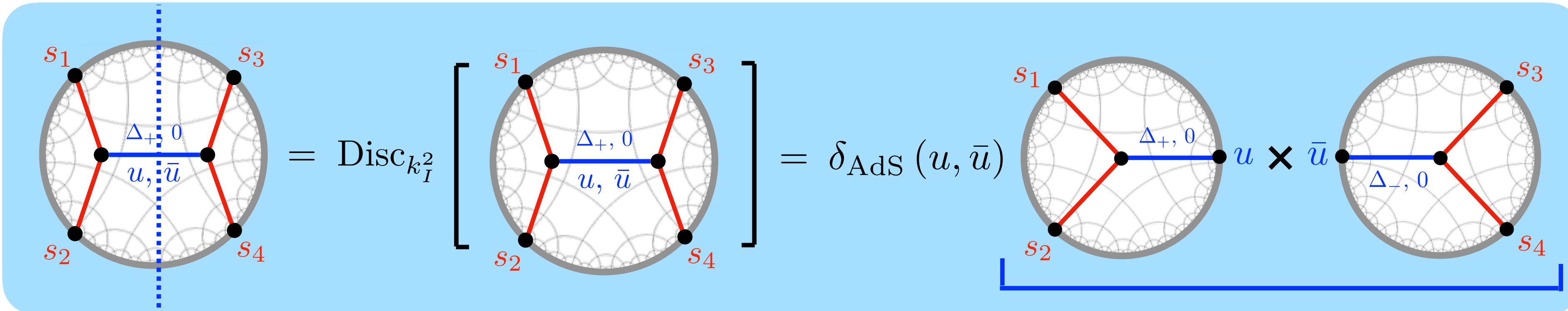
Characteristic signal of a single particle exchange!



integration contour

Mellin representation of AdS exchanges

On type 1 poles the exchange **factorises**



CPW 1

Their residues generate **non-analytic** terms in k_I :

$$\sim \# \left(k_I^2\right)^{\Delta_+ - \frac{d}{2}} \left(1 + k_I^2 + \left(k_I^2\right)^2 + \ldots\right) + \text{analytic}$$

Energy of exchanged single-particle state

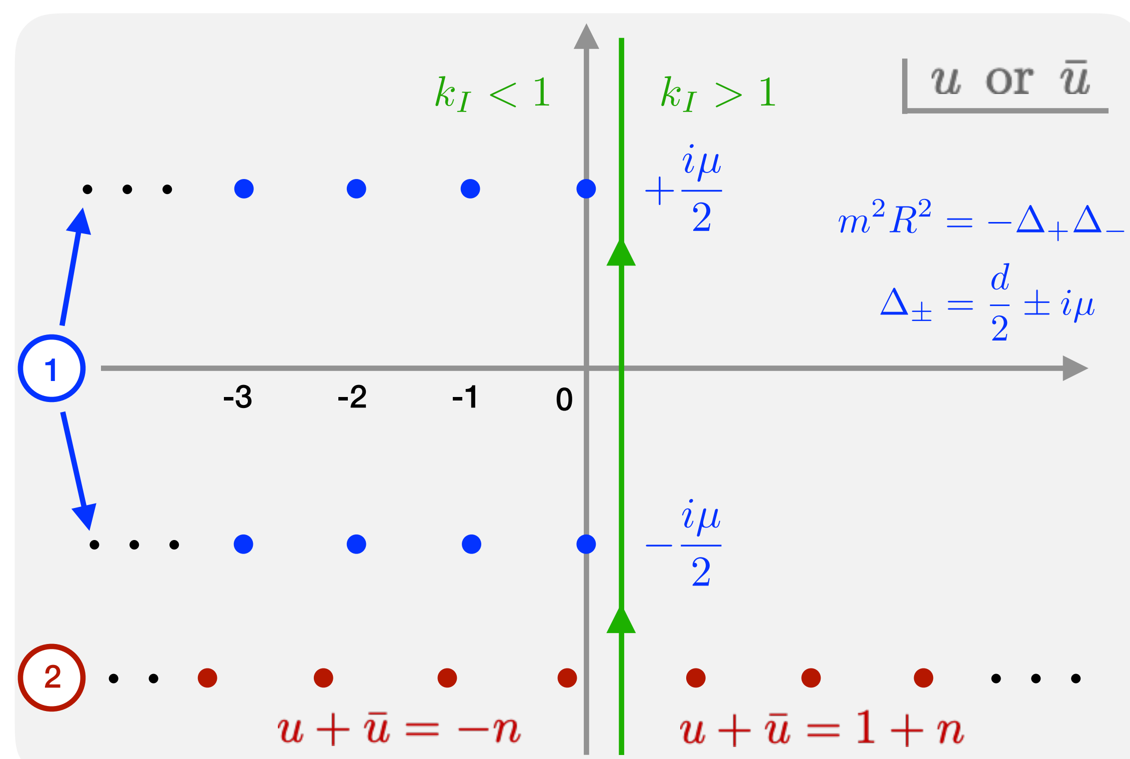
descendent contributions

Characteristic signal of a single particle exchange!

$$\left(k_I^2\right)^{\Delta_- - \frac{d}{2}} \left(1 + k_I^2 + \left(k_I^2\right)^2 + \ldots\right)$$

Note: CPWs generate **two families** of non-analytic terms

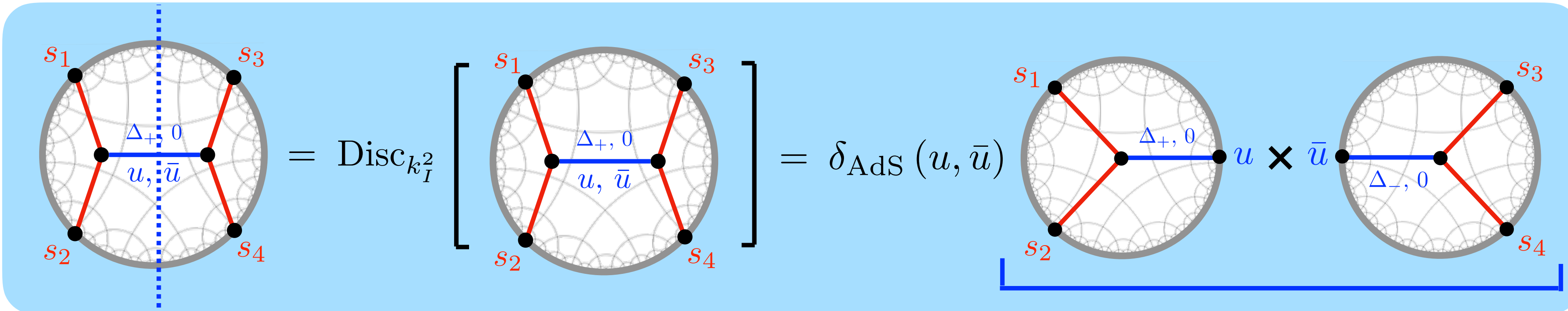
$$\left(k_I^2\right)^{\Delta_+ - \frac{d}{2}} \left(1 + k_I^2 + \left(k_I^2\right)^2 + \ldots\right)$$



integration contour

Mellin representation of AdS exchanges

On type 1 poles the exchange **factorises**



CPW 1

Their residues generate **non-analytic** terms in k_I:

$$\sim \# \left(k_I^2\right)^{\Delta_+ - \frac{d}{2}} \left(1 + k_I^2 + \left(k_I^2\right)^2 + \ldots\right) + \text{analytic}$$

Energy of exchanged single-particle state

descendent contributions

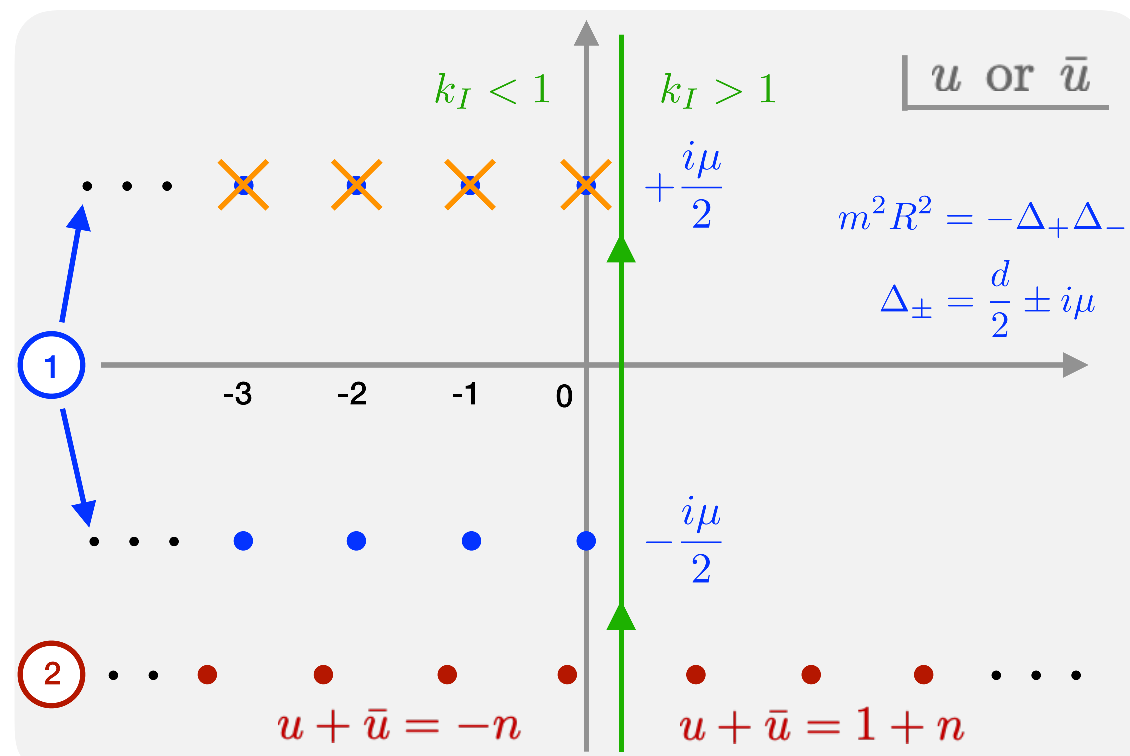
Characteristic signal of a single particle exchange!

× = zeros of $\delta_{\mathrm{AdS}}(u, \bar{u})$

$$\cancel{\left(k_I^2\right)^{\Delta_- - \frac{d}{2}} \left(1 + k_I^2 + \left(k_I^2\right)^2 + \ldots\right)}$$

Note: CPWs generate **two families** of non-analytic terms

$$\left(k_I^2\right)^{\Delta_+ - \frac{d}{2}} \left(1 + k_I^2 + \left(k_I^2\right)^2 + \ldots\right)$$



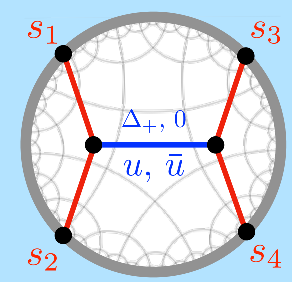
integration contour

$\delta_{\mathrm{AdS}}(u, \bar{u})$ **projects away** the shadow contributions!

Mellin representation of AdS exchanges

The type ② poles encode bulk contact terms, which give only **analytic terms** in k_I,

$$\mathrm{Res}_{u+\bar{u}=-n}\left[\vphantom{\int}\right.$$



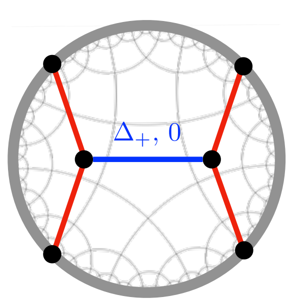
$$\left.\vphantom{\int}\right] \propto \left(k_I^2\right)^n$$

Coefficient admits expansion in $1/m^2$

$m =$ mass of exchanged particle

These comprise the EFT expansion of the exchange amplitude

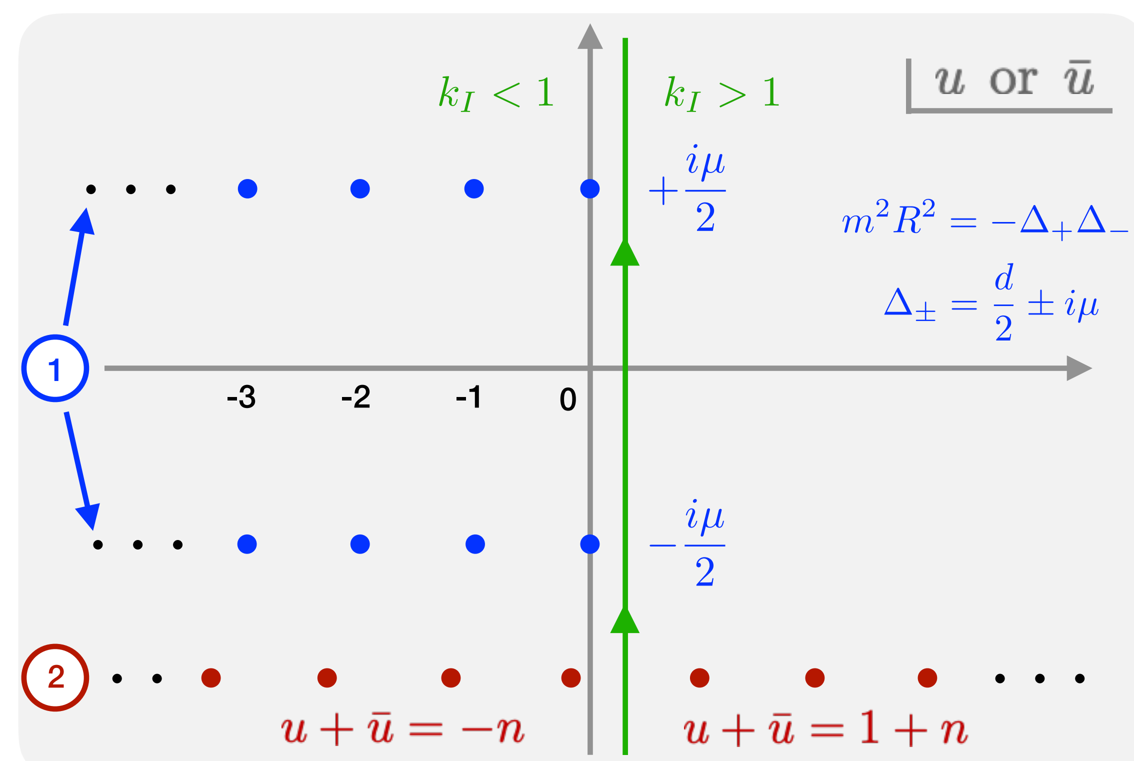
Example: External conformally coupled scalars



$$\Bigg|_{u+\bar{u}=-n} = \frac{1}{k_1k_2k_3k_4}\frac{1}{k_{12}^{d-2}}\left[\sum_{m=0}^{\infty} c_{mn}\left(\frac{k_{34}}{k_{12}}\right)^m\right]\left(\frac{k_I^2}{k_{12}^2}\right)^n$$

with $\quad c_{mn} = \dfrac{(-1)^m}{2^{2n}m!}\dfrac{(d-3+2n+m)!}{\left(\frac{\frac{d}{2}+i\mu+m-1}{2}\right)_{n+1}\left(\frac{\frac{d}{2}-i\mu+m-1}{2}\right)_{n+1}}$

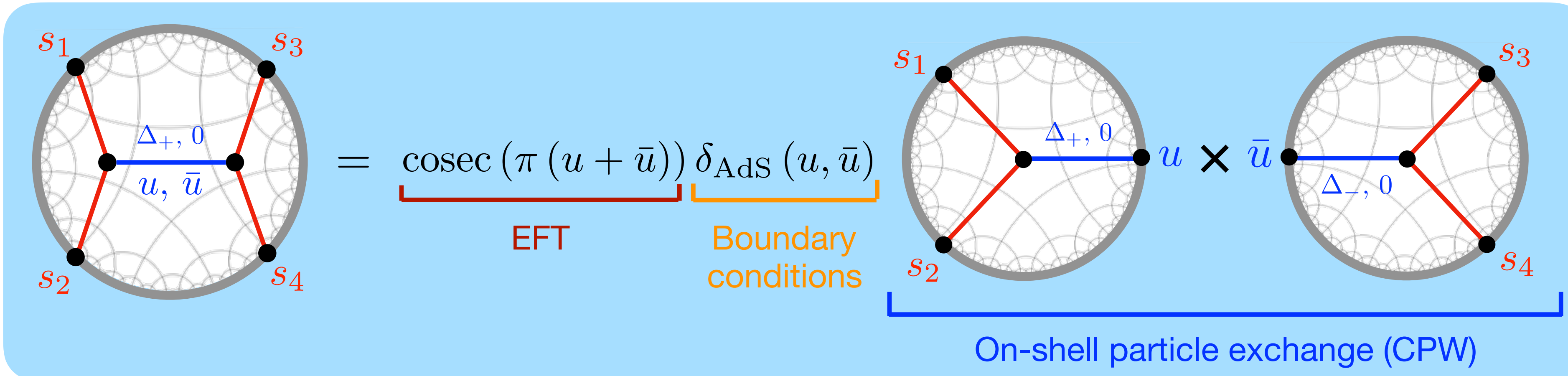
$$m^2R^2 = -\left(\tfrac{d}{2}+i\mu\right)\left(\tfrac{d}{2}-i\mu\right)$$



integration contour

Mellin representation of AdS exchanges

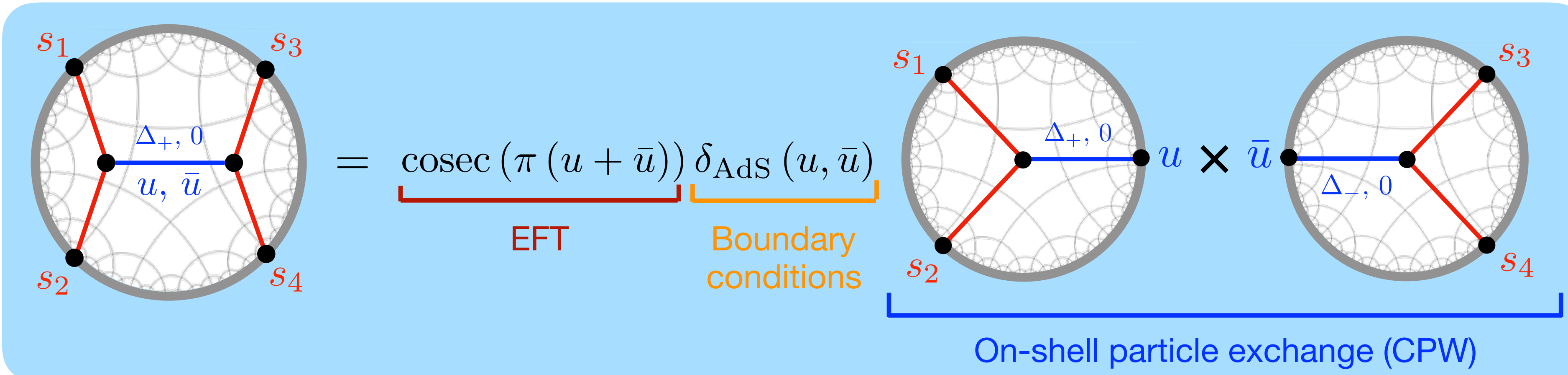
Four-point exchange of generic scalars



This form is fixed by a combination of **Boundary Conditions** and **Conformal Symmetry**!

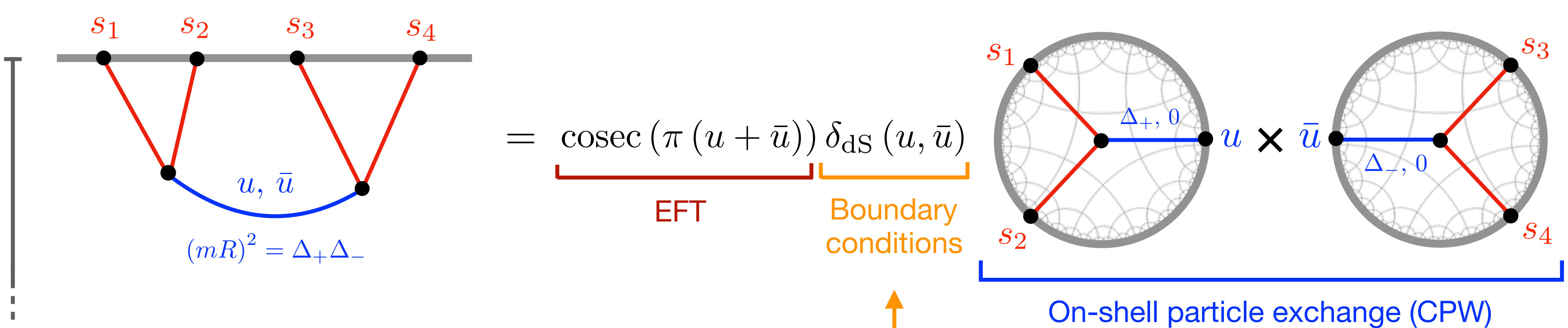
Mellin representation of (A)dS exchanges

Four-point exchange of generic scalars



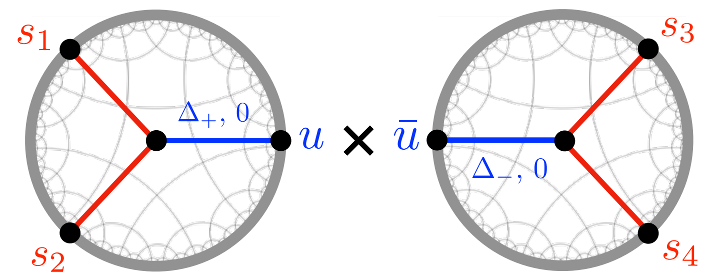
This form is fixed by a combination of **Boundary Conditions** and **Conformal Symmetry**!

The corresponding exchange amplitude in **de Sitter** takes the **same form**

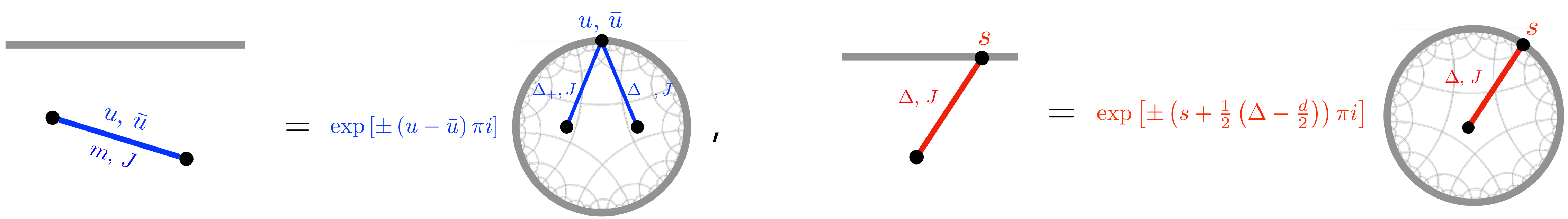


An entire function which now implements
the **early-time** boundary condition (Bunch Davies)

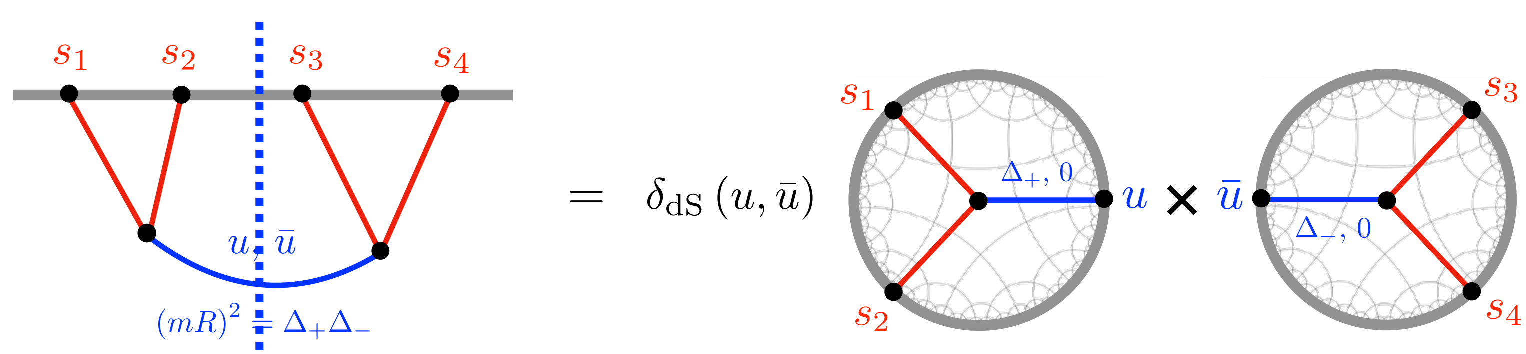
Mellin representation of (A)dS exchanges



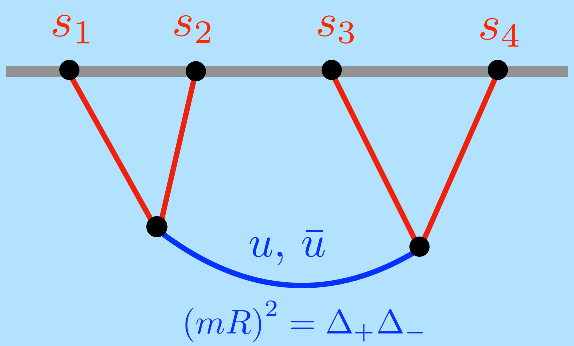
Contributions from each branch of the in-in contour are obtained by multiplying each leg with the appropriate phase:



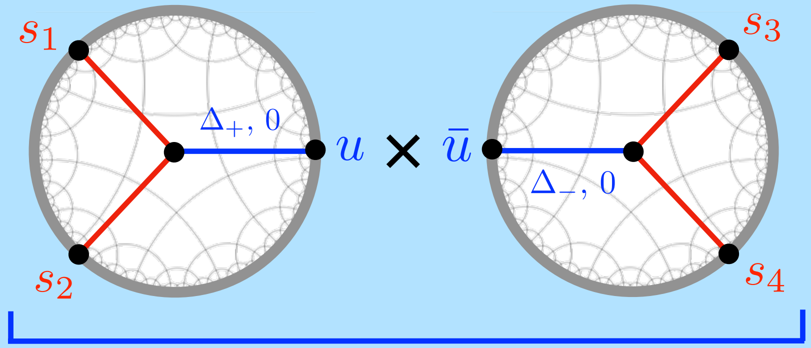
Sum contributions from each branch of the in-in contour, keeping the two bulk points separated:



Mellin representation of dS exchanges

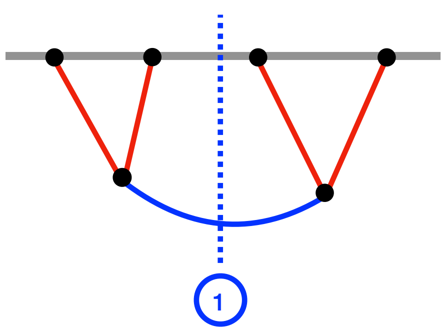


$$= \underbrace{\operatorname{cosec}\left(\pi\left(u+\bar{u}\right)\right)}_{2}\,\delta_{\mathrm{dS}}\left(u,\bar{u}\right)$$



Conformal Partial Wave 1

The exchanged single particle state is now signaled by **two families** of **non-analytic** terms



$$\sim \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_+-\frac{d}{2}}\left(1+k_I^2+\ldots\right)$$

$$+\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_--\frac{d}{2}}\left(1+k_I^2+\ldots\right)$$

This reflects the boundary (late-time) behaviour of fields in dS:

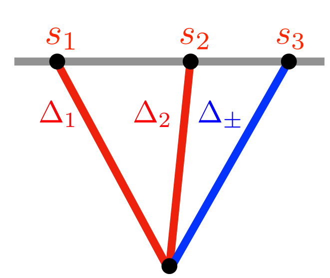
$$ds^2 = R^2\frac{-d\eta^2+d\mathbf{y}^2}{\eta^2}$$

$$\lim_{\eta\to 0}\varphi\left(\eta,\mathbf{k}\right) = O_{\Delta_+}\left(\mathbf{k}\right)\eta^{\Delta_+} + O_{\Delta_-}\left(\mathbf{k}\right)\eta^{\Delta_-}$$

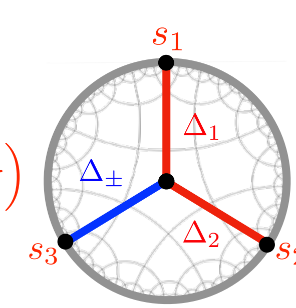
forbidden by Dirichlet b.c. in AdS

$$\eta = -e^{-t}$$

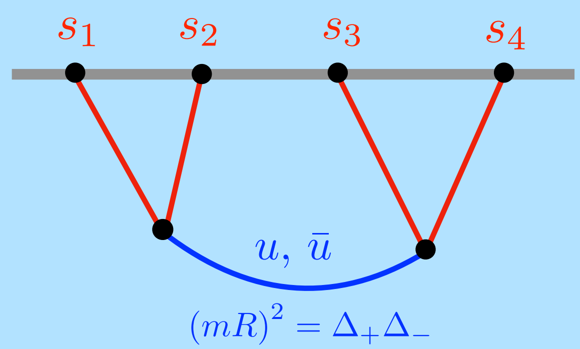
The sine factors originate from the constituent 3pt diagrams:



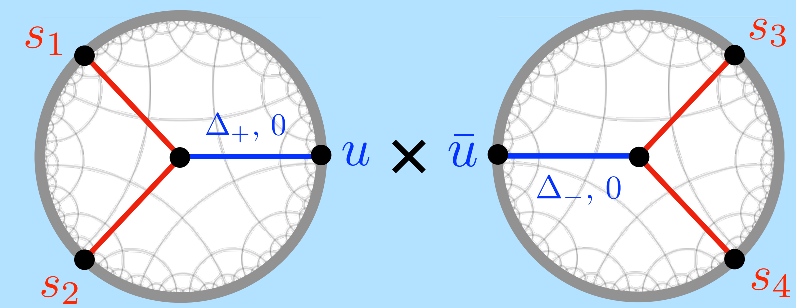
$$= \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_\pm-d}{2}\right)\pi\right)$$



Mellin representation of dS exchanges

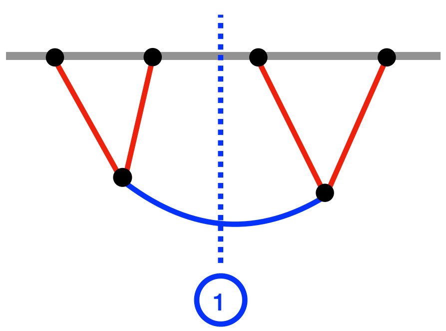


$$= \underbrace{\operatorname{cosec}\left(\pi\left(u+\bar{u}\right)\right)\delta_{\mathrm{dS}}\left(u,\bar{u}\right)}_{2}$$



Conformal Partial Wave 1

The exchanged single particle state is now signaled by **two families** of **non-analytic** terms



$$\sim \sin\left(\left(\tfrac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right)\sin\left(\left(\tfrac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_+-\frac{d}{2}}\left(1+k_I^2+\ldots\right)$$

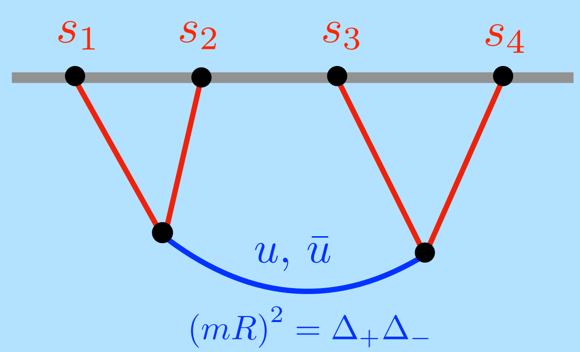
$$+\sin\left(\left(\tfrac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right)\sin\left(\left(\tfrac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_--\frac{d}{2}}\left(1+k_I^2+\ldots\right)$$

In de Sitter there are two types of exchanged particles:

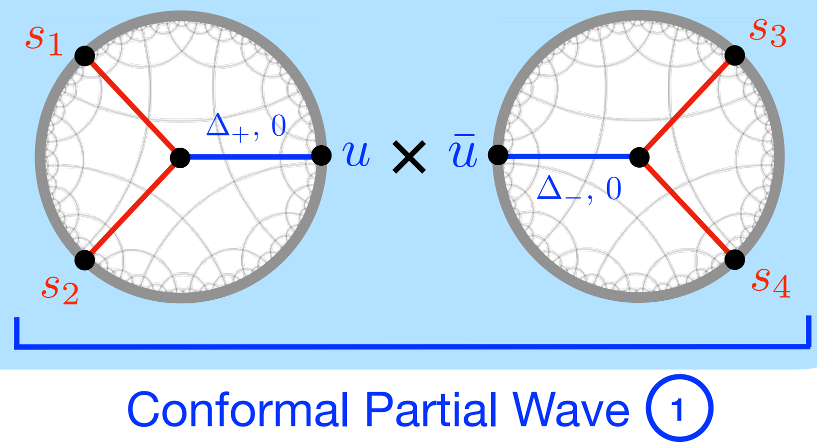
- Light particles: $0 \leq m/H < \frac{d}{2} \longrightarrow \frac{d}{2} < \Delta_+ \leq d \longrightarrow$ power law $\left(k_I^2\right)^{\Delta_\pm-\frac{d}{2}}$
- Massive particles: $m/H \geq \frac{d}{2} \longrightarrow \Delta_\pm = \frac{d}{2} \pm i\nu,\ \nu \in \mathbb{R} \longrightarrow$ oscillations $\left(k_I^2\right)^{\pm i\nu}$

$(m^2 = H^2\Delta_+\Delta_-)$

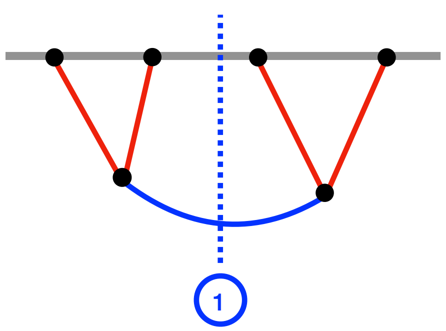
Mellin representation of dS exchanges



$$= \underbrace{\operatorname{cosec}\left(\pi\left(u+\bar{u}\right)\right)}_{2}\, \delta_{\rm dS}\left(u,\bar{u}\right)$$



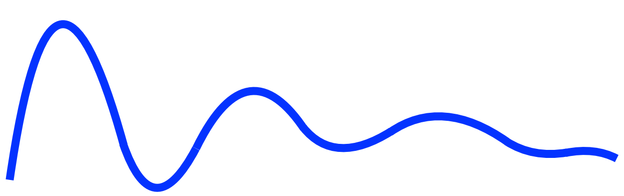
For an exchanged massive particle $m^2 = H^2\left(\frac{d}{2}+i\nu\right)\left(\frac{d}{2}-i\nu\right)$ we have



$$\sim \sin\left(\left(\frac{\Delta_1+\Delta_2-\frac{d}{2}+i\nu}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4-\frac{d}{2}+i\nu}{2}\right)\pi\right)\left(k_I^2\right)^{i\nu}\left(1+k_I^2+\ldots\right)$$

$$+\sin\left(\left(\frac{\Delta_1+\Delta_2-\frac{d}{2}-i\nu}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4-\frac{d}{2}-i\nu}{2}\right)\pi\right)\left(k_I^2\right)^{-i\nu}\left(1+k_I^2+\ldots\right)$$

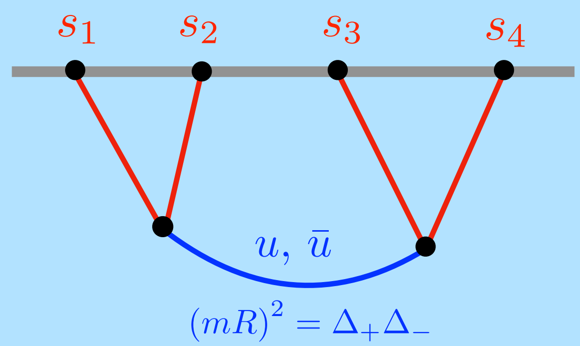
$$\propto \sin\left[\delta\left(\nu\right)+\nu\log\left(k_I^2\right)\right]+\ldots$$



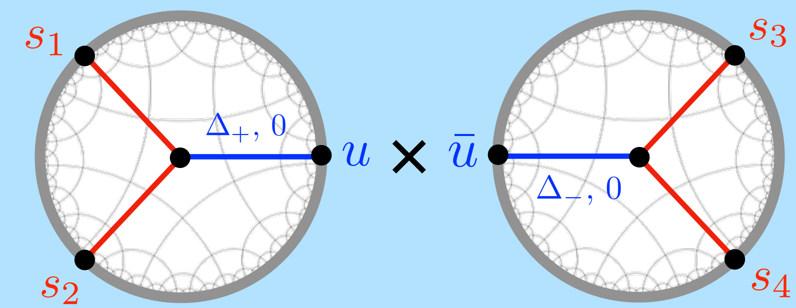
Recovers and effortlessly generalises the analysis of

Arkani-Hamed and Maldacena [hep-th] 1503.08043!

Mellin representation of dS exchanges



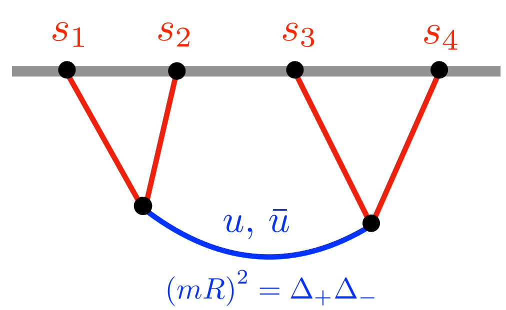
$$= \underbrace{\operatorname{cosec}\left(\pi\left(u+\bar{u}\right)\right)\delta_{\text{dS}}\left(u,\bar{u}\right)}_{2}$$

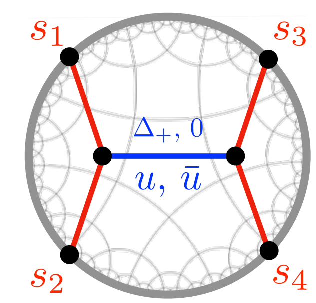


Conformal Partial Wave 1

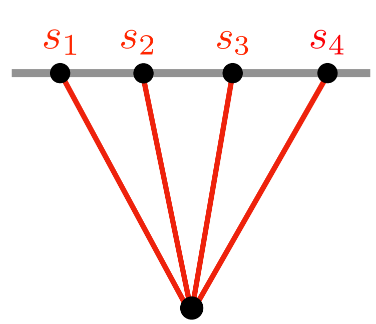
The **EFT part** of the **dS exchange** is **equal** to that of the **AdS exchange**, up to a sine factor

$$\mathrm{Res}_{u+\bar{u}=-n}\left[\;\right] = \sin\left[\left(-d+\sum_{j=1}^{4}\Delta_j\right)\frac{\pi}{2}\right]\mathrm{Res}_{u+\bar{u}=-n}\left[\;\right]$$

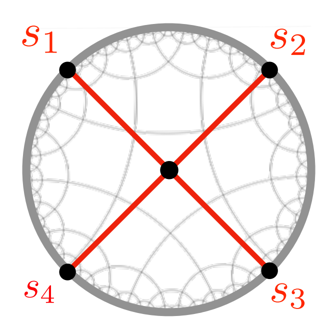




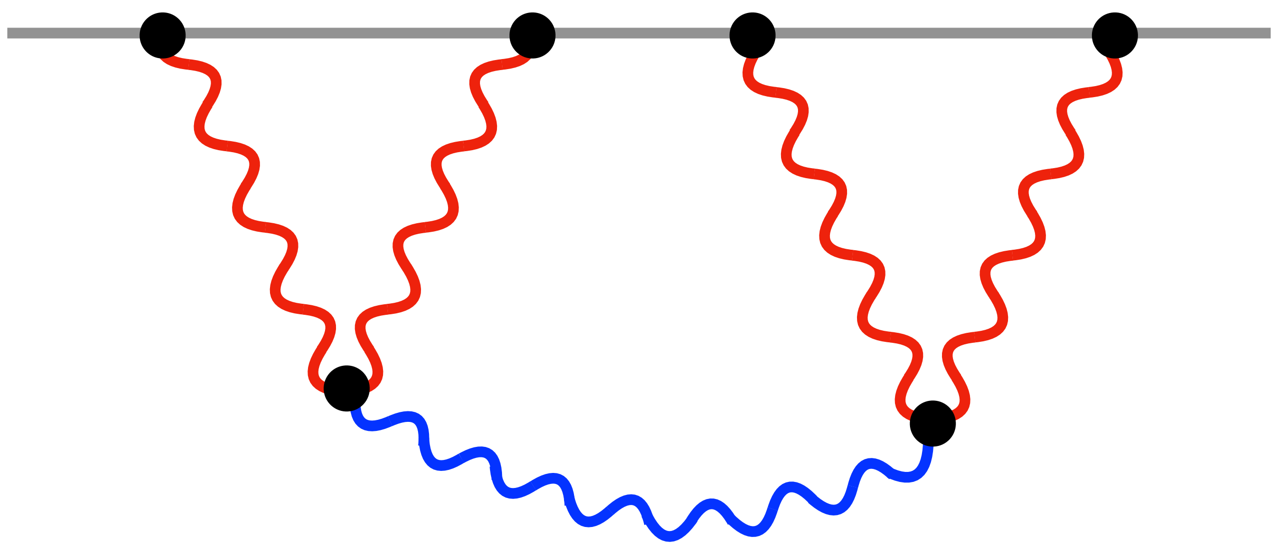
This is to be expected since the EFT expansion is a sum of contact terms, and we know that:



$$= \sin\left[\left(-d+\sum_{j=1}^{4}\Delta_j\right)\frac{\pi}{2}\right]$$

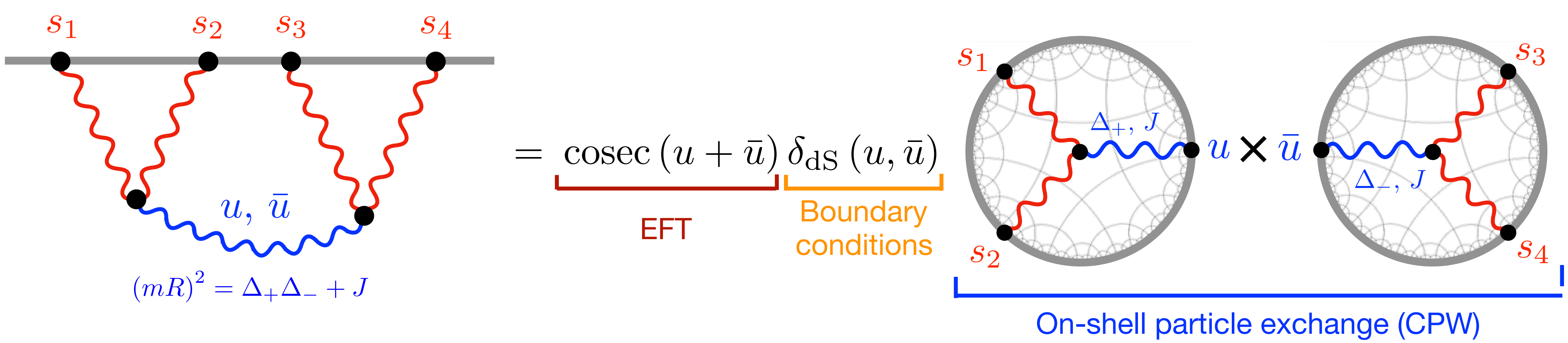


Particles with spin



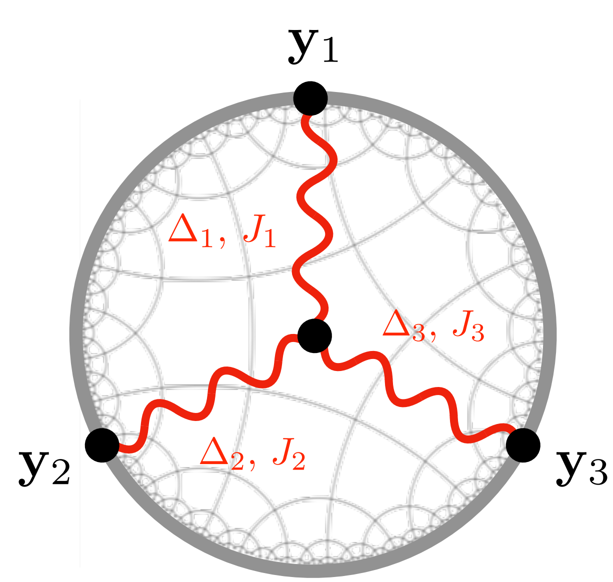
Particles with spin

Conformal Symmetry + **Boundary Conditions** imply



3pt Witten diagrams are known for **any triplet of spinning fields** in position space

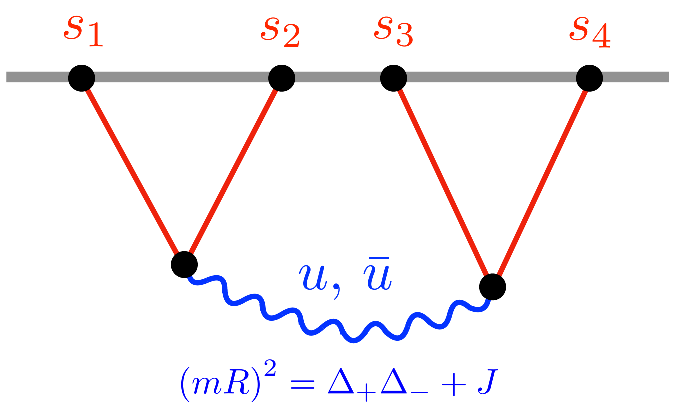
C.S. and M. Taronna [hep-th] 1603.00022
C.S. and M. Taronna [hep-th] 1702.08619



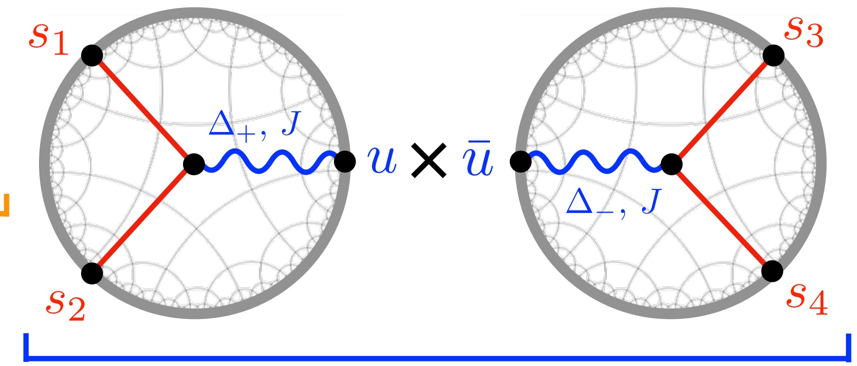
We just need to determine their Mellin-Barnes representation in momentum space!

Particles with spin

The simplest case is a **spin-J field** exchanged between **scalars**

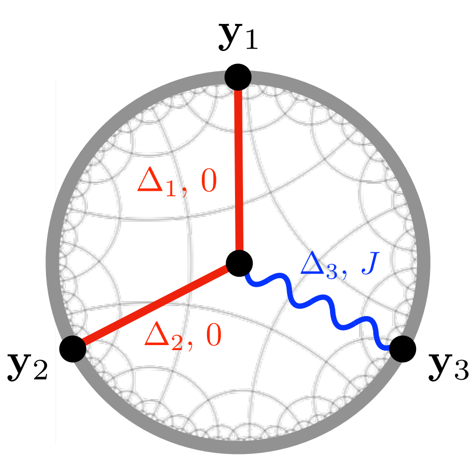


$$= \underbrace{\operatorname{cosec}\left(u+\bar{u}\right)}_{\text{EFT}}\,\underbrace{\delta_{\mathrm{dS}}\left(u,\bar{u}\right)}_{\text{Boundary conditions}}$$



On-shell particle exchange (CPW)

The constituent 3pt Witten diagrams in this case are proportional to **a single conformal structure**

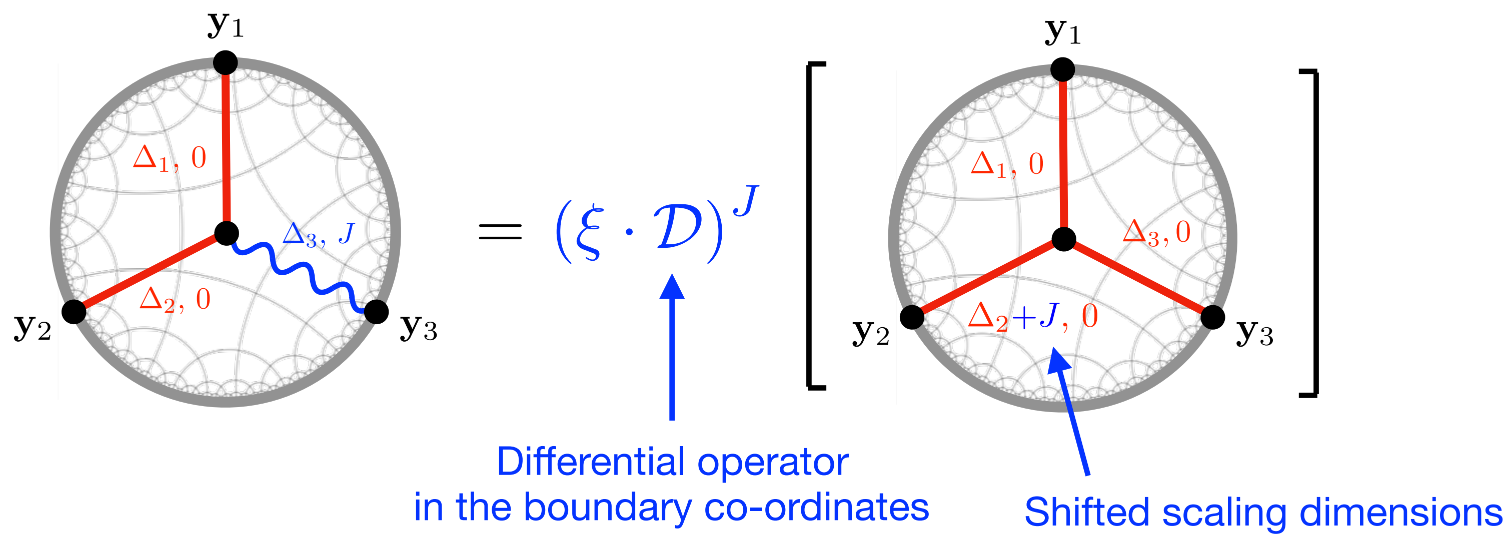


$$\propto \frac{\left(\dfrac{\mathbf{y}_{12}\cdot\xi}{|\mathbf{y}_{12}|^2} - \dfrac{\mathbf{y}_{13}\cdot\xi}{|\mathbf{y}_{13}|^2}\right)^J}{|\mathbf{y}_{12}|^{\Delta_1+\Delta_2-\Delta_3+J}|\mathbf{y}_{23}|^{\Delta_2+\Delta_3-\Delta_1-J}|\mathbf{y}_{31}|^{\Delta_1+\Delta_3-\Delta_2-J}}\,, \quad \xi^2 = 0$$

Polarization

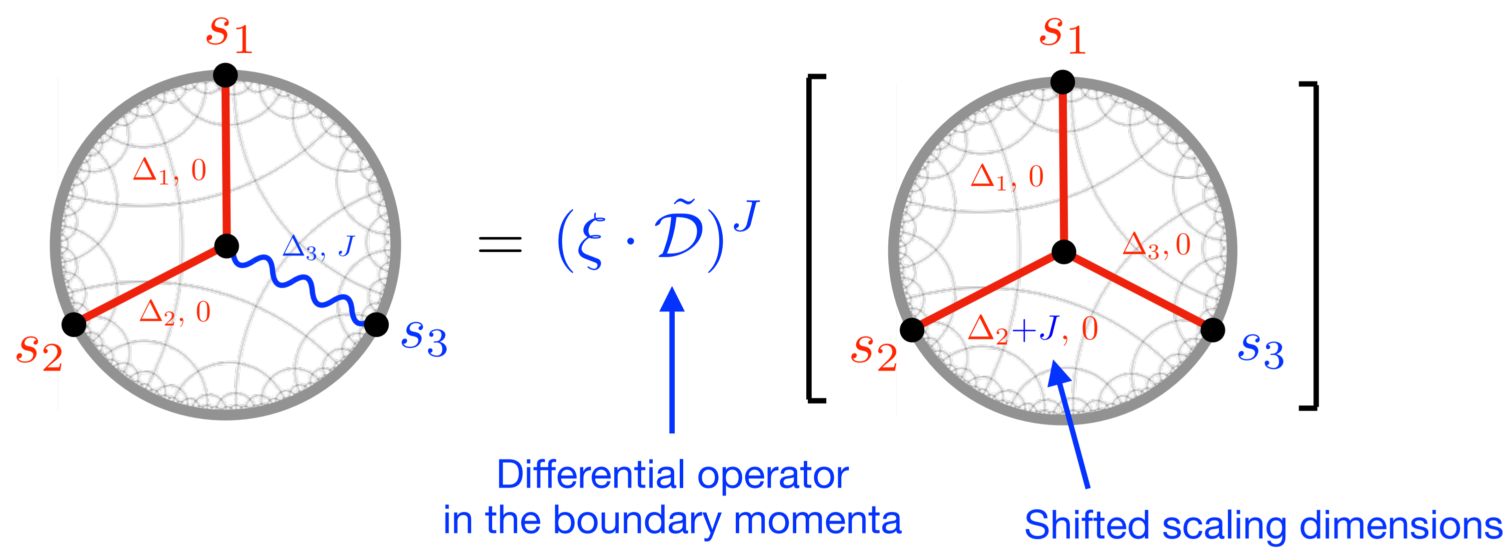
Particles with spin

Spinning Witten diagrams can be obtained by acting with differential operators on a **scalar seed**



Particles with spin

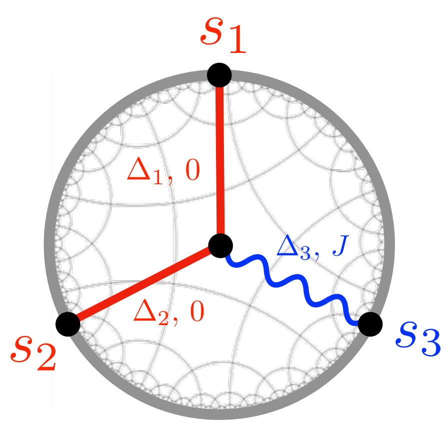
Spinning Witten diagrams can be obtained by acting with differential operators on a **scalar seed**



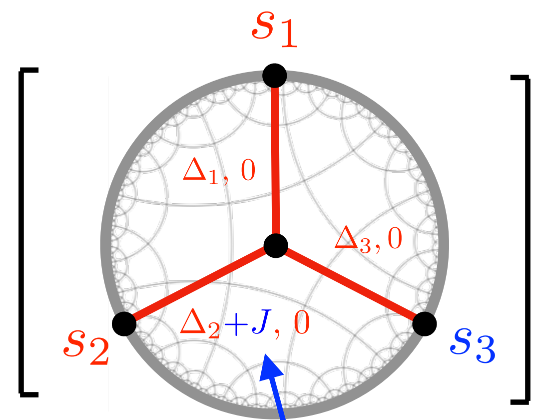
Their **Mellin-Barnes** representation is **inherited** from that of the scalar seed!

Particles with spin

Spinning Witten diagrams can be obtained by acting with differential operators on a **scalar seed**



$$= (\xi \cdot \tilde{\mathcal{D}})^J$$

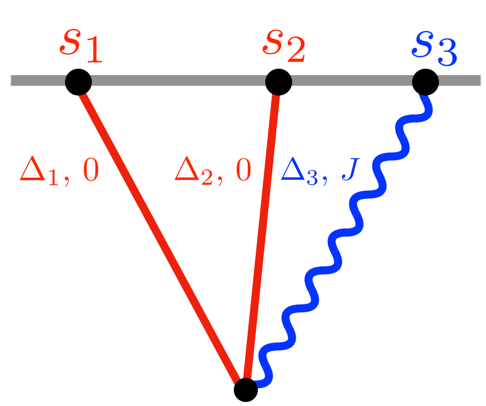


Differential operator
in the boundary momenta

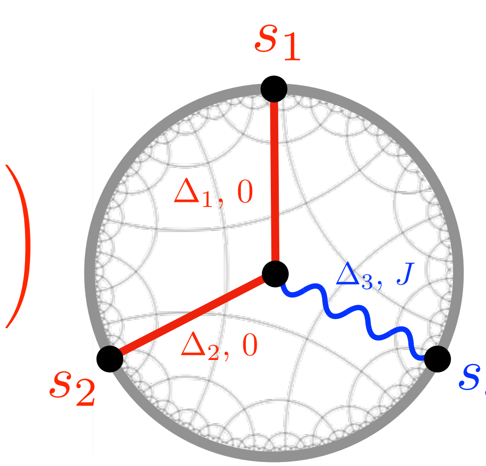
Shifted scaling dimensions

Their **Mellin-Barnes** representation is **inherited** from that of the scalar seed!

For the corresponding amplitude in **de Sitter** we have:

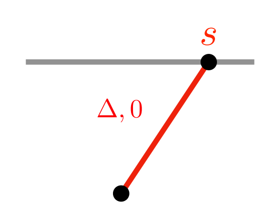


$$= \sin\left(\left(J - d + \sum_{j=1}^{3} \Delta_j\right)\frac{\pi}{2}\right)$$

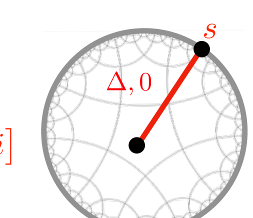


since $\Delta_2 \to \Delta_2 + J$
in the scalar seed

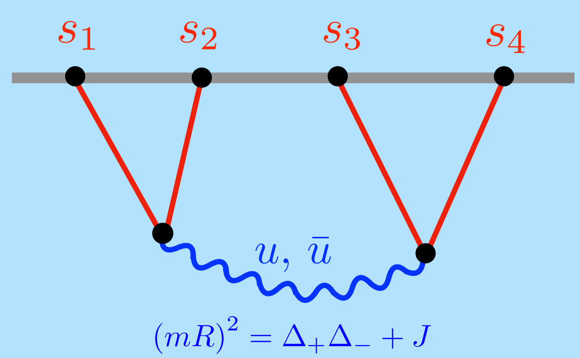
Recall:



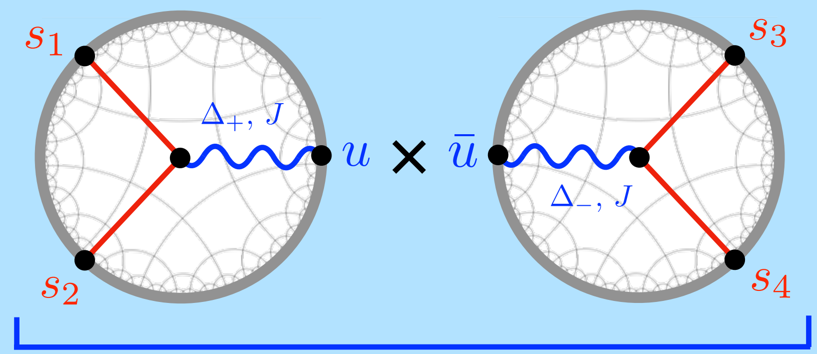
$$= \exp\left[\pm\left(s + \tfrac{1}{2}\left(\Delta - \tfrac{d}{2}\right)\right)\pi i\right]$$



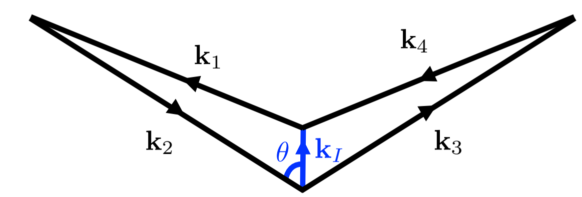
Particles with spin



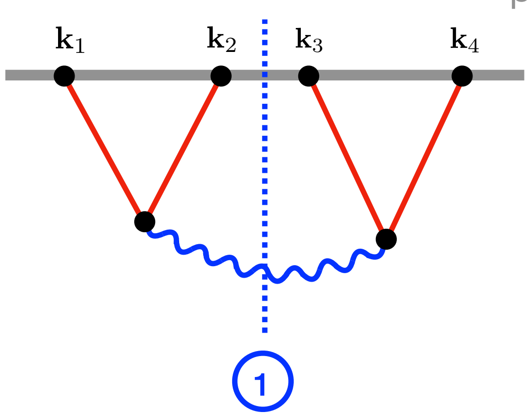
$$= \underbrace{\mathrm{cosec}\,(\pi\,(u+\bar{u}))}_{2}\,\delta_{\mathrm{dS}}\,(u,\bar{u})$$



In the soft limit $k_I \to 0$ we have

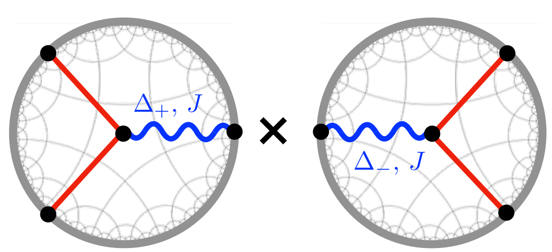


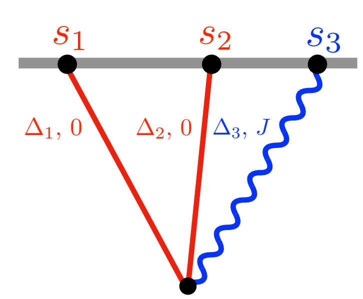
Gegenbauer polynomial



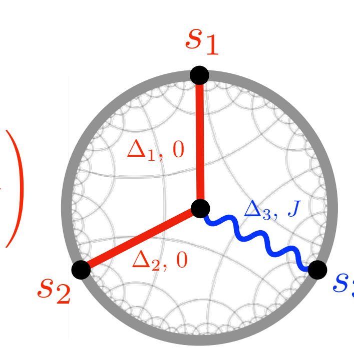
$$\sim C_J^{\left(\frac{d-2}{2}\right)}(\cos\theta)\left[\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_++J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_++J-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_+-\frac{d}{2}}\right.$$
$$\left.+\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_-+J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_-+J-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_--\frac{d}{2}}\right]+\ldots$$

The angular dependence arises from the contraction of tensor structures:

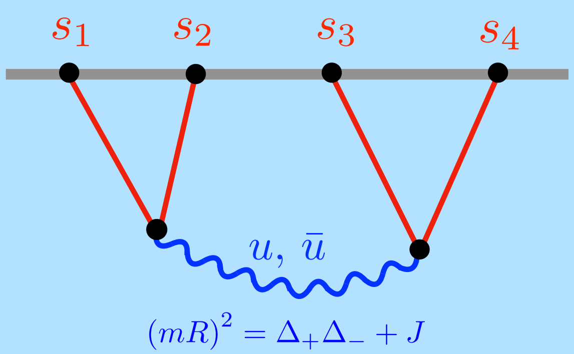




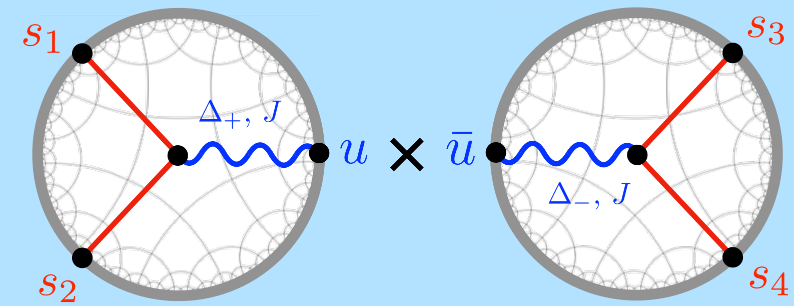
$$= \sin\left(\left(J-d+\sum_{j=1}^{3}\Delta_j\right)\frac{\pi}{2}\right)$$



Particles with spin

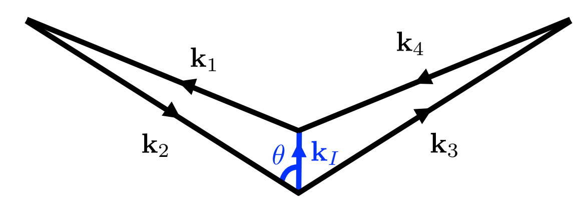


$$= \underbrace{\operatorname{cosec}\left(\pi\left(u+\bar{u}\right)\right)}_{2}\,\delta_{\mathrm{dS}}\left(u,\bar{u}\right)$$

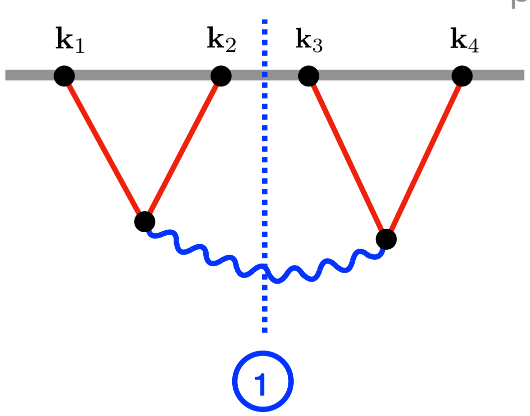


Conformal Partial Wave 1

For an exchanged massive spin J particle, $\Delta_\pm = \frac{d}{2} \pm i\nu$, we have



Gegenbauer polynomial



$$\sim C_J^{\left(\frac{d-2}{2}\right)}\left(\cos\theta\right)\left[\sin\left(\left(\frac{\Delta_1+\Delta_2-\frac{d}{2}+i\nu+J}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4-\frac{d}{2}+i\nu+J}{2}\right)\pi\right)\left(k_I^2\right)^{i\nu}\right.$$

$$\left.+\sin\left(\left(\frac{\Delta_1+\Delta_2-\frac{d}{2}-i\nu+J}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4-\frac{d}{2}-i\nu+J}{2}\right)\pi\right)\left(k_I^2\right)^{-i\nu}\right]+\ldots$$

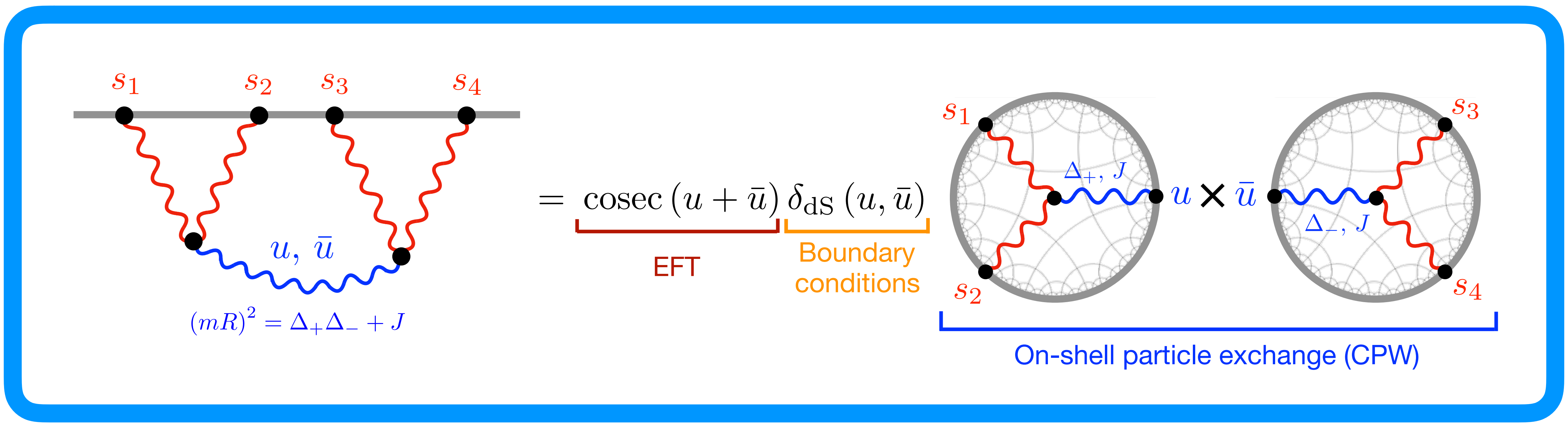
$$\propto C_J^{\left(\frac{d-2}{2}\right)}\left(\cos\theta\right)\,\sin\left[\delta\left(\nu\right)+\nu\log\left(k_I^2\right)\right]+\ldots$$

$$\Big\downarrow{\scriptstyle d=3}$$

$$P_J\left(\cos\theta\right)$$

Recovers and effortlessly generalises the analysis of

Arkani-Hamed and Maldacena [hep-th] 1503.08043!



s_1 s_2 s_3 s_4
$u,\ \bar{u}$
$(mR)^2 = \Delta_+\Delta_- + J$
$= \underbrace{\mathrm{cosec}\,(u+\bar{u})}_{\text{EFT}}\ \underbrace{\delta_{\mathrm{dS}}\,(u,\bar{u})}_{\text{Boundary conditions}}$
$\Delta_+,\ J$
$u \times \bar{u}$
$\Delta_-,\ J$
On-shell particle exchange (CPW)

Thank you!